AdS AND dS ENTROPY FROM STRING JUNCTIONS or THE FUNCTION OF JUNCTION CONJUNCTIONS*

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Flux compactifications of string theory exhibiting the possibility of discretely tuning the cosmological constant to small values have been constructed. The highly tuned vacua in this discretuum have curvature radii which scale as large powers of the flux quantum numbers, exponential in the number of cycles in the compactification. By the arguments of Susskind/Witten (in the AdS case) and Gibbons/Hawking (in the dS case), we expect correspondingly large entropies associated with these vacua. If they are to provide a dual description of these vacua on their Coulomb branch, branes traded for the flux need to account for this entropy at the appropriate energy scale. In this note, we argue that simple string junctions and webs ending on the branes can account for this large entropy, obtaining a rough estimate for junction entropy that agrees with the existing rough estimates for the spacing of the discretuum. In particular, the brane entropy can account for the (A)dS entropy far away from string scale correspondence limits.

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* Apologies to *Schoolhouse Rock* circa 1973, http://www.schoolhouserock.tv/conjunction.html

1. Introduction

One of the most interesting recent developments is the stabilization of moduli and construction of large classes of de Sitter and anti de Sitter flux compactifications [1,2,3]. These models include cases in which the size of the compactification is hierarchically smaller than that of the (A)dS, by realizing the mechanism suggested in [4] (see also similar works [5,6,7,8]). The recent models of KKLT [3] are of particular interest, as they produce four dimensional de Sitter as well as anti de Sitter vacua in a relatively well studied geometrical framework [9] admitting a low energy effective supersymmetric field theory description.[a]

It is of interest to look for holographic duals of these new flux compactifications. In the de Sitter case, such a description could teach us a lot about the nature of dark energy (which in the real world is roughly seventy percent of the observed universe) as modeled in existing constructions.[b] Even in the AdS case the new examples provide an interesting challenge. For four or fewer large dimensions, previous nonperturbative formulations such as matrix theory [11] and AdS/CFT [12] examples obtained via near horizon limits (such as $AdS_2 \times S^2 \times X$) have broken down due to infrared problems.

Unlike the flux compactifications on large Einstein spaces which have played a role in the AdS/CFT correspondence [12], the new examples are not (known to be) realized via a near horizon limit of any simple brane systems. Nonetheless, there are general arguments suggesting a similar holographic dual description. In the AdS case at least one expects a field theoretic dual via the relation [13,14] mapping gravitational Feynman diagrams in AdS_{d+1} to conformally invariant Greens functions of a d-dimensional quantum field theory. This dictionary does not depend on the existence of a larger theory from which the AdS background is obtained as a near horizon limit.

In the dS case one also has a strong hint of a dual description, in that the Gibbons–Hawking entropy [15] associated with the horizon suggests a microphysical statistical mechanical origin that may well be associated to a holographic dual theory. Steps toward such a duality proposal using analo-

[a] The space of models described in [1] should be taken into account in any attempt to bound the number of vacua, and in comparing numbers of low energy SUSY vacua to vacua without low energy SUSY. Ultimately it is quite possible that nonsupersymmetric nongeometrical noncritical string backgrounds may be more generic than the better studied geometrical low energy SUSY backgrounds of critical string theory.

[b] One may think about the problem of dark energy in string theory analogously to the problem of understanding black hole physics in string theory. There is no sense in which we try to "explain" the black hole mass independently of anything else, but we learn a lot about the physics of black holes by understanding the microscopic origin of their entropy [10].

gies to AdS/CFT have been made in [16] based on symmetries and the structure of quantum field theory in the global de Sitter geometry, and in [17] based on entropy counts and geometry of brane configurations realizing motion on the Coulomb branch of (A)dS flux compactifications of string theory. In [18] some important issues were raised that need to be addressed in any duality proposal in the dS case. In [19] some proposals based on novel quantum gravity constructions have been made and investigated. In my view, our best hope for finding a dual formulation if one exists is to study the workings of explicit models.

In [17,20], a method for obtaining the dual theories for flux compactifications has been proposed, as summarized in [21] (see also [2]). The idea is simply to deform the system to the Coulomb branch, which introduces explicit brane domain walls [22] whose worldvolume content corresponds to that of the dual field theory on its Coulomb branch. For the AdS case, the solutions obtained by trading all the flux for branes in the infrared region of the geometry have the property that the solution caps off in the infrared, eliminating the AdS horizon. In the well understood AdS/CFT examples, the brane degrees of freedom at the scale of the VEV in this solution [23] account for the full set of degrees of freedom of the known dual field theory. In general flux compactifications, we would like to understand if this is the case.

In [17], we noted that the Bousso–Polchinski tuning available for flux compactifications suggests dual field theories with entropy that is much greater than quadratic in the flux (and therefore brane) quantum numbers. This makes more pressing the question of whether the branes in a generic Coulomb branch configuration can account for such a large entropy when the (A)dS space is much larger than string scale in size. (When the (A)dS space is string scale in size, there is a "correspondence point" (cf [24]) at which the brane entropy scales like that of the (A)dS space if the brane entropy is quadratic in the flux quantum numbers [17].)

In this note, we show that the best current estimates for the number of flux vacua in the KKLT system [4,25,17,26] agrees with a simple estimate of the number of degrees of freedom available on the branes realizing the Coulomb branch of the system. That is, from an estimate of the number of flux vacua, one obtains an estimate of the smallest cosmological constant and therefore the largest (A)dS radius scale available in the models. Translating this to an entropy using the Bekenstein/Hawking, Susskind/Witten, and Gibbons/Hawking arguments, one can compare the result to an estimate of the number of degrees of freedom available on brane domain walls in the

Coulomb branch configuration. The latter count requires the inclusion of string junctions and webs. We find that the two estimates agree within their theoretical error bars, though both estimates are most reliably considered as lower bounds. In this way we relate the statistics of flux vacua with the statistical mechanics of individual flux vacua.

We further present a heuristic explanation of why this comparison works (in our case and in the original case of AdS/CFT on the Coulomb branch) based on the Susskind Witten analysis of entropy in AdS vacua as a function of energy.

This result supports the idea that one can figure out the dual theory from the information about its Coulomb branch available directly on the gravity side, part of a program to determine the duals under current development [20] (see also [2,27]).[c] It improves our understanding of the (A)dS entropy discussed in [17] for the cases in which the cosmological constant is tuned to be very small.

This note is organized as follows. In Section 2 we review the Bousso–Polchinski style estimate for the number of KKLT flux vacua. In Section 3 we review the deformation of the system to the Coulomb branch via brane domain walls and present our estimate for the number of degrees of freedom of the dual theory visible on the branes. We also present a heuristic explanation of the agreement between Section 2 and Section 3 based on the Susskind Witten analysis.

2. Statistics of Flux Vacua

The Bousso–Polchinski mechanism predicts exponentially many vacua as a function of multiple input flux quantum numbers, as follows [4,25,17,26]. A systematic approach to the problem of counting flux vacua was recently developed in [26]. The basic idea is the following. One expects a limit on the strength of flux quantum numbers from back reaction on the geometry. There are b_3 RR flux quantum numbers $Q_i, i = 1, \dots, b_3$ and b_3 NS flux quantum numbers $N_i, i = 1, \dots, b_3$. If one expresses the expected limitation in the form

$$R^2 \equiv \sum_{i=1}^{b_3} \gamma_i Q_i^2 + \alpha_i N_i^2 < R_{max}^2 \tag{2.1}$$

[c] Other aspects of the analysis [20] include the relation between the vacua with fixed moduli on the gravity side and the structure of renormalization group fixed points on the field theory side, constraints on the quantum numbers on the two sides, and their structure under monodromies of the compactification.

for some order one coefficients α_i and γ_i, then one obtains a total number of vacua which is of order

$$N_{vac} \sim \frac{R_{max}^{2b_3}}{b_3!} \tag{2.2}$$

from the volume of the sphere in flux space containing the fluxes consistent with (2.1). (This assumes that each choice of flux leads to of order one vacua.)

In the KKLT models, this estimate may be given in terms of the quadratic form

$$L \sim \int_{CY} H \wedge F \tag{2.3}$$

as follows. Dimensional reduction on a space with flux produces contributions to the four dimensional effective potential from the flux kinetic terms for the NS flux H_{NS} and the Ramond flux F_{RR}

$$\Lambda_{flux} \sim \int_{CY} \frac{1}{l_4^2} \frac{g_s^4}{V^2} \sqrt{g} (|F_{RR}|^2 + \frac{1}{g_s^2} |H_{NS}|^2). \tag{2.4}$$

where we are in 4d Einstein frame and V is the compactification volume in string units. This contribution takes the form

$$\Lambda_{flux} \sim \sum_{i=1}^{b_3} (c_i Q_i^2 + a_i N_i^2)\,, \tag{2.5}$$

where a_i and c_i are functions of the moduli, which in turn depend on the fluxes, and Q_i and N_i are the RR and NSNS flux quantum numbers on the 3-cycles in the compactification. (In asymmetric orbifold models such as [1] the dependence of a_i, c_i on the moduli is eliminated for the geometrical moduli by using asymmetric orbifolding to freeze them at the string scale.)

If we pick the maximum flux scale R_{max} such that the moduli-dependent coefficients a_i and c_i do not take extreme values in the solutions to the equations of motion, then one can relate L to a positive definite quadratic form for each point on the moduli space solving the equations of motion.

That is, in the no scale models [9] appearing in KKLT, the Gauss' law relation between $L \sim \int H \wedge F$ and orientifold 3-plane and D3-brane charge

$$\frac{1}{2(2\pi)^4(\alpha')^2} \int H \wedge F = \frac{1}{4}(N_{O3} - N_{\overline{O3}}) - N_{D3} + N_{\overline{D3}} \tag{2.6}$$

translates via supersymmetry into a relation between the orientifold +D3-brane tension and L. In a zero energy vacuum of the no-scale approximation [9] to the effective potential, this tension $\int H \wedge F$ cancels the positive terms (2.5) in the potential. So for every solution to the equations of motion we wish to consider, a relation of the form

$$\sum a_i N_i^2 + c_i Q_i^2 \sim L \leq R_{max}^2 \tag{2.7}$$

holds, with a_i and c_i order one coefficients that depend on the fluxes. So rewriting R_{max}^2 as L_{max} we can rewrite (2.2) as

$$N_{vac} \sim \frac{L_{max}^{b_3}}{b_3!}\,. \tag{2.8}$$

By integrating the number of vacua solving the equations of motion over the flux choices and moduli space with a suppression factor introduced for large fluxes to take into account (2.7), [26] found an estimate

$$N_{vac} \sim \frac{(2\pi L_{max})^K}{12\pi^n n! K!} f(K)\,, \tag{2.9}$$

where $K = b_3$ is the number of independent complex fluxes in the compactification, and where $n = b_3/2 - 1$ is the complex dimension of the complex structure moduli space of the Calabi-Yau threefold associated to the F theory compactification. $f(K)$ is an integral of flux-independent quantities over a fundamental domain of the moduli space.

If we take these vacua to be distributed roughly uniformly between cosmological constants of $\pm\frac{1}{l_4^2}$ (where l_4 is the four-dimensional Planck length), this predicts a minimum cosmological constant of magnitude

$$\Lambda_{min} \sim \frac{1}{l_4^2 N_{vac}} \tag{2.10}$$

corresponding to a maximum curvature radius $L_{(A)dS}$ of order

$$(L_{(A)dS}^{max})^2 \sim l_4^2 N_{vac} \tag{2.11}$$

among the elements of the discretuum of vacua predicted by the estimate (2.8)(2.9). This curvature scale in turn corresponds to an entropy of order

$$S_{max} \sim \frac{(L_{(A)dS}^{max})^2}{l_4^2} \sim N_{vac} \tag{2.12}$$

as we will review in the next section. Taking the vacua to be uniformly distributed is a nontrivial assumption, since the vacua could instead accumulate around some particular values of the cosmological constant. We will see that this naive assumption fits with what we find for the entropy, though a much more thorough analysis of the distribution of vacua will ultimately be required.

This estimate, which may ultimately prove accurate as a count of the number of vacua, appears at least to be a lower bound on this number. For example, we expect more solutions to the equations fixing the complex structure and dilation moduli at the no scale level than the $DW = 0$ solutions so far counted [28]. In addition, when we saturate Gauss' law with some number of threebranes as well as fluxes, the number of vacua of the threebrane theory comes into play and has not yet been estimated accurately while at the same time fixing the moduli.[d] There are almost certainly other classes of vacua such as [1] to be included in a full count as well, though the corresponding entropies for these may be studied independently.

3. Statistical Mechanics of Flux vacua

Given an (A)dS vacuum of radius $L_{(A)dS}$, we can associate a maximal entropy of order L_{AdS}^{d-2}/l_d^{d-2} to a region of the spacetime contained in a 2-sphere of radius $L_{(A)dS}$. In the dS case, this is simply the Gibbons Hawking entropy [15]. In the AdS case, this follows from applying the Bekenstein/Hawking entropy bound to AdS space, as was studied for AdS/CFT by Susskind and Witten [29].

We will apply the Susskind–Witten analysis to the Coulomb branch configurations of our flux vacua in the AdS case. Let us first briefly review their analysis, generalizing trivially from the $AdS_5 \times S^5$ context in which they applied it. One begins with an AdS_d/CFT_{d-1} dual pair, for which the CFT has n_{CFT} degrees of freedom and therefore of order $E^{(d-2)n_{CFT}}$ states in its spectrum as a function of energy scale E. Cut off this theory at a scale of order $1/(L_{CFT}\delta)$ for some $\delta < 1$, where L_{CFT} is the size of the sphere on which the CFT lives. The corresponding operation on the gravity side is to place an infrared cutoff in global AdS at a sphere of area $L_{AdS}^{d-2}/\delta^{d-2}$ surrounding the origin. Since precise coefficients are not obtained by this analysis, for simplicity we may take δ somewhat smaller than but of order 1, so that the cutoff restricts us to of order one mode per degree of freedom on the S^{d-2} on which the CFT lives. The area of this S^{d-2} in Planck units, L_{AdS}^{d-2}/l_d^{d-2}, bounds the entropy that can fit inside the sphere on the gravity side. Susskind and Witten checked that this entropy is indeed N^2 in the gravity dual to the $\mathcal{N} = 4$ $U(N)$ super Yang-Mills theory, using the relations $L_{AdS} \sim L_{S^5} \sim (g_sN)^{1/4}l_s$.

[d] We thank S. Kachru for this caveat.

Said differently, the cutoff requires each degree of freedom to be excited with energy at most of order $1/L_{AdS}$. The total energy allowed below the cutoff is then $E_T = n_{CFT}/L_{AdS}$. From the corresponding gravity side cutoff at a sphere of area L_{AdS}^{d-2}, we can independently identify this total energy E_T as the mass $M_{BH}^{(L_{AdS})}$ of the largest black hole fitting within the region bounded by this area. In the $AdS_5 \times S^5$ case, these two formulas for the energy scale of the cutoff agree, once we identify n_{CFT} with N^2. This result is consistent with a naive extrapolation of the weak coupling relation $n_{CFT} \sim N^2$ into the strong 't Hooft coupling regime.

This analysis keeps track of the moding of states on the sphere as well as the total entropy, and it illustrates a basic aspect of how the entropy is distributed in the AdS/CFT duality[e]: from the cutoff on the sphere, allowing only of order one mode on the S^{d-2} for of each of the $n_{CFT} = N^2$ degrees of freedom, one obtains the entropy which is numerically equal to one degree of freedom per Planck area but organized as $n_{CFT} = N^2$ degrees of freedom per L_{AdS}^3.

The Susskind–Witten analysis just reviewed was in the global AdS solution. We can apply it in the Poincare patch, corresponding to the CFT on Minkowski space M_{d-1}. We do this by enforcing the Bekenstein bound corresponding to black brane solutions extending in the M_{d-1} directions. This leads again to N^2 degrees of freedom per L_{AdS}^{d-2} area along the $d-2$ spatial directions of M_{d-1}.

We would like to see how many of the n_{CFT} degrees of freedom of the system become manifest on its Coulomb branch. Let us first review how the Coulomb branch arises from the gravity side point of view. It is obtained by introducing brane domain walls separated radially from the horizon. This reduces the flux in the bulk region on the side of the brane toward the horizon (let us call this the "IR side" since it corresponds to the IR region from the field theory point of view). The simplest such configuration, obtained in [23] for the $AdS_5 \times S^5$ case, is to trade all the flux in this region for branes at a radial scale of order L_{AdS}. There being no flux supporting the compactification on the IR side of the branes, it shrinks down and caps off the solution at a finite radius in the IR direction, removing the horizon. In the solution [23], this region turns out to be smooth (in fact flat) ten dimensional space. In a general flux compactification, we do not know the precise solution but the absence of flux in this region means that the AdS horizon will be removed generically. This corresponds to the fact that a generic Coulomb

[e] Emphasized for example by S. Shenker.

branch configuration will lift most of the degrees of freedom of the theory to a scale of order the scale set by the VEVs. Also in a generic system there will be a potential on the Coulomb branch, so that the physical solutions are time dependent. I expect this will not preclude the counting and identification of degrees of freedom from the brane content on the gravity side.[f]

In the $AdS_5 \times S^5$ case, we see N^2 degrees of freedom from stretched strings ("W bosons") at the mass scale

$$\langle \phi \rangle \sim \frac{L_{AdS}}{l_s^2} \tag{3.1}$$

of the VEVs of the diagonal scalar matrix elements. (There are also string oscillation modes on top of these including some at of order this energy scale, which have the same N scaling.) These are electric degrees of freedom from the point of view of the spontaneously broken $U(N)$ gauge group on the manifest D3-branes of the [23] solution, and become massless as we return to the origin of the moduli space. In this sense, the N^2 degrees of freedom have become manifest on the Coulomb branch directly on the gravity side of the correspondence.

Let us clarify the energy scales involved in this analysis. We put the Susskind–Witten cutoff originally at the radius L_{AdS} corresponding to the total energy scale N^2/L_{AdS}. Exciting the stretched string "W bosons" individually fits within this cutoff, so we can exhibit the count of degrees of freedom by exciting them individually. But exciting all N^2 of them at the scale (3.1) would of course not fit inside the above cutoff, which as we discussed allows states up to to a total energy scale corresponding to N^2 degrees of freedom each excited only up to energy $1/L_{AdS}$. So if one wants to apply the Susskind–Witten analysis to the system on the Coulomb branch, including energies up to the scale

$$E_{C\ branch} \sim N^2 \langle \phi \rangle \tag{3.2}$$

we need a larger cutoff (smaller δ).

We can now ask in the more general flux compactifications of interest here [3,1] whether the brane degrees of freedom continue to account for the black hole entropy. That is, when we count the elementary degrees of freedom n_B on the brane domain walls replacing all the flux to the IR end of the branes, at the mass scale of the VEV suggested by the geometry, is n_B of order L_{AdS}^{d-2}/l_d^{d-2}? We will see that for the KKLT models, at the level of the estimate in section 2 this saturation holds as well in our case, when we take

[f] Another work that used off shell bubbles to illustrate a physics point is [30].

into account string junction degrees of freedom living on the branes in the Coulomb branch of that system.

The branes in the KKLT construction consist of $D5$ and $NS5$ branes wrapped on b_3 3-cycles of the compactification manifold of type IIB string theory, as well as of order $\int H\wedge F$ D3-branes ending on them according to the Gauss' law constraint (2.6). Q_i $D5$ and N_i $NS5$ branes wrapped on the same cycle C_i reduce to J_i (p_i, q_i) fivebranes where $(p_i, q_i) = (Q_i/J_i, N_i/J_i)$ are relatively prime integers. We will be interested in the highly tuned situation described in section 2 in which the size of the Calabi-Yau is much smaller than the curvature radius of the AdS_4. In particular, let us consider all the length scales within the Calabi–Yau to be somewhat bigger than string scale for control but not parametrically bigger as a function of the flux quantum numbers. Similarly, let us consider a situation with g_s somewhat smaller than one but of order one.

The degrees of freedom on the branes consist of strings and string webs (combinations of string junctions) which are at a mass scale of order

$$m_{\langle\phi\rangle} \sim \frac{1}{l_s} \tag{3.3}$$

which is the analogue of (3.1) in our system. The string and string web degrees of freedom can be electric from the point of view of the gauge group on each bunch of branes. When the classical mass and binding energy formulas are a good approximation, some string webs are stable at an energy scale of order (3.3) by virtue of being the lightest degrees of freedom with their quantum numbers. We will estimate the number of such degrees of freedom n_B coming from string junctions that we can reliably obtain ending on these various branes, and see that they account for the entropy predicted on the gravity side (2.9)–(2.12),

$$n_B \sim N_{vac} \tag{3.4}$$

with N_{vac} given by (2.9).

We are interested in the number of degrees of freedom available on the $N_5 >> b_3$ 5-branes and $N_3 >> b_3$ 3-branes obtained from the AdS_4 solution by trading all the flux in the IR region of the geometry for branes. There will be multifundamental states arising from string webs (connected combinations of string junctions) with multiple external strings ending on the branes. String webs, discussed in many interesting papers such as [31] (including one relating them to black hole entropy [32]) are combinations of (p, q) strings connected through three-string junction vertices. They satisfy a basic charge conservation condition

$$\sum_I (p_I, q_I) = 0\,, \tag{3.5}$$

where I runs over the strings entering any vertex (and therefore applies to the sum over external strings entering a string web).

We will start by studying junctions with one endpoint on each set of branes (indexed by their type and by the cycle they wrap or end on). This will produce an entropy accounting for the gravity side prediction. We will not analyze the details of the flux compactifications necessary to produce Coulomb branch configurations with sufficiently stable solutions to (3.5). It is clear that some such configurations exist, and our main goal is to explain how large entropies of order N_{vac} (2.12)(2.9) can arise in any Coulomb branch configuration of branes.

Before proceeding to the count of junctions, let us note two issues we have not completely resolved. Firstly, junctions with more endpoints on each bunch of branes, which could be viewed as bound states of lower junctions, would lead to an entropy greater than the gravity side prediction (2.9)(2.12) if such states were considered separately. Secondly, the junctions we do consider with one endpoint on each bunch of branes can themselves be viewed as bound states of strings. The question is what degrees of freedom are elementary in the effective field theory at the energy scale determined by the Coulomb branch VEVs. These issues are similar to those in a somewhat similar situation in the black hole context in [32], where just the lowest junction connecting three sets of branes accounted for all the expected entropy. There is a heuristic argument, along the lines of the arguments in [32], that higher bound state junctions are less likely to constitute valid independent effective fields than the basic junctions with one endpoint on each bunch of branes, since the ratio of the binding energy of one constituent to the mass of the bound state decreases as the number of endpoints increases. Because of this, the lowest junctions are reliably counted but the higher ones are less and less under control as we increase the number of endpoints, keeping the size of the Calabi–Yau fixed. Another argument due to [32] is that in some cases the lowest junction states connecting different bunches of branes ($U(N)$ factors in the brane system gauge group) can be related to elementary string states in dual quiver theories. These are not proofs that only the lowest junctions need be considered however, and leaves open the possibility that *more* entropy is available in the system than the naive Bousso–Polchinski tuning predicts. In any case, we will account for at least the [4,26] estimate with the simplest controlled junction states, addressing the puzzle raised in [17].

Let us study the junctions ending on the 3-branes and the fivebranes. First consider the endpoints on the 3-branes. Consider a generic situation where the ends of these 3-branes are distributed roughly uniformly over the $b_3/2$ pairs of dual intersecting A and B cycles contributing to the anomalous $\int H \wedge F$ 3-brane charge. Because of Gauss' Law (2.6), there are of order $N_3 \sim L$ D3-branes, and so of order $L/(b_3/2)$ per pair of dual A and B cycles. A junction with one end on each of the $b_3/2$ groups of $2L/b_3$ D3-branes has

$$n_3 \sim \left(\frac{2L}{b_3}\right)^{b_3/2} \tag{3.6}$$

ways to end on the threebranes. In our estimates we will account for the L-dependence and not reliably keep track of the prefactor's dependence on b_3 (which is much smaller than L in the regime of validity of the analysis), though the strongest factorial dependence on b_3 evident in (2.9) will arise naturally also in our estimate.

If the junction also ends on the (p,q) fivebranes in all possible ways (again with a single endpoint per bunch of branes), then there is another factor in the entropy coming from the fivebranes, which we now compute. Since we have of order N_5/b_3 (p,q) 5-branes per 3-cycle, we have of order $n_5 \sim (N_5/b_3)^{b_3}$ ways the endpoints can end on fivebranes.

Let us relate this to the quantity L with respect to which the gravity side estimate (2.9) is expressed. Noting that

$$2N_5 \sim \sum_{i=1}^{b_3}(|Q_i| + |N_i|) \tag{3.7}$$

and recalling from section 2 that $L \sim \sum_{i=1}^{b_3}(c_i Q_i^2 + a_i N_i^2)$ and using the fact that on average $Q_i \sim N_i \sim N_5/b_3$, we obtain the relation

$$L^{\frac{1}{2}} \sim \frac{N_5}{b_3}\sqrt{2b_3}\,. \tag{3.8}$$

This translates the fivebrane factor in the entropy to

$$n_5 \sim (L^{1/2}/\sqrt{b_3})^{b_3}. \tag{3.9}$$

Putting the threebrane and fivebrane factors together, we obtain

$$S_{junctions} \sim n_3 n_5 \sim \left(\frac{L}{b_3}\right)^{b_3}. \tag{3.10}$$

The gravity side estimate (2.9) does not determine the function of $K = b_3$ multiplying the $L^K/K!$ factor, though [26] offered some arguments that it was subdominant in its K dependence to the factorial in the denominator. At this level, (3.10) from the lowest junctions with charge on all the sets of branes agrees with the gravity side estimate (2.8)(2.9)(2.12).

Incidentally, one obtains the same estimate if one considers junctions ending only on 5-branes if one decomposes the (p, q) 5-branes back into separate D and NS 5-branes.

Having recovered the entropy directly on the branes of our system, a result similar to that obtained above for the $\mathcal{N} = 4$ SYM theory on its Coulomb branch, it is interesting to ask if this is a coincidence or should have been expected. The following is a heuristic argument for the agreement based on the above Susskind–Witten analysis on the Coulomb branch.

Let us first consider the $\mathcal{N} = 4$ super Yang-Mills theory. If we consider the gravity side geometry out on the Coulomb branch in a configuration in which all of the IR flux has been traded for branes [23], this corresponds to a field theory configuration in which the off diagonal matrix degrees of freedom have been lifted to the scale m_ϕ of the VEVs. The density of states of the system for energies $E < N^2 m_\phi$ has a much slower growth with energy than the undeformed CFT. This means that in the dual [23] solution for the Coulomb branch, the black hole solutions saturating the entropy at a given energy $E_0 < N^2 m_\phi$ contain parametrically fewer states (i.e. have entropy of order N rather than N^2) than those in the full $AdS_5 \times S^5$ geometry at the same energy scale E_0.

If we now move the VEV scale m_ϕ down in energy to somewhat below E_0/N^2, so that the branes go behind the black hole horizon, then the two solutions (pure AdS and Coulomb branch) agree for energies above E_0. In particular, the branes contribute enough entropy to enhance the Coulomb branch black hole density of states to that of the CFT at energy E_0. This makes it clear why the brane states saturated the order N^2 entropy (up to order one factors we do not control by these considerations).

Now in the more general flux compactifications of interest here, we are again applying the procedure of trading all the IR flux for branes. This again removes the flux stabilizing the compactification, and probably caps off the solution in the IR. This again suggests that the black holes in the capped off Coulomb branch solution will have parametrically fewer states than in the full AdS geometry at a given energy scale. As in the above discussion of the AdS_5 case, pushing the branes back behind the horizon will produce again a black hole saturating the entropy bound up to the energy scale E_0. I therefore find it very plausible that the branes in a KLT-like Coulomb branch configuration in a general flux compactification will saturate the entropy and will provide a reliable indicator of the content of the holographic dual theory. This bolsters considerably the case for obtaining the content of the dual quantum field theory from the branes on the Coulomb branch of the background [20].

There is a simple lesson from this analysis regarding the distribution of the entropy. As emphasized above, in the Susskind–Witten analysis in ordinary AdS/CFT, the entropy is organized as n_{CFT} degrees of freedom per $L_{(A)dS}$ area. Both in AdS and in dS flux compactifications, one can obtain numerological agreement with the expected entropy in a situation where the entropy is organized into one mode per *string* area per intrinsic degree of freedom, rather than being organized into n_{CFT} degrees of freedom each excited by one mode per $L^{d-2}_{(A)dS}$ area (as discussed in section 7 of [17]). This estimate is based on there being of order Q^2 degrees of freedom in a system with Q branes coming from open string degrees of freedom. In this paper, we have seen that because their number scales like larger than quadratic powers of the flux (brane) quantum numbers, junction states can account for the expected entropy $n_{junction} \sim N_{vac}$, arranged in the expected way as $n_{junction}$ states per $(A)dS$ area rather than as one per string area.

It will be very interesting to see if and how refinements of the statistical analysis (keeping track of the specific configurations required to tune the cosmological constant to be very small) continue to lead to agreement between the two sides of the putative duality. In this note we have not addressed any aspect of the distribution of flux vacua admitting large numbers of degrees of freedom, but have only seen that it is possible and very natural for string junction states to account for the large entropy predicted for some vacua by the Bousso–Polchinski mechanism.

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