This document is a guide to writing solutions to problems in your introductory physics class. You may be thinking that the concept of this type of assignment is fairly self-explanatory, and you’re probably right, but the information you can find here will help you to organize your thoughts, solve the problems, and present your work so that the grader knows what you are thinking. Your instructor will tell you that just giving “the answer” to a problem without any work will earn you little or no credit, even if you are correct. This is a general outline of what your instructor wants in your solution, and what will earn you as much credit as possible.

Purpose

This raises the question of what, then, does your instructor want; what is the purpose of composing a problem solution? This purpose is twofold: your instructor wants you to practice the thought processes and review the knowledge involved in solving the problem, and your instructor wants to see those processes and that knowledge in your solution. That means that when you compose a problem solution, you should be trying both to solve the problem and to communicate to your instructor how you solved it.

It may be helpful to consider your problem solution a persuasive document. You are trying to convince your instructor that you know how to solve the problem. This is usually accomplished by simply solving it. If you don’t seem to get all the credit you think you should for “correct” work, though, it might be helpful to keep this in mind as you solve subsequent problems.

Audience

Given the purpose stated above, the audience of your problem solution is your instructor — in principle. In practice, your audience may be a separate grader. This is OK; for your purposes, you can usually consider the two to be interchangeable audiences. Both should have a solid understanding of physics and also of what material your class has covered, and the two should be in good communication about how grading is done in your class.

Style

Your problem solution should be written in technical style. This is usually fairly natural when solving problems mathematically, but keep it in mind when adding textual commentary or when drawing diagrams. When helpful, present information in the form of idealized diagrams, tables, lists, mathematical derivations, and so forth. Make certain that your audience knows what you are doing at every point in your solution. Stick to objective, factual information and to the methods used in your class.

Solution Structure - An Example

Let’s look at an example problem and solution to get an idea for the structure that you should use to find and communicate solutions to problems.

An extreme bicyclist is going to attempt to jump the Canyon of Doom. He will begin at the top of a large ramp, ride down it and then back up a second, smaller ramp (to get airborne), fly over the canyon, and then land on and ride down a third ramp the same height as the second. The Canyon of Doom is 30m wide, the first ramp is 10m higher than the others, and the second ramp makes an angle $\pi/4$ above the horizontal. Will he make it?
Focus the Problem

Begin your solution by presenting the situation it describes and all of the information it gives you, i.e., by presenting your understanding of the problem definition. Drawing a picture is usually the most effective (and time efficient — very important on exams) way to do this; incorporate the given information into the picture by labeling it. In our case, we have drawn the ramps and the canyon; and we have labeled the lengths and the angle given in the prompt.

You will be making a lot of decisions about what you think the problem is saying and about how you should solve it without even realizing that you are doing so. For example, should you be considering the situation from the top or from the side? Where do objects sit in relation to one another? At what time should you be considering the situation? If your setup does not seem to help you move on from this stage, try looking for these implicit decisions and reconsidering whether or not they are correct.

Next, state the objective of the problem. What is it asking you; what are you trying to find out? Our problem is asking us to determine whether or not our bicyclist will successfully jump over the canyon.

Describe the Physics

Convert the problem that you have described into physics terms. Write down related equations that you think will be useful. Define variables. Eliminate details that you think are not physically relevant. State your assumptions. It may also be helpful to turn your picture into a technical diagram reducing the situation to only the physically relevant parts. For example, if our problem were about forces, we would probably draw a free-body diagram, perhaps of the bicyclist sailing through the air. As it stands, our particular problem doesn’t really require a technical diagram. We have, however, defined some directions and the target quantity, and we have written down three equations we expect to use. We have called the displacement $s$ to distinguish any general displacement from the target quantity $x$. 
We will call the horizontal direction the "x" direction and the vertical direction the "y" direction. We will call the horizontal distance that he flies \( x \).

\[
\begin{align*}
 s &= \frac{1}{2}at^2 + v_0t + s_0 \\
 E_K &= \frac{1}{2}mv^2 \\
 E_I &= mgh
\end{align*}
\]

**Plan the Solution**

Next, use mathematics to find a symbolic expression giving the target quantity in the terms of the given information. Provide textual commentary as you go, narrating what you are doing and explaining anything that might not be immediately obvious to your audience. You will frequently have to introduce new quantities as you go; that’s OK. We use kinematics and the conservation of energy to find \( x \) in our problem; along the way, we introduce a vertical distance, a velocity, a time, a mass, and the acceleration due to gravity.

In the air, there is no force acting in the horizontal direction, so he moves with constant velocity \( v_x \) for some time \( t \).

\[
x = v_x t
\]

We now need to know \( v_x \) and \( t \). We can find \( v_x \), the speed when he first launches into the air, from conservation of energy:

\[
\begin{align*}
 \frac{mgx}{2} &= \frac{1}{2}mv^2 \\
 mgh &= \frac{1}{2}mv^2 \\
 gh &= \frac{1}{2}v^2 \\
 2gh &= v^2 \\
 \sqrt{2gh} &= v
\end{align*}
\]

Where \( h \) is the height of the first ramp above the second.

The horizontal component \( v_x \) is given by the cosine of the angle \( \theta \) made by the second ramp with the ground.

\[
\begin{align*}
 v_x &= v \cos \theta \\
 &= \sqrt{2gh} \cos \theta
\end{align*}
\]
We still need \( t \), the time of flight. We can find \( t \) by calculating the time for him to fall back to the height of the third ramp. Setting the height of the second and third ramps to zero, we get

\[
0 = 0 + v_y t - \frac{1}{2} gt^2
\]

\( v_y \) is given by the sine of \( \theta \) and the speed \( v \).

\[
v_y = v \sin \theta
\]

\[
\frac{1}{2} gt^2 = v \sin \theta t
\]

\[
t = \frac{2v \sin \theta}{g}
\]

\[
= \frac{2 \sqrt{2gh} \sin \theta}{g}
\]

Putting everything back into our equation for \( x \),

\[
x = v_x t
\]

\[
= \left( \frac{2 \sqrt{2gh} \cos \theta}{g} \right) \left( \frac{2 \sqrt{2gh} \sin \theta}{g} \right)
\]

\[
= \frac{2 (2gh) \sin \theta \cos \theta}{g}
\]

\[
= 4h \sin \theta \cos \theta
\]

Do not substitute numerical values into your expressions as you go. This is not what your audience needs to see. It will prevent you from earning partial credit, and you will often waste time calculating with numbers that will disappear from your final expression. Solve with symbols first.

This step is called “Plan the Solution” in the sense that your final, numerical expression is the solution and you are planning the symbolic expression that will allow you to find it.

On a side note, you will notice that we have made and crossed out a mistake. In a homework problem, you should present a clean solution, with such mistakes removed (unless, perhaps, you aren’t sure what was right and would like to ask your instructor about it later). On an exam, don’t waste your time trying to make your solution pretty. Just make your logic clear, and clearly cross out any mistakes so that your grader
can recognize and exclude them. Do not assume that your grader will not take away credit for mistakes sitting in an otherwise correct solution or that your grader will give you credit for correct statements sitting in an otherwise incorrect solution.

**Execute the Plan**

Substitute any given numerical values (or generally known ones, like \( g \)) into your symbolic solution to find a numerical one. (This part will not be present for problems that don’t provide numerical values; that’s OK.) If the problem is not just asking for the numerical value of some pre-defined symbol, but actually wants you to answer a question, then interpret your solution and answer that question.

\[
\begin{align*}
\text{Substituting in the numerical values } & \ h = 10\ m, \ \theta = \frac{\pi}{4} \\

x &= 4(10\ m) \sin \left( \frac{\pi}{4} \right) \cos \left( \frac{\pi}{4} \right) \\
&= 40\ m \left( \frac{1}{\sqrt{2}} \right) \left( \frac{1}{\sqrt{2}} \right) \\
&= \frac{40\ m}{2} \\
&= 20\ m
\end{align*}
\]

He will only fly 20 m, but the canyon is 30 m wide. He will fall to a violent, bloody death on the sharp rocks below.

**Evaluate the Answer**

Finally, do some quick reasonableness checks to evaluate the answer. If you are having a hard time evaluating your answer, try comparing it to some quantities that you understand well already. For example, the speed of an arrow should not be any significant fraction of the speed of light.

Meters are the correct unit for a distance.

20 m is comparable to the lengths in the problem, so it is a reasonable value.

The question has been completely and explicitly answered.

You may not always write out this step; it is OK to do it just for yourself. However, if you are not sure of your answer, this is a good way to communicate to your audience that you have a basic understanding of what the answer should be (and earn some partial credit).
But That Was Really Long

The example we worked through was admittedly a lot of writing for the given problem. You don’t always have to go through the entire process in that much detail. On an exam, writing that much might prevent you from being able to finish in time. The example did everything in detail so that you could see the entire problem-solving process and all of the logic that your instructor wants from you. If you think that your audience will understand what you are doing and what your symbols mean, you can omit writing down much of your problem-solving process. In particular, you can use the conventions of physics and mathematics to communicate much information implicitly. (For example, we did not explain in the example that \( g \) was the acceleration due to gravity, but you knew that anyway because it is conventional and because of the context.) You can see an example of a much briefer (but riskier!) solution to the same problem below.

\[ x = v_x t \]

\[ \frac{1}{2} m v^2 = mgh \]

\[ \frac{1}{2} v^2 = gh \]

\[ v^2 = 2gh \]

\[ v = \sqrt{2gh} \]

\[ \frac{1}{2} g t^2 = v_y t \]

\[ t g = v_y \]

\[ gt = 2v_y \]

\[ t = \frac{2v_y}{g} \]

\[ x = (v_x) \left( \frac{2v_y}{g} \right) \]

\[ = 2v_x \cos \theta \cdot \frac{v \sin \theta}{g} \]

\[ = \frac{4gh \cos \theta \sin \theta}{g} \]

\[ = \frac{4h \cos \theta \sin \theta}{g} \]

\[ = 4 \cdot (10 \text{m}) \cos \left( \frac{\pi}{6} \right) \sin \left( \frac{\pi}{6} \right) \]

\[ = 20 \text{m} < 30 \text{m} \]

[He will not make it.]
Be careful writing terse solutions, though, because if you make a mistake or do something unexpected, a lack of explanation will probably mean that you won’t earn much partial credit. In addition, writing everything out and trying to communicate what you are doing will help to organize your thoughts in tricky problems, so slow down and explain what you are doing if you are having trouble. You are discouraged from writing solutions this brief unless you are very certain that you understand what you are doing very well, but most graders will give a solution like this full credit as long as it is completely correct.

Read *The Competent Problem Solver*

This document is based heavily on *The Competent Problem Solver*, ©Kenneth Heller and Patricia Heller, 1995. You are *highly* encouraged to read that document. It is available at no cost on the Physics Department’s website.