THE QUEST FOR UNIFICATION

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The Standard Model partially unifies the strong, electromagnetic and weak interactions, suggesting a common origin for them. A more fundamental theory, a Grand Unified theory or a string theory, can complete this unification and explain many of the features which in the Standard Model are put in by hand. However many possible implementations of such a theory have been suggested. We emphasize the importance of the prediction for gauge coupling unification in distinguishing between these implementations and how it can select a particular string profile. We discuss how the superstring can extend the unification to include gravity leading to a testable relation between the gravitational and Standard Model interactions. A calculation of the threshold effects to be expected from the superheavy modes shows that the relation given by the weakly coupled heterotic string is in good agreement with experiment. We discuss the resulting profile of the supersymmetric extension of the Standard Model which is consistent with these unification predictions. Its phenomenological implications should be testable by future precision experiments looking for rare flavor changing processes and also more directly by direct supersymmetric particle searches at the Large Hadron collider.

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1. Introduction – towards a final theory

Although the Standard Model can rightly be considered to be a triumph of 20th century physics, providing as it does a remarkably successful theory of the strong, weak and electromagnetic interactions, many consider it to be only a step on the way to a final "Theory of Everything".

The reason is that the Standard Model appears incomplete with much of its structure put in by hand. For example, there is no explanation for the choice of the local gauge group $SU(3) \times SU(2) \times U(1)$ on which it is based. There *is* partial unification of the fundamental forces in the sense that they are all mediated by gauge bosons and that the photon and Z boson are linear combinations of the $SU(2) \times U(1)$ neutral gauge bosons. However this unification is incomplete because the three separate gauge group factors require three separate gauge coupling constants. These couplings are parameters of the Standard Model which must be put in by hand to fit the observed strengths of the strong, electromagnetic and weak interactions. The matter sector looks even more contrived with no explanation for the complicated multiplet assignments of the quarks and leptons, for the fact that the charged weak interactions are purely left-handed and for the large number of additional parameters needed to specify the Yukawa couplings of the theory which lead to the quark and lepton masses and mixing angles. Finally the scalar sector, needed to generate the masses of the weak bosons and the fermions, introduces further parameters and also leads to the so called "hierarchy problem", the problem of explaining why the electroweak breaking scale, which is related to the scalar masses, is so much smaller than the other fundamental scale in the theory - the Planck scale some 10^{16} time the electroweak breaking scale. The problem arises because in a theory with a large fundamental scale radiative corrections drive the SM Higgs scalar mass close to the scale. Since the electroweak breaking scale is proportional to the Higgs mass the weak gauge bosons are also driven to be superheavy. Attempts to solve the hierarchy problem fall into three classes.

One possibility is that the scalars, and possibly other states of the Standard Model, are composite with a scale of compositeness close to electroweak breaking scale. In this case the radiative corrections involving very massive states, perhaps even at the Planck scale, are small due to the cutoff in the relevant form factors at the composite scale. It has proved very hard to realize a composite explanation in a way that does not lead to observable effects in conflict with experimental limits although our inability to solve strongly coupled theories means the composite solution may still be viable. However composite theories do not offer obvious answers to the complexity found in the Standard Model.

A second possibility is that scalars are light due to a new symmetry, supersymmetry [1]. Supersymmetry ($SUSY$) *does* offer some hope that the theory may become simpler at a high scale because the low energy scalar sector is protected by supersymmetry against large corrections coming from a new and more unified theory applicable at a high scale. The prototype unified theory is a Grand Unified theory (GUT) based on the simple group $SU(5)$, a rank 5 group with just the neutral generators needed to accommodate those of the rank 5 Standard Model [2]. In $SU(5)$ the states of a single family are accommodated in just two representations, a $\overline{5}$ and a 10. Thus the representation content of the Standard Model is simplified and there is a (partial) unification of quarks and leptons. Moreover some of the features of the Standard Model structure are explained from the structure of the $\overline{5}$ and 10. These representations are such that the charged weak interactions are purely left-handed because for these representations only the left-handed quark and lepton states are doublets of the electroweak $SU(2)$. Moreover $SU(5)$ has only a single gauge coupling constant and the couplings of the Standard Model are related in a definite way. As we shall discuss this leads to detailed testable predictions. Of course $SU(5)$ is not the only possibility. An even more attractive Grand Unified group is $SO(10)$ because the 15 fermions fermions of a single family fit into a 16 dimensional representation. The 16th state is a right-handed neutrino which restores the left-right symmetry between quark and leptons. Moreover the presence of a right-handed neutrino states leads to an elegant mechanism for generating small neutrino masses - the so-called see-saw mechanism.

These facts strongly support the idea of an underlying unification but its implementation may differ in detail. A particularly attractive possibility is provided by the (weakly coupled) heterotic string in which there is a stage of Grand Unification but it may occur only at the level of the full 10 dimensional theory. Below the compactification scale, in 4 dimensional space, the theory may be just that of the (supersymmetric) Standard Model. However the underlying unification still ensures that quark and leptons should belong to complete GUT representations and preserve the GUT connection between gauge couplings. As we shall discuss it also leads to a testable prediction relating the strength of gravity to those of the gauge interactions. If this prediction should prove to be correct it will provide the first quantitative evidence for unification of all the fundamental forces including gravity.

The third possible explanation of the hierarchy problem, motivated by considerations of theories in more than three space dimensions, is that the hierarchy problem is absent because there is *no* fundamental scale much higher than the electroweak scale [3] or that if there is a higher scale it is protected from us by a "warp factor" [4] in the higher dimensions. Surprisingly present experimental measurements do not directly exclude new space dimensions with a size as large a 0.1 mm. It has been speculated that such theories could also accommodate a more fundamental unified theory capable of simplifying the structure of the Standard Model, although as we shall discuss, the detailed predictions may differ from the original GUT or string predictions..

Of course only experiment can tell us whether any of these ideas are realized in nature. We do know that the original formulation of the Standard Model is not correct because there is now significant evidence for neutrino masses. Restoring the symmetry between quarks and leptons by adding a right handed neutrino provides a very elegant way to generate these masses by the see-saw mechanism [5]. In this case the neutrino masses are anomalously light due to the large Majorana mass of the right handed neutrinos. The latter are the only states which can acquire such a Majorana mass as they are the only Standard Model singlet fields. The observed smallness of the neutrino masses then implies the Majorana mass must be very heavy giving support to the idea of an underlying high scale of new physics. Unfortunately, while this is the most elegant explanation, it is not the only way to explain small neutrino masses and cannot by itself distinguish between the candidate unified theories.

Unified theories *do* yield one quantitative prediction going beyond the Standard Model which does offer some hope for distinguishing between the different possibilities. This is the prediction for gauge coupling unification. As pointed out by Georgi, Quinn and Weinberg [6] the continuation of the gauge couplings to high energies using the renormalization group equations with beta functions calculated using just the Standard Model states shows that the couplings approach each other, perhaps suggesting an underlying unification. Of course this conclusion rests on having an underlying theory to normalize the $U(1)$ gauge coupling and the original investigations assumed that the underlying unified gauge group is $SU(5)$. Since the original analysis was performed the couplings have been measured to a high accuracy and now the couplings are more that ten standard deviations from the minimal $SU(5)$ prediction. Due to the hierarchy problem, however, nonsupersymmetric $GUTs$ with a high unification scale are inconsistent with perturbative unification so perhaps one should not be surprised at the failure of gauge unification in the original calculation. In fact the modified

$SU(5)$ calculation, including the supersymmetric contributions to the beta functions, has been known for 20 years [7]. Originally it gave a worse prediction for the weak mixing angle than non-supersymmetric version but the improved measurements of the gauge couplings have steadily gone towards the SUSY prediction and now the agreement is at better than the 1% level! It is largely because of the success of this prediction that so much attention has been paid to the study of SUSY phenomenology.

The fact that the prediction is for a single number has stimulated some to speculate that the simple SUSY prediction is an accident. We take the contrary view that nature is not so perverse and that the result is our first quantitative the indication of what lies beyond the SM. Taking this to its limit leads us to ask what the *very precise* prediction implies for the nature of the underlying unified theory. As we shall argue it leads to a remarkable precise determination of the nature of the underlying "Theory of Everything" and its low energy implications.

We start with a review of the precision of the prediction relating the gauge couplings and demonstrate the it is even better than the 1% level. If one is to maintain this level of accuracy in the presence of high scale threshold effects the nature of the underlying (string) theory is severely restricted. We use this condition to determine the profile of such an underlying string theory. In such a theory there is a new prediction relating the gauge coupling unification scale to the string scale or the Planck scale which determines the strength of gravity. Such a prediction if verified would be the first quantitative evidence for unification of the fundamental forces *including* gravity. Initial studies suggested the prediction coming from the weakly coupled heterotic string, while qualitatively encouraging, failed in detail. We discuss the possible explanations for this failure and emphasize that the calculation of the heavy threshold effects, in particular those due to the breaking of the underlying GUT at the compactification scale, is crucial. We describe a new calculation of the threshold effects coming from Wilson line braking and show that, including their effects, the resulting prediction for the unification scale given by the weakly coupled heterotic string can be in excellent agreement with experiment. Finally we consider the profile of the supersymmetric extension of the Standard Model (SSM) that emerges from an underlying string theory in which the accuracy of the gauge coupling unification prediction is maintained. The most significant low-energy phenomenological signals coming from such a theory are flavor changing neutral currents, close to the present experimental limits, and the new Higgs and supersymmetric states required for the SSM. We briefly discuss the expectation for the spectrum of such states and their characteristic signals.

2. Gauge coupling unification

The unification of gauge couplings remains the most significant piece of quantitative evidence for physics beyond the Standard Model. At the two loop level the RGE equations for the SM gauge couplings [8] have the solution for $\alpha_i(\mu)$, evaluated at the scale μ, given by

$$\alpha_i^{-1}(\mu) = \alpha_s^{-1} + \frac{b_i}{2\pi} \ln \left[\frac{M_X}{\mu}\right] + \frac{1}{4\pi} \sum_{j=1}^{3} \frac{b_{ij}}{b_j} \ln \left[\frac{\alpha_s}{\alpha_j(\mu)}\right] , \tag{1}$$

where b_i (b_{ij}) are the one (two) loop beta functions. Given the multiplet content of the SM the beta functions are completely determined. The normalization of the $U(1)$ gauge coupling, g_1, of the SM is dependent on the details of unification so in general we have $\alpha_1 = g_1^2\gamma^2/4\pi$ where γ is to be determined by the underlying unified theory.

What is the implication of Eq. (1) for the gauge couplings? There are three unknown constants, namely the value of the universal coupling, α_s, the unification scale, M_X, and the $U(1)$ normalization, x. As there are only 3 gauge couplings measured a prediction requires further input. In a given unification scheme the normalization of the $U(1)$ factor is known. In $SU(5)$ it is given by $\gamma = 1$ and the same value applies in many other $GUTs$. This value is also found in level-1 compactified heterotic string theories even though there may be no stage of Grand Unification below the compactification scale. Once x is determine there is a prediction relating the observed gauge couplings because there are then only two unknown constants in Eq. (1). In string theories the scale of unification may also be predicted. We will discuss these predictions in detail as they provide the only quantitative evidence for unification.

Using the $SU(5)$ normalization and the SM values of the beta functions the prediction for the gauge couplings was first tested by Georgi, Quinn and Weinberg [6]. The result was encouraging, the couplings closely approaching each other at a scale of $O(10^{16}$ GeV). Unfortunately, with the improved measurement of the low energy gauge couplings, the agreement has proved to be illusory and now the couplings are more than 10 standard deviations from meeting at a point.[a]

[a] Strictly, due to threshold effects, the running couplings do not meet at a point at the unification scale [9]. The quoted discrepancy correctly includes these threshold effects.

However it is known that $GUTs$ which involve a new large Grand Unified scale, M_X, suffer from the hierarchy problem due to large radiative corrections driving the SM Higgs scalar mass close to M_X. Since the electroweak breaking scale is proportional to the Higgs mass the GUT is inconsistent unless some mechanism of eliminating these corrections is included. As discussed in the introduction several ideas have been proposed. The only one which is consistent with a large GUT scale while preserving the perturbative unification of gauge couplings given by Eq. (1), is if the SM is extended to include supersymmetry. Supersymmetry limits the radiative corrections to be proportional to the SUSY breaking scale. In this sense the only consistent GUT is a supersymmetric GUT and the gauge unification prediction should be calculated including the corrections arising from the new SUSY states.

The minimal supersymmetric extension of the SM (the $MSSM$) assigns the SM states to ($N = 1$) supermultiplets and requires the existence of new states, superpartners of the SM states, to complete these representations. Of course the addition of these states means the beta functions in Eq. (1) change when the scale, μ, is greater than the mass of the new states and so the prediction of gauge unification will change. One might worry that the introduction of the new mass scale, associated with these new SUSY states, spoils the predictivity of the theory because there are now three unknown parameters and only three measureables. However this is not the case because the solution to the hierarchy problem requires that the scale of SUSY breaking is limited to $\leq O(1\ \text{TeV})$ and, as we will discuss, this introduces a very small uncertainty in the prediction.

To determine the prediction of gauge unification in the MSSM it is convenient to combine the standard model gauge couplings to eliminate the one-loop dependence on the unification scale and the value of the unified coupling. This leads to a relation which depends only on the threshold effects associated with the unknown masses of the supersymmetric states,

$$\alpha_3^{-1}(M_Z) = \left\{ \frac{15}{7} \sin^2\theta_W(M_Z) - \frac{3}{7} \right\} \alpha_{em}^{-1}(M_Z) + \frac{1}{2\pi} \frac{19}{14} \ln \frac{T_{eff}}{M_Z} + (\text{two-loop})\,. \tag{2}$$

Note that the leading dependence on the SUSY thresholds comes from T_{eff}, an effective supersymmetric scale, while two loop corrections, which have a milder dependence on the scale, through (gauge and matter) wave-function renormalization $\ln Z \sim \ln \alpha_i(\mu)$ can be ignored to a first approximation. In terms of the individual SUSY masses T_{eff} is given by [10]

$$T_{eff} = m_{\widetilde{H}} \left(\frac{m_{\widetilde{W}}}{m_{\widetilde{g}}}\right)^{\frac{28}{19}} \left(\frac{m_H}{m_{\widetilde{H}}}\right)^{\frac{3}{19}} \left(\frac{m_{\widetilde{W}}}{m_{\widetilde{H}}}\right)^{\frac{4}{19}} \left(\frac{m_{\widetilde{l}}}{m_{\widetilde{q}}}\right)^{\frac{9}{19}} \quad (3)$$

provided the particles have mass above M_Z. The precision of the relation, Eq. (2), between Standard Model couplings is limited by the dependence on T_{eff}. However the uncertainty in T_{eff} is limited by the fact that the supersymmetry masses are bounded from below because no supersymmetric states have been observed and from above by the requirement that supersymmetry solve the hierarchy problem. From Eq. (3) we see the dependence on the squark, slepton and heavy Higgs masses is very small, the main sensitivity being to the Higgsino, Wino and gluino masses.

If all the supersymmetric partner masses are of O(1 TeV) then so is T_{eff}. However in most schemes of supersymmetry breaking radiative corrections split the superpartners. In the case of gravity mediated supersymmetry breaking with the assumption of universal scalar and gaugino masses at the Planck scale the relations between the masses imply $T_{eff} \simeq m_{\widetilde{H}} \left(\alpha_2(M_Z)/\alpha_3(M_Z)\right)^2 \simeq |\mu|/12,$. For μ of order the weak scale T_{eff} is approximately 20 GeV. From this we see the uncertainty in the SUSY breaking mechanism corresponds to a wide range in T_{eff}. In what follows we shall take 20 GeV $< T_{eff} <$ 1 TeV as a reasonable estimate of this uncertainty. Using this one may determine the uncertainty in the strong coupling for given values of the weak and electromagnetic couplings using Eq. (2). The result (including two-loop effects) as a function of α_3 and $\sin^2\theta_W$ is plotted in Figure 1 [11].

The precision of the prediction is remarkable. A measure of this is given by the area between the two curves in Figure 1. If one assumes that a random model not constrained by unification may give any value for α_3 and $\sin^2\theta_W$ between 0 and 1 the relative precision is an impressive 0.002! Of course this estimate is sensitive to the measure chosen. Changing to α_3 and $\sin\theta_W$ makes very little difference. Changing to α_3^{-1} and $\sin^2\theta_W$ (and restricting the possible range of α_3^{-1} to be $1 < \alpha_3^{-1} < 10$) increases the relative precision by a factor of 10. Using this variation as an estimate of the uncertainty associated with the measure we conclude that a reasonable estimate for the precision of the prediction for the correlation between the Standard Model couplings is in the range $(0.2-2)\%$. One may also use the result of Eq. (1) to predict one of the couplings given the other. However, as may be seen from Figure 1(a), the prediction is not equally precise for each coupling. This effect may be seen directly in Figure 1(a) since the curve is

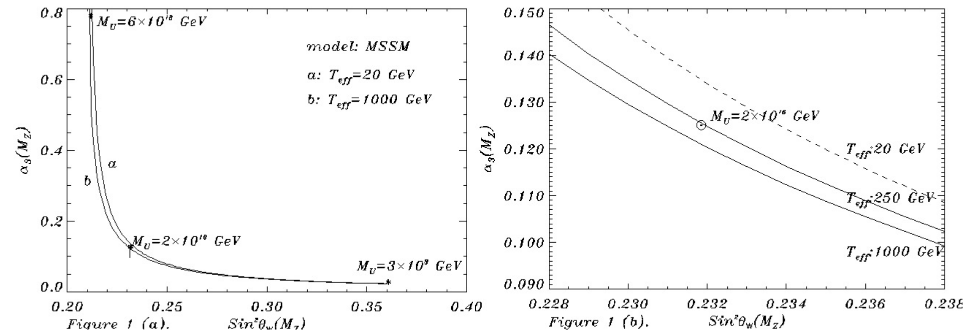


Figure 1. Plots of $\alpha_3(M_Z)$ versus $\sin^2\theta_W$ calculated in the MSSM for two values of the effective supersymmetric threshold, $T_{eff} = 20$ GeV and $T_{eff} = 1000$ GeV. The limits correspond to requiring $\alpha_3(M_Z)$ remains in the perturbative domain and unification occurring above the supersymmetry threshold. The area between the two curves provides a measure of the predictivity of the theory. The experimental range of values is also shown.

more steeply varying in the α_3 direction than in the $\sin^2\theta_W$ direction close to the experimental point. The optimal combination of α_3 and $\sin^2\theta_W$ with minimal uncertainty normal to the curve can be determined numerically but is relatively close to $\sin^2\theta_W$. Quantitatively the error in α_3 is $\approx 10\%$ for a change in T_{eff} from 20 to 1000 GeV and the corresponding error in $\sin^2\theta_W$ is 1.3%. Thus the prediction for $\sin^2\theta_W$ provides a more realistic measure of the precision of the unification prediction.

Using the latest measurements of the relevant parameters one finds the couplings unify at the scale $M_X = (3.5 \pm 2)10^{16}$ GeV and and the value of $\sin\theta_W$ is predicted to be $\sin^2\theta_W = 0.2334 \pm 0.0025$ to be compared to the experimental value of $\sin^2\theta_W = 0.2311 \pm 0.0007$. (For comparison using the measured values of $\sin^2\theta_W$ one finds the couplings unify at the scale $M_X = (4.3 \pm 3)10^{16}$ GeV [b] and $\alpha_s(M_Z) = 0.134 \pm 0.005$ to be compared to the experimental measurement $\alpha_s(M_Z) = 0.117\pm\ 0.017$.)

The agreement between the prediction and experiment is impressive and as we will discuss leads to strong restrictions in the underlying theory if it is to be maintained [11]. First however we turn to a discussion of the possibility that compactified string theories may predict the value of the unification scale.

[b] The unification scale is slightly larger than early estimates due to the measured top quark being larger than the estimates following from radiative corrections.

3. String theory – unification with gravity

As mentioned above, the string makes an important additional prediction which goes beyond Grand Unification, namely it determines the unification scale in terms of the Planck scale. If this could be checked it would provide the first quantitative test of the unification of the strong, electromagnetic and weak forces with gravity which the superstring provides.

The prediction for the gauge unification scale in the weakly coupled heterotic string follows from the general form of the 4D Lagrangian

$$\begin{aligned} L_{eff} &= -\int d^{10}x\,\sqrt{g}\,\alpha_{10}^{-1}\Big(\frac{4}{\alpha'^4}R+\frac{1}{\alpha'^3}TrF^2+...\Big) \\ &= -\int d^4x\,V\,\sqrt{g}\,\alpha_{10}^{-1}\Big(\frac{4}{\alpha'^4}R+\frac{1}{\alpha'^3}TrF^2+...\Big)\,. \end{aligned} \tag{4}$$

In this we may see that Newton's constant, G_N, and the value of running gauge couplings at the unification scale, α_{GUT}, are given in terms of the 10D string coupling α_{10}, the string tension $\alpha' \sim \frac{1}{M_{string}}$ and the volume of the 6D compactified space V by

$$G_N = \frac{\alpha_{10}\alpha'^4}{64\pi V}, \qquad \alpha_{GUT} = \frac{\alpha_{10}\alpha'^3}{16\pi V}\ . \tag{5}$$

For the case that α_{10} is small the volume V is approximately M_{string}^{-6} and one obtains Eq. (6) eliminating V between the two equations. This leads to the prediction is [12]

$$M_{string} \approx g_{string} \times (5.2 \times 10^{17}\ \mathrm{GeV}) \approx 3.6 \times 10^{17}\ \mathrm{GeV} \tag{6}$$

which is only a factor of 10 above the "observed" gauge unification scale. This is remarkably close and gives considerable encouragement to the idea that gravity is unified with the other fundamental forces. Given the importance of the result it is crucial to understand the origin of the residual discrepancy. This has been examined in some detail. There are various effects that can cause the prediction of the Grand Unified scale to vary [13]. One should remember that the Grand Unified scale is the argument of the log and thus, in order to get it correctly, one has to work to very high precision. Possible problems in determining this scale are :

- SUSY threshold effects. It seems that although these could make the agreement with the prediction for the strong coupling better this only can be achieved with a very peculiar supersymmetry spectrum at low energies in which the gluino is lighter than the Wino [14].

Moreover it does not change the predicted value of the unification scale.

- A second possibility is that there is a Grand Unified group below the string scale so that the scale at which the gauge couplings unify should be identified with the scale of GUT breaking and the scale, Eq. (6) at which gravity unifies with the gauge coupling lies higher. Unfortunately this explanation means we will never be able to test the string prediction for M_{string} leaving just the prediction for gauge coupling unification. Also there are inevitably further threshold corrections coming from new states lying at and above the Grand Unification scale. These states include the heavy gauge bosons and Higgs bosons needed to make up complete GUT multiplets.
- Another possibility is that there are string threshold effects which in a given string theory are calculable [15] and amount to including the effect of the heavy Kaluza–Klein modes which are themselves split when the Grand Unified theory is broken. Unfortunately it has proved difficult to calculate these effects in realistic string compactifications. We will return to a discussion of these effects shortly.
- There is another way of exhibiting the implication of Eq. (5). If one uses the "measured" value for $V = O((3.10^{16}\ \mathrm{GeV})^{-6})$ one may use Eq. (5) to obtain the value of α_{10} instead. This gives an enormous value, quite inconsistent with the assumption of weak coupling that went into the derivation of Eq. (5). If this is the explanation, rather than the threshold effects just discussed, the failure of the prediction of Eq. (6) is not surprising - it was the wrong calculation. Instead one should go to the strongly coupled heterotic string case [16]. In this case the string unification scale has an additional dependence on the compactification scale of the 11th dimension and can be brought into agreement with the gauge unification scale if the size of this dimension is $(10^{15}\ \mathrm{GeV})^{-1}$ [17]. Of course, unless this compactification scale is known, one loses the prediction for the gauge unification scale. Subsequently there has been an explosion of interest in gauge unification at much lower scales associated with low scale compactification [3] in which the SM fields also propagate in additional dimensions at a scale below the gauge unification scale. This has the effect of modifying the rate at which the gauge couplings evolve causing them to run like a power of the scale rather than a logarithm. As a result the couplings can unify at a low scale, perhaps even at the TeV scale.

How will we be able to distinguish between these various possibilities?

This may be easy if the origin is due to low-scale physics accessible to experimental measurement. However most of the potential explanations involve effects due to very heavy states well above the scale we can directly probe. In this case we will have to rely on circumstantial evidence. I will argue that we already have been provided with just such evidence through the success of the prediction of gauge coupling unification! The fact that experiment and theory agree to a precision better than 1% seems to me unlikely to be just a happy accident. In this case, given that heavy threshold effects of the types listed above also affect gauge coupling unification, we may hope that we can identify their origin from the condition that the *accuracy* of this prediction is not spoiled.

3.1. *High-scale threshold sensitivity of the unification predictions*

Including the effect of thresholds the value of the gauge couplings at M_Z is of the form

$$\alpha_i^{-1}(M_Z) = -\delta_i + \alpha^{-1}(\Lambda) + \frac{\overline{b}_i}{2\pi}\Delta(\Lambda,\mu_0) + \frac{b_i}{2\pi}\ln\frac{\Lambda}{M_Z} + \frac{3T_i(G)}{2\pi}\ln\left[\frac{\alpha(\Lambda)}{\alpha_i(M_Z)}\right]^{1/3} - \sum_\phi \frac{T_i(R_\phi)}{2\pi}\ln\ Z_\phi(\Lambda,M_Z)\,. \quad (7)$$

Here $b_i = -3T_i(G) + \sum_\phi T_i(R_\phi)$ are the one-loop beta function coefficients and Δ and the coefficient $\overline{b}_i$ give the massive threshold correction corresponding to the particular theory considered - they are non-zero only for the N=2 massive SUSY states and vanish for the N=4 spectrum. The parameter μ_0 is the mass of the heavy states, often the compactification scale in string theories, and Z_ϕ and $(\alpha(\Lambda)/\alpha(M_Z))$ are the matter and gauge wavefunction renormalization coefficients respectively. In Eq. (7) there are additional effects due to low energy supersymmetric thresholds δ_i [c] which were discussed above.

3.1.1. *Grand Unified Models*

In a realistic Grand Unified theory one is forced to introduce a number of additional gauge non-singlet multiplets in order to spontaneously break the

[c] The definition of T_{eff} Eq. (3) in terms of δ_i is $\ln T_{eff}/M_z = -(28\pi/19)(\delta_1 b_{23}/b_{12} + \delta_2 b_{31}/b_{12} + \delta_3)$, with $b_{ij} = b_i - b_j$.

gauge symmetry and to give a realistic pattern of quark and lepton masses and mixing angles. These states include the heavy gauge bosons and Higgs bosons needed to make up complete GUT multiplets. These states introduce significant threshold uncertainty, Δ, which depend on the unknown mass of the heavy states. For reasonable ranges of the unknown heavy particle masses these corrections [9] can readily be much greater than 1% and so in such theories the precision of prediction found in the MSSM must be considered accidental in such schemes. Taking the extreme view that this accuracy is not accidental one must conclude that the troublesome massive states must be absent and discard the possibility that there is a stage of Grand Unification below the string unification scale. This illustrates the power of the requirement that the precision prediction be maintained.

3.1.2. *Weakly coupled heterotic string models*

In the heterotic string there need be no Grand Unified group below the compactification scale and then the uncertainties associated with the breaking of the Grand Unified group can be much reduced. The calculation of the threshold effects of states at the compactification scale in string theories requires one specifies the specific compactification. Given the huge number of candidate string theories this looks an impossible task. However, as we will discuss, the threshold corrections come principally from those states below the string scale and these must be limited in number if the accuracy of the gauge unification prediction is not to be disturbed. This means the classification of the various possibilities becomes tractable.

In comparing the threshold effects at the unification scale to the SUSY threshold effects discussed above we will restrict ourselves to a measure of the sensitivity of the prediction to these scales for either the strong coupling or $\sin^2\theta_W$ while the other is maintained fixed. In leading order we have from Eq. (7)

$$\delta\alpha_3^{-1}(M_Z) - \frac{15}{7}\delta\sin^2\theta_W = \frac{1}{2\pi}\left[\overline{b}_1\frac{b_{23}}{b_{12}} + \overline{b}_2\frac{b_{31}}{b_{12}} + \overline{b}_3\right]\delta\Delta(\Lambda,\mu_0)\,, \qquad (8)$$

where $b_{ij} = b_i - b_j$, $b_1 = 33/5$, $b_2 = 1$, $b_3 = -3$. The relative sensitivity of α_3 (keeping $\sin^2\theta_W$ fixed) to changes in μ_0 and T_{eff} is given by

$$\mathcal{R} \equiv \left|\frac{\delta(\ln(\alpha_3(M_Z)))}{\delta(\ln(\alpha_3(M_Z)))_{MSSM}}\right| = \frac{14}{19}\left|\left\{\frac{5}{7}\overline{b}_1 - \frac{12}{7}\overline{b}_2 + \overline{b}_3\right\}\frac{d\Delta}{d(\ln\mu_0)}\right|\,, \qquad (9)$$

where we have assumed that the predicted value for $\alpha_3(M_Z)$ in the model considered is close to that of the MSSM.[d] One obtains the same result for the *relative* threshold sensitivity if we compute the prediction for $\sin^2\theta_W(M_Z)$ (with $\alpha_3(M_Z)$ fixed).

To illustrate this we first consider the case of the weakly coupled heterotic string with an N=2 sector (without Wilson lines present) in which six of the dimensions are compactified on an orbifold, T^6/G. In such models the spectrum splits into N=1, N=2 and N=4 sectors, the latter two associated with a $T^2 \times T^4$ split of the T^6 torus. Due to the supersymmetric non-renormalization theorem, the N=4 sector does not contribute to the running of the holomorphic couplings.[e] The N=1 sector gives the usual running associated with light states but does not contain any moduli dependence. The latter comes entirely from the N=2 sector. For the heterotic string all states are closed string states and at one loop the string world sheet has the topology of the torus T^2. For the case of a six-dimensional supersymmetric string vacuum compactified on a two torus T^2 the string corrections take the form [12, 18]. Here $T \propto R_1R_2$ and $U \propto R_1/R_2$ where T, U are moduli and R_1, R_2 are the radii associated with T^2. We consider the case of a two torus T^2 with $T = iT_2$ (the subscript 2 denotes the imaginary part) and $U = iU_2$. Making the dimensions explicit T_2 should be replaced by $T_2 \to T_2^o/(2\alpha') \equiv 2R^2/(2\alpha')$. Performing a summation over momentum and winding modes and integrating over the fundamental domain of the torus gives the following result for the string threshold correction Δ^H [18]

$$\begin{aligned}\Delta^H &= -\frac{1}{2}\ln\left\{\frac{8\pi e^{1-\gamma_E}}{3\sqrt{3}}\, U_2T_2|\eta(iU_2)|^4\ |\eta\,(iT_2)|^4\right\}\\ &= -\frac{1}{2}\ln\left\{4\pi^2\,|\eta(i)|^4\left(\frac{M_s}{\mu_0}\right)^2\left|\eta\left[\frac{i\,3\sqrt{3}\pi}{2e^{1-\gamma}}\,\left(\frac{M_s}{\mu_0}\right)^2\right]\right|^4\right\}\end{aligned} \tag{10}$$

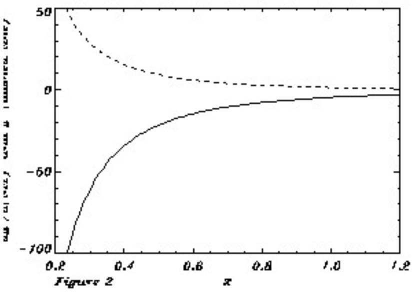
with the choice $U_2 = 1$ in the last equation. M_s is the string scale, $\mu_o \equiv 1/R$, $\eta(x)$ is the Dedekind eta function and we have replaced α' in terms[f] of M_s. For large[g] T_2 the eta function is dominated by the leading exponential and so

[d] For a model to be viable one must in first instance predict the right value for $\alpha_3(M_Z)$ and only after would the question of threshold sensitivity be relevant.

[e] Actually even in the case of N=4, two-loop corrections to the effective gauge couplings may introduce substantial threshold corrections [11].

[f] From [12] $M_s = 2\,e^{(1-\gamma_E)/2}\,3^{-3/4}/\sqrt{2\pi\alpha'}$ where g_s is the string coupling at the unification.

[g] Note that $T_2 \approx 5.5(M_sR)^2$ in $\overline{DR}$ scheme, so one can easily have $T_2 \approx 20$ while R is still close to the string length scale, to preserve the weakly coupled regime of the heterotic string. In this section "large" R corresponds to values of T_2 in the above range.



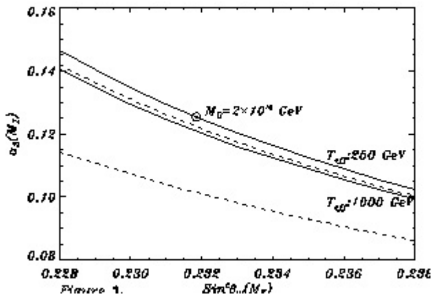


Figure 2. The values of the derivative $d\Delta^H/d(\ln\mu_0) = 1 - 2x\ln'_x\eta\left(i3\sqrt{3}\pi e^{\gamma_E-1}/(2x^2)\right)$ and that of Δ^H for x close to 1. Figure 3. The values of $\alpha_3(M_Z)$ for the model with gauge bosons in the N=2 sector with a change of x from 1 to 1/2, leading to an area (between dashed lines) of values available to $\alpha_3(M_Z)$ larger by a factor of ≈ 5 compared to the MSSM case (continuous lines).

one finds the power law behavior $\Delta \propto T_2 \propto R^2$ which has a straightforward interpretation as being due to the decompactification associated with T^2. This contribution is basically due to the tower of Kaluza–Klein states below the string scale and can be understood at the effective field theory level [19] whose result is regularized by the string world-sheet [19]. The presence of power-like running, although taking place over a very small region of energy range,[h] can significantly affect the sensitivity of the unification prediction for $\alpha_3(M_Z)$ with respect to changes of the compactification scale (which, being determined by a moduli vev, is not fixed perturbatively and hence is not presently known). To demonstrate this explicitly, consider the variation of the threshold Δ^H with respect to T_2

$$\frac{\delta\Delta^H}{\delta\ln T_2} = -\frac{1}{2}\left\{1 + 4\frac{d\ln|\eta(iT_2)|}{d(\ln T_2)}\right\}. \tag{11}$$

This gives the following relative threshold sensitivity with respect to the compactification scale μ_0 ($x \equiv \mu_0/M_s \equiv R_s/R_c$)

$$\mathcal{R} = \frac{14}{19}\left|\left\{\frac{5}{7}\overline{b}_1 - \frac{12}{7}\overline{b}_2 + \overline{b}_3\right\}\left\{1 - 2x\frac{d}{dx}\ln\eta\left(i\sigma x^{-2}\right)\right\}\right|, \tag{12}$$

where $\sigma = 3\sqrt{3}\pi e^{\gamma_E-1}/2$ and $x \equiv \mu_0/M_s \equiv R_s/R_c$ with R_s the string length and R_c the compactification radius.. In Figure 2 we plot the second factor in curly brackets (which is equal to $\delta\Delta^H/\delta(\ln x)$).

[h] This takes place essentially between the compactification and the string scale [19]

To understand the importance of Δ^H and the implications it has for the threshold sensitivity of $\alpha_3(M_Z)$, we note that Δ^H plays a central role in the attempts to bridge the well-known "gap" (of a factor of ≈ 10) [13, 20] between the heterotic string scale and the MSSM unification scale. Since $\mathcal{R}$ gives the relative sensitivity of the gauge coupling predictions to the N=2 threshold and the SUSY threshold we see that even for $x = 0.5$, corresponding to the compactification radius being twice the string length, the precision of the gauge coupling prediction is substantially reduced. The requirement that this precision should not be lost constrains $x \approx 1$ (i.e.$\mu_0 = M_{string}$). However this fails to "bridge the gap". This example clearly illustrates the general problem that weakly coupled heterotic string models have to reconcile two conflicting constraints, namely the need for a large Δ^H to solve the scale mismatch between the MSSM unification scale and the heterotic string scale and the need for a small derivative of Δ^H to avoid a large threshold sensitivity of the gauge couplings. The latter constraint is introduced by the power-law dependence[i] of the thresholds on the compactification scale, and even though such running takes place over a small energy range it still gives significant effects. The problem can be avoided if $x \approx 1$, corresponding to the compactification scale being very close to the string scale. On the other hand the former constraint seems to require a small value for x. As we shall discuss this conclusion may be evaded in theories with Wilson line breaking.

3.1.3. *Low scale string theories*

The remarkable precision of the prediction for gauge coupling unification has led us to consider the nature of an underlying (string) theory that can maintain this accuracy. Due to the non-decoupling of the contribution of massive states to renormalizable terms in the low energy effective Lagrangian, the requirement that the gauge predictions be undisturbed places strong constraints on the massive sector. We determined the contribution of states transforming non-trivially under the Standard Model gauge group in compactified string models. Requiring that these contributions leave the MSSM predictions intact leads to a constraint on the magnitude of the compactification radius associated with the propagation of such states to limit the number of states in the Kaluza–Klein tower. It is straightforward to extend the discussion given above to a variety of string theories [19] This shows that due to the sensitivity of the gauge coupling predictions to the compactifi-

[i] This is due almost entirely to the presence of the towers of Kaluza–Klein modes rather than to winding modes, [19].

cation scale in the case the gauge couplings have power law running rather than logarithmic running the compactification radius should be very close to the inverse of the string scale (well within a factor of 2). These conclusions apply to string theories with an $N = 2$ sector. It is possible to construct compactified string theories in which this sector is absent in which case there is no significant one-loop sensitivity of the unification prediction for the gauge couplings to changes of the high scale/moduli fields. The downside is that then one does not have the large Δ^H needed to "close the gap". Even if this problem is solved in another way the $N = 4$ sector necessarily present introduces strong threshold dependence at two loop order which proves to lead to almost as restrictive a bound on the compactification scale.

The implication of this result is quite far-reaching. One immediate one is that unification at a low scale through power law running of the gauge couplings cannot maintain the precision of the MSSM predictions. Indeed, due to the different contributions to the beta functions of massive compared to massless SUSY representations, low scale unification requires a very different multiplet content from the MSSM in order to obtain the same gauge unification prediction. As a result the precision prediction for gauge couplings must be considered a fortuitous accident, something we find hard to accept given the remarkable precision of the prediction. Even if we do accept this, and the N=2 sector happens to give the same beta function as the N=1 MSSM spectrum, power law running introduces a strong dependence on the heavy thresholds so that the precision of the "prediction" is lost. For both these reasons we consider a low scale of unification due to power law running to be unlikely. Models with a low scale of unification due to a non-$SU(5)$ hypercharge normalization do not require power law running. Nonetheless it turns out that they too have enhanced threshold dependence due to the need for additional light states and again this loses the predictive power of the MSSM.

3.2. *A string theory profile*

The profile of our string model which preserves the precision prediction for gauge couplings found in the MSSM therefore requires a large scale of unification with an $SU(5)$ normalisation for the weak hypercharge. Even so, there is still a strong constraint on the compactification scale because the cutoff of the contribution of heavy states is at the string scale or higher. To avoid the same power law running corrections that degraded the predictions in the case of low scale unification, the radius of compactification of those dimensions in which Standard Model gauge non-singlet fields propagate must not be large compared to the cutoff radius. This means the compactified string theory lies far from the Calabi–Yau limit and close to the superconformal limit. This is quite attractive in that many aspects of the effective low-energy field theory, such as Yukawa couplings, are amenable to calculation in the superconformal limit.

Of course our analysis does not preclude the existence of large new dimensions not associated with the propagation of Standard Model states. A particular example is the strongly coupled heterotic string in which gravity but not the Standard Model states propagate in the eleventh dimension. However the size of the extra dimensions is still severely constrained by the need to have a high unification scale, the minimum occurring for just one additional dimension as in the strongly coupled heterotic case. Thus even in the case of large new dimensions in which Standard Model states do *not* propagate, the new dimension cannot be larger than 10^{-14} fm.

4. Unification with gravity – closing the gap

The need for a high scale of unification broadly fits the expectation in the (weakly coupled) heterotic string. However in detail there is a discrepancy between the MSSM value for the unification scale of $M_X = (3.5\pm2)10^{16}$ GeV and the prediction in the weakly coupled heterotic string, approximately a factor of 10 larger.

As discussed above one possibility is to have a GUT below the string compactification scale, the GUT breaking scale being the gauge unification scale. Even in this case it is still necessary for the compactification scale to be very close to the string scale to keep the two loop contributions from the Kaluza–Klein states small. If this theory comes from the weakly coupled heterotic string the compactification and string scales must be close to the Planck scale. Unfortunately if this is the explanation the string prediction for the compactification scale does not lead to a testable prediction. Moreover, as discussed above, this also diminishes the accuracy of the prediction to well above the 1% level.

The strongly coupled version of the heterotic string provides another explanation for the discrepancy found in the weakly coupled case. For the case the eleventh dimension is some three orders of magnitude larger than the string length, corresponding to a compactification radius of $O(10^{-14} fm)$, the gauge unification scale may be reduced to that found in the MSSM. This provides a way to reconcile the predicted unification scale with that needed in the MSSM. However, unless one is able to predict that the radius of the eleventh dimension is indeed of $O(10^{-14} fm)$, the prediction for the unification scale is lost, being just given in terms of this unknown compactification scale.

To me a more attractive possibility is that calculable heavy threshold effects associated with the breaking of the underlying string GUT reduces the weakly coupled string prediction for the unification scale to its "measured" value. The requirement that the precision of the gauge coupling prediction be maintained severely limits the magnitude of these threshold effects and precludes explanations requiring large radii. However in models with Wilson line breaking it is possible to have very large threshold corrections to the unification scale while keeping the threshold contribution to gauge couplings small. Given that such Wilson line breaking is very often *needed* to break the underlying gauge symmetry of the heterotic string, this explanation seems very reasonable. Although these have been calculated in specific string theories [21], the analysis has not been done in realistic string theories. However the indication coming from the toy models is that Wilson line effects can be very significant even though the compactification scale, which is related to the scale of Wilson line breaking for discrete Wilson lines, is close to the Planck scale. For this reason it is worth having a closer look.

4.1. *Wilson line breaking*

The Wilson line operator is defined as

$$W_i = e^{i\int_{\gamma_i} dy A^I_{y_m} T^I}, \quad I = 1, \cdots, r_k\, G\,, \tag{13}$$

where A_{y_m} are the higher dimensional components of the gauge field, y_m are the compact dimensions, $m = 1$ ($m = 1, 2$) for one (two) compact dimensions respectively. A summation over m and I is understood. γ_i are one-dimensional cycles of compactification. T_I are the generators of the Cartan sub-algebra of the Lie algebra

Continuous Wilson lines have their magnitude determined by a moduli which can have any vacuum expectation value. In this case they act just as if the breaking was spontaneous and due to a Higgs scalar multiplet transforming as the adjoint. The Wilson line breaks the associated gauge group and gives a mass to those gauge bosons not commuting with the Wilson line. Ignoring matter and the KK excitations, the mass of these excitations, M_X, defines the unification scale as above it there is no further relative evolution of the gauge couplings of the unbroken gauge group factors. Of course the overall gauge coupling continues to run until the string cutoff scale but this effect can be absorbed in the starting value of the gauge coupling used at M_X. For matter fields the effect of Wilson lines is determined by their gauge transformation properties. Chirality protects those fields which are massless before Wilson line breaking from acquiring a mass. However their KK modes can be shifted in mass by the Wilson line and this can lead to further corrections to the gauge couplings.

If one is to discuss the effect of threshold effects close to the string scale it is necessary to know the full spectrum of states. We will be concerned in determining these effects for the minimal set of fields consistent with obtaining the MSSM at low scales. The main uncertainty in this spectrum in a theory with an underlying GUT is the origin of the doublet triplet splitting needed if one is to have the light Higgs doublets of the MSSM necessary for electroweak breaking. In the case of continuous Wilson lines this must come from some mechanism, such as the missing partner mechanism [22], which significantly complicates the minimal spectrum needed and gives rise to significant uncertainties in the prediction following from gauge unification from the associated threshold corrections. For this reason we concentrate on the possibility that the Wilson lines are discrete because this does offer an elegant explanation of the doublet triplet splitting problem without complicating the spectrum [23].

Discrete Wilson lines are associated with a discrete group, D, which acts on the coordinates of the compactified $d-4$ dimensional space In the absence of Wilson lines the states of the theory, which may have non-trivial intrinsic transformation properties under D, correspond to the discrete group singlet states. Thus the massless modes, which have no dependence on the $4+d$ coordinates, must be intrinsic D singlets. The discrete Wilson line provides a representation, $\overline{D}$, of the discrete group acting on the gauge quantum numbers and the orbifold projection is modified to require that the states of the theory should be singlets under the simultaneous action of the discrete group action on both the internal and gauge quantum numbers. Thus if there is massless representation, R, of the gauge group, G, that transforms as $\widetilde{R}$ under the discrete group before Wilson line breaking then, with Wilson line breaking, only the component of the representation that transforms as $\widetilde{R}^{-1}$ under $\overline{D}$ remains, thus splitting the multiplet.

In the case of $SU(5)$ the electroweak breaking Higgs fields belong to the fundamental five dimensional representation ($R=5$). If only the $SU(2)$ doublet component transforms as $\widetilde{R}^{-1}$ under the discrete Wilson line then only the doublet will remain light while the remaining color triplet components are heavy, offering an origin for the doublet triplet splitting. In contrast to the Higgs, the quark and lepton matter multiplets fill out complete $SU(5)$ representations. It is therefore necessary that these states should not be split. This is explained naturally in the case of discrete Wilson lines. Since division by a freely acting discrete group changes the Euler number by the factor, N, the order of the group, in the theory before compactification there must be an excess of $3N$ in the number of left-handed states transforming as the $(\overline{5} \oplus 10)$ compared to the conjugate representation. These states form complete N dimensional representations of the group D. As a result, after modding out by the diagonal subgroup of $(D \oplus \overline{D})$, one is left with 3 families which form complete representations of $\overline{D}$ and hence fill out complete $(\overline{5} \oplus 10)$ representations of $SU(5)$. However note that the members of a family corresponding to different representations of $\overline{D}$ originate from different D representations in the underlying manifold.

In the discrete Wilson line case the massive gauge bosons, associated with the broken generators, acquire mass determined by the compactification scale. The same is true of the heavy partners of the Higgs doublets and the Kaluza–Klein excitations. However our condition that the precision prediction for gauge coupling unification should not be spoiled by power law running says that the compactification scale should be very close to the string scale. Thus, in this case, it is the string scale that determines the cutoff of the contribution of the zero modes and provides the gauge unification scale. For a given compactified heterotic string model with discrete Wilson line breaking no new parameters are introduced and therefore the gauge unification scale remains a prediction.

4.2. *Orbifold calculation*

In order to estimate the effect of Wilson lines it is useful first to consider (discrete) Wilson lines in orbifold compactifications for the weakly coupled heterotic string. This means we have to consider the KK and winding states that affect the running of the gauge couplings. However in practice the fact that the gauge unification scale is lower than the string scale means that the compactification radius, R, is greater than the string length, M_{string}^{-1}. In this case the KK states, whose mass is determined by integer multiples of $1/R$, are less massive than the winding states with mass integer multiples of $R\ M_{string}^2$. In the heterotic string contributions from states above the string scale are exponentially suppressed and as a result the winding modes have a contribution suppressed by a factor of $O(e^{-2\pi R M_{string}})$. To make this point more explicitly we note that the momentum modes alone give the contribution [19]

$$\Delta_i = \frac{\overline{b}_i}{4\pi}\int\limits_{-1/2}^{1/2} d\tau_1 \int\limits_{\sqrt{(1-\tau_1^2)}}^{\infty} d\tau_2 \frac{1}{\tau_2}\left\{\sum_{m_1,m_2\in Z} exp\left[-\frac{\pi\tau_2}{T_2}\left(m_1^2+m_2^2\right)\right]-1\right\}$$

$$= \frac{\overline{b}_i}{4\pi}\sum_{(m_1,m_2)\neq(0,0)}\int\limits_{-1/2}^{1/2} d\tau_1\, E_1\left[\kappa_{\vec{m}}\sqrt{1-\tau_1^2}\right] \tag{14}$$

with $|\vec{m}|^2 = m_1^2 + m_2^2$ and where

$$\kappa_{\vec{m}} \equiv \frac{\pi|\vec{m}|^2}{T_2}\ . \tag{15}$$

It is straightforward to compare the result of directly evaluating Eq. (14) with the *full* string result of Eq. (10). For example at $T_2 = 2$, corresponding to $R_c/R_s = 2$, the KK states alone give 99% of the *full* threshold correction [19]. This means that one may determine the threshold corrections to an excellent accuracy simply by including the contribution of states at or below the string scale. With this motivation we now consider the KK threshold corrections in simple orbifold examples.

4.2.1. *One compact dimension*

In the case of one additional dimension the effect of Wilson line breaking is to change the mass of the states in the KK tower according to

$$M_n^2(\sigma) = \chi^2 + \frac{1}{R^2}(m+\rho_\sigma)^2, \qquad \rho_\sigma = -R{<}A_y^I{>}\sigma_I, \quad (\text{sum over } I)\,, \tag{16}$$

where ρ_σ is derived using for one compact dimension with constant A_y^I. R is the compactification radius, σ_I is the weight or root of the representation that the higher dimensional field (of charge σ in Cartan–Weyl basis) belongs to. χ is the bare mass of the KK tower that henceforth we take to be 0. The general correction introduced to the gauge couplings by the combined effect of KK states and Wilson lines is given by the general formula [12] valid whether or not supersymmetry is present

$$\Omega_i^* = \sum_r \sum_{\sigma=\lambda,\alpha}\Omega_i(\sigma), \quad \Omega_i(\sigma) \equiv {\sum_{m\in Z}}' \frac{\beta_i(\sigma)}{4\pi}\int_0^\infty \frac{dt}{t}\, e^{-\pi\, t\, M_m^2(\sigma)/\mu^2}\Big|_{\text{reg.}}\ . \tag{17}$$

$\Omega_i(\sigma)$ is the contribution to the one loop beta function for the gauge coupling g_i of a tower of KK modes associated with a state of charge σ in the

Weyl–Cartan basis and of mass "shifted" by $\rho(\sigma)$ real. Here $\sigma = \lambda, \alpha$ are the weights/roots belonging to the representation r. The sum over m runs over all non-zero integers and accounts for the effects of KK modes of mass $M_m(\sigma)$ associated with the compact dimension. The regulated sum over the KK tower can be performed and the gauge group dependent piece is regularization independent.

If the gauge symmetry group G is a Grand Unified group before the Wilson lines are "turned on", the overall divergent part of Ω_i^* is the same for all group-factors i that G is broken into. As a result the σ independent part of Ω_i^* can be absorbed into the redefinition of the tree level coupling, similar to the case with one compact dimension. The resultant form for the gauge couplings is [24]

$$\frac{1}{\alpha_i(R)} = \frac{1}{\alpha_o} - \sum_r \sum_{\sigma=\alpha,\lambda} \frac{\beta_i(\sigma)}{4\pi} \ln \frac{|\sin \pi \Delta\rho_\sigma|^2}{\pi^2 \rho_\sigma^2}, \qquad \rho_\sigma = -R\,\sigma_I \langle A_y^I \rangle\,, \quad (18)$$

where $\rho = [\rho] + \Delta_\rho$, $[\rho] \in Z$ One only needs to add here the contribution of zero modes (before Wilson line breaking), not included in Ω_i^* and whose presence is in general model dependent.

In the limit of turning off the Wilson lines vev's $\Delta_\rho = \rho_\sigma \to 0$ the correction in (18) coming from the KK excitations vanishes and no splitting of the gauge couplings is generated. The interesting case is what happens when ρ_σ is non zero.

For the case of continuous Wilson lines the breaking can be continuously taken to zero. The Wilson line acts in the same way as a physical Higgs field. providing the longitudinal component of the broken gauge bosons, forming a massive $N = 2$ supermultiplet. Thus for continuous Wilson lines there is also a contribution from the $(m = 0)$ would-be zero mode which acquires a mass ρ_σ/R after Wilson line breaking. Including it gives the result

$$\frac{1}{\alpha_i(R)} = \frac{1}{\alpha_o} - \sum_r \sum_{\sigma=\alpha,\lambda} \frac{\beta_i(\sigma)}{4\pi} \ln \frac{|\sin \pi \Delta\rho_\sigma|^2}{\pi^2 M_{string}^2 R^2}\,. \quad (19)$$

For the case that $\Delta\rho_\sigma$ is small this corresponds to a contribution $\frac{\beta_i(\sigma)}{4\pi} \ln (\Delta\rho_\sigma/M_{string}R)^2$. The interpretation of this is straightforward. As may be seen from Eq. (16) it corresponds to the contribution of the KK state that has been made anomalously light, with mass $\Delta\rho/R$ through a cancellation of the Wilson line contribution to the mass and the contribution associated with a non-zero KK level. The string imposes a cutoff M_{string} on the one loop contribution of this state to the gauge coupling evolution. The contribution from all the higher levels can be seen to be small corresponding also to the to the string cutoff implicit in Eq. (17).

4.2.2. *Two compact dimensions*

Assuming constant $A_{y_{1,2}}$, one computes the Wilson lines operator W_i of Eq. (13) corresponding to each one-cycle γ_i of the two torus of compactification, $\rho_{i,\alpha}$ [24],

$$\rho_{1,\alpha} = -R_1 \alpha_I \langle A^I_{y_1} \rangle, \quad \rho_{2,\alpha} = -R_2\, \alpha_I \left[\langle A^I_{y_1} \rangle \cos\theta + \langle A^I_{y_2} \rangle \sin\theta \right], \tag{20}$$

where θ is the angle between the two cycles and is fixed by the type of orbifold considered ($\theta = 2\pi/N$ for T^2/Z_N compactifications). Using the Klein-Gordon equation in 6D we find the mass of the 4D KK fields in the adjoint (α) and fundamental (λ) representations as [24]

$$M^2_{n_1,n_2}(\sigma) = \frac{\mu^2}{T_2\, U_2} \, |n_2 + \rho_{2,\sigma} - U(n_1 + \rho_{1,\sigma})|^2, \quad \sigma = \alpha, \lambda\,. \tag{21}$$

with the notation familiar in string models

$$U \equiv U_1 + iU_2 = R_2/R_1\, e^{i\theta}, \quad (U_2 > 0); \qquad T_2(\mu) = \mu^2 R_1 R_2 \sin\theta\,. \tag{22}$$

For $\theta = \pi/2$ the two compact dimensions "decouple" from each other and in this case one finds $M^2_{n_1,n_2}(\sigma) = (n_1 + \rho_{1,\sigma})^2/R_1^2 + (n_2 + \rho_{2,\sigma})^2/R_2^2$.

For generality we keep the θ angle arbitrary. If the gauge symmetry group G is a grand unified group before the Wilson lines are "turned on", the overall divergent part of Ω_i will be the same for all group-factors i that G is broken into. As a result the σ independent part of Ω_i^* can be absorbed into the redefinition of the tree level coupling, similar to the case with one compact dimension. In that case one obtains the following splitting of the gauge couplings in 4D [24]

$$\frac{1}{\alpha_i} = \frac{1}{\alpha_o} + \sum_r \sum_{\sigma=\alpha,\lambda} \frac{\beta_i(\sigma)}{4\pi} \{ \ln \frac{\pi e^\gamma |\rho_{2,\sigma} - U\rho_{1,\sigma}|^2}{(R_2 \sin\theta)^2} \\ - \ln \left| \frac{\vartheta_1(\Delta_{\rho_{2,\sigma}} - U\Delta_{\rho_{1,\sigma}}|U)}{\eta(U)} \right|^2 + 2\pi\, U_2 \Delta^2_{\rho_{1,\sigma}} \}\,, \tag{23}$$

where the functions ϑ_1 and η are defined in [24] and with

$$|\rho_{2,\sigma} - U\rho_{1,\sigma}|^2/(R_2 \sin\theta)^2 = |\sigma_I \left(\langle A^I_{y_2} \rangle - i \langle A^I_{y_1} \rangle \right)|^2\,. \tag{24}$$

As in the one dimensional case, the splitting of the gauge couplings is induced by the combined effects of the compact dimensions and Wilson lines vev's in a particular direction in the root space of the initial gauge group G. One may need to add to the above equation the correction from a zero mode $(0,0)$ which acquires a mass after Wilson line breaking and the massless states. Only a logarithmic correction will then be present with the power-like corrections (divergences) "absorbed" into $1/\alpha_o$.

For the case that only $\rho_{2,\sigma}$ is non-zero the correction approximately reduces to the form of Eq. (19) after absorbing σ independent terms in a redefinition of the coupling.

4.3. *Gauge Coupling Unification with Wilson line breaking*

In this section we will use the results discussed above to determine the effects of Wilson line breaking on gauge coupling unification. These corrections have been determined in the context of the weakly coupled heterotic string with orbifold compactification but the general structure is indicative of the effects in more general compactifications because it is driven by states which are made anomalously light by Wilson line breaking and this happens in general compactification schemes.

4.3.1. *Kaluza–Klein gauge excitations*

We start with a discussion of the effect of the gauge sector of KK modes. The Wilson lines do not affect the gauge boson excitations associated with the unbroken gauge group. The remaining X and Y gauge boson excitations acquire mass corrections according to the form given in Eq. (21).

For the case of one additional dimension the contribution of the massive X and Y gauge bosons is given by Eq. (19). On the other hand the contribution of the KK modes of the unbroken gauge bosons is given by Eq. (18) with $\rho_{3,2,1} = 0$ and the normalization chosen is such that this vanishes. The X and Y contribute to the relative gauge evolution in the opposite way to that of the $3,2,1$ gauge bosons. From Eq. (19)[j] we see they contribute between an "effective mass scale" given by $\frac{|\sin \pi\Delta\rho_\sigma\,|}{\pi R}$ and the cutoff scale M_{string}. Once they start to contribute the relative evolution of the gauge couplings ceases corresponding (up to matter contributions) to a reduction in the unification scale by the factor $|\sin \pi\Delta\rho\,|\,/\pi R M_{string}$. Such a reduction is what is needed if the unification scale in the weakly coupled heterotic string is to agree with the gauge coupling unification scale. Note that this is a general conclusion independent of the initial gauge group or the Wilson line.

[j] Although we are considering discrete Wilson lines the equation still applies because the $m = 0$ field still contributes to the first massive level after Wilson line breaking

Given the importance of this systematic trend it is of interest to see how it arises directly from the form of the spectrum in Eq. (21). From this equation it is clear that the effect of the Wilson line is systematically to shift pairs of states with opposite signs of $n_{1,2}$ up and down in mass keeping the mean, m, unchanged. However in field theory calculation. In this the logarithmic corrections coming from individual massive states come in pairs with mass $m+\rho$ and $m-\rho$ giving the contribution $\log(m+\rho)+\log(m-\rho) = \log(m^2-\rho^2)$. We see that the lighter state dominates and the net effect is a reduction from m^2 to $m^2 - \rho^2$ in the effective mass squared at which the states start to contribute and systematically reducing the unification scale. In string theory the string regularization further reduces the contribution of the more massive state going further in the direction of reducing the unification scale.

In the calculation of the precise contribution of the massive states it is necessary to compute their beta functions. As noted above, in the calculation of the relative evolution of the gauge couplings, the contribution of the X and Y gauge bosons is the same magnitude as the contribution of the $SU(3) \otimes SU(2) \otimes U(1)$ Standard Model gauge bosons but has the opposite sign. The contribution to the gauge coupling running from massive KK modes comes only from the $N = 2$ supermultiplets, the $N = 4$ supermultiplets do not contribute at all. An $N = 2$ gauge supermultiplet contributes only 2/3 of the contribution of an $N = 1$ gauge supermultiplet because it includes both an $N = 1$ gauge supermultiplet and an $N = 1$ chiral supermultiplet. The number of KK excitations contributing to the beta function is model dependent, depending on how many (large) bulk dimensions the gauge field propagates in and whether the KK modes fill out $N = 2$ or $N = 4$ representations. To determine the number we need to know the specific string theory. In the absence of this the best we can do is to estimate the magnitude of the reduction in the string prediction for the unification scale to be expected from Wilson lines, parameterizing our ignorance in the specific number by varying the number of KK excitations. In this we are helped by the fact we are working in the limit where R_c is larger than R_s. As discussed in [25], in this limit one can use either the full string theory or an effective field theory to determine the KK spectrum.

4.3.2. *Kaluza–Klein matter excitations*

The minimal set of matter fields is that of the MSSM with 3 generations of quarks and leptons and two Higgs doublets. Again the structure of their KK excitations is model dependent depending on whether the propagate in the bulk and if so in how many dimensions. In particular if they correspond to twisted states about orbifold fixed points, they have no KK excitations. We consider the various possibilities in turn.

Twisted matter. If all the matter fields correspond to twisted states then only the gauge KK modes need be included. As discussed above the effect of the Wilson lines is to reduce the scale at which the gauge contribution runs. However the contribution of the matter fields is still cutoff at the string scale so there is a mismatch between these scales. It is straightforward to determine the net effect. At one loop order the quarks and leptons fill out complete representations of $SU(5)$ and so do not change the relative evolution of the gauge couplings which determine the unification scale and the precise value of one of the three gauge couplings at low scales. For these predictions, at this order, only the Higgs zero mode contribution and the contribution of the gauge bosons need be included. If the Higgs contribution were cut-off at the same reduced scale as the gauge bosons the prediction would be just that in the MSSM with a reduced cut-off scale. However the Higgs contribution is not cutoff and its contribution must be included between the reduced unification scale and the original cut-off scale. This changes the gauge coupling evolution by causing the electroweak coupling to run more slowly. As a result, if the strong coupling is still to unify with the other couplings, its value at low scales must be increased. In the MSSM the value needed for the strong coupling is already somewhat larger than the measured value so this change goes in the wrong direction. For this reason we do not consider this possibility further.

Quark, lepton Kaluza–Klein modes. If the quarks and leptons all have Kaluza–Klein modes the situation is more complicated as they all contribute to gauge coupling running. However for the case of (discrete) Wilson lines their effect on the relative evolution of the coupling constants vanishes because the massive quark and lepton modes fill out complete multiplets of $SU(5)$ which are degenerate. The reason is that these multiplets carry the same $(D \oplus \overline{D})$ intrinsic charge as is necessary if they are to give complete multiplets of zero modes. As a result the massive excitations also have the same dependence on the compactified coordinates and hence the same mass.

Higgs Kaluza–Klein modes. If the Higgs fields are untwisted states they have KK excitations whose effects need to be included. We consider the case that the doublet triplet splitting is due to discrete Wilson line breaking. There are two cases to consider.

If the Higgs doublet fields are discrete group, D, singlets they must also be $\overline{D}$ (Wilson line) singlets. This means that for them ρ_σ is zero in Eq. (20) and so their KK excitations are unshifted. However this is not the case for their color triplet partners which are not D singlets. At one loop order the contribution of the color triplet KK contribution to the relative gauge coupling evolution acts in the opposite way to the Higgs doublets (the two together give no one loop contribution). As a result the colour triplet states reduce the unwanted increase in the strong coupling just discussed coming from the contribution of the light Higgs above the gauge boson cutoff scale. The massive color triplet states belong to $N = 2$ supermultiplets which means that they consist of two $N = 1$ chiral multiplets and thus they have a beta function coefficient of magnitude twice that of the Higgs (but opposite in sign). As a result they can readily dominate over the "excess" Higgs contribution above the reduced gauge boson unification scale and actually reduce the value of the strong coupling, bringing it into better agreement with experiment. We will present numerical estimates of this effect below.

The other possibility is that the Higgs doublet fields come from non singlet D fields. In this case the net effect of the KK doublet and color triplet fields is model dependent as both may contribute to the running of gauge couplings.

4.3.3. *Wilson line breaking of $SU(5)$.*

Our discussion to date applies to a general Grand Unified gauge group, G, before Wilson line breaking. However when making a quantitative estimate of the effects we will illustrate the effects to be expected by considering the case that $G = SU(5)$ and in this case the discrete group is restricted to be Z_3 [26]. The Wilson line group element is $Diagonal(a^2, a^2, a^2, a^{-3}, a^{-3}) = e^{iY\theta}$ where $Y = Diag[2, 2, 2, -3, -3]$.This breaks $SU(5)$ to $SU(3)\otimes SU(2)\otimes U(1)$ giving the X and Ygauge bosons a mass. The condition that this Wilson line should be a representation of Z_3 is $\theta = 2\pi n/3$. In this case the Higgs doublet fields are clearly $\overline{D}$ singlets as required.

For clarity of presentation we discuss the case of one additional dimension but it is easy to generalize it to the case of two additional dimensions using the results given above. To include the effects of the KK modes we use Eq. (18). For our $SU(5)$ example with Wilson line breaking we have $\rho_{X,Y} = 5/3R$. At one loop order this affects the running of the couplings and hence the unification predictions

$$\alpha_i^{-1}(Q) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi}\log\frac{Q}{M_X} - \frac{2}{3}\frac{b_i(3,2,1)}{4\pi}\ln\frac{\left|\sin\pi\Delta\rho_{24_{X,Y}}\right|^2}{\pi^2 M_s^2 R^2}$$
$$+2\sum_{Higgs}\sum_j \frac{b_i(5_j)}{4\pi}\ln\frac{\left|\sin\pi\Delta\rho_{5_j}\right|^2}{\pi^2 M_s^2 R^2} \equiv \alpha_{GUT}^{-1} + \frac{b_i}{2\pi}\log\frac{Q}{M_X} + \frac{b_i'}{2\pi} \quad (25)$$

where the one loop beta function coefficient, b_i, is just that of the MSSM, $b_i(3,2,1)$ is the coefficient coming from the unbroken (MSSM) gauge supermultiplet, $b_i(5_j)$ is the coefficient coming from the Higgs supermultiplet sector and the sum $j = 1, 2$ is over the $(2, 1)$ and $(1, 3)$ $SU(2) \otimes SU(3)$ components of the 5. In this equation we have absorbed terms independent of the group factor, i, in α_{GUT}^{-1} and we have $b_i = b_i(3, 2, 1) + b_i(5_1)$. The scale M_X is the unification scale taking account of the KK thresholds (of course in the string we expect $M_X = M_s$).

We wish to determine the change in the unification scale and strong coupling in this scheme relative to the MSSM in which

$$\alpha_{MSSM,i}^{-1}(Q) = \alpha_{MSSM,GUT}^{-1} + \frac{b_i}{2\pi}\log\frac{Q}{M_X^0}\,, \quad (26)$$

where M_X^0 is the unification scale in the MSSM. In both cases $\alpha_{1,2}(M_Z)$ are input as their measured values. The unification scale is found from the relative evolution of $\alpha_{1,2}$. Combining Eqs. (25,26), we find

$$\log\left(\frac{M_X}{M_X^0}\right) = \frac{b_2' - b_1'}{b_2 - b_1} \quad (27)$$

and

$$\Delta\alpha_3^{-1} = -\frac{1}{b_1' - b_2'}\left[(b_2' - b_3')\Delta b_1 + (b_3' - b_1')\Delta b_2 + (b_1' - b_2')\Delta b_3\right]\frac{1}{2\pi}\log\left(\frac{M_X^0}{M_X}\right), \quad (28)$$

where Δb_i are defined to be

$$\Delta b_i = b_i' - b_i\,. \quad (29)$$

As discussed above the massive matter multiplets in a given representation of $SU(5)$ are degenerate and so do not contribute to Δb_i. Thus in determining M and $\Delta\alpha_3^{-1}$ the only contributions are the KK modes of the gauge bosons and the Higgs multiplets.

In Table 1 we give the results for the case $M_s R = 2$. This the smallest value consistent with our neglect of winding modes and yet not so small that

	$n=1$	$n=2$
$\frac{M_X^0}{M_X}$	4.1	5.9 (24)
$\Delta\alpha_3^{-1}$	-0.27	-0.34 (-0.51)
$\frac{M_X^0}{M_X}$	3.1	4.7 (14.7)
$\Delta\alpha_3^{-1}$	0.54	0.3 (0.85)

Table 1 The change in the string prediction for the unification scale and the strong coupling for $M_sR = 2$. The Wilson line group element, Eq. (16), is specified by $a = e^{2\pi n/3}$ and the result for two choices for the Wilson line (n=1 and n=2) are shown. Also shown in parenthesis is the result including the light winding mode which occurs for n=2 only. The first two rows give the results for the case the Higgs multiplets have no KK modes and the last two rows give the results for the case the Higgs have KK excitations. The calculation refers to the 1D case, Eq. (18) and is very close to the 2D case with $\rho_1 = \rho$ and $\rho_2 = 0$.

the precision is greatly reduced. The numbers quoted apply to the case of a $N = 2$ massive spectrum in one extra dimension using the form of Eq. (25). The results for additional extra dimensions are similar and can be found in [27].

From the Table one may see that, as expected, the unification scale is increased bringing it closer to the string prediction. Without Higgs KK modes there is an increase in the strong coupling which makes the agreement with experiment worse. However with Higgs KK modes the strong coupling is decreased improving the fit (the correction $\Delta\alpha_3^{-1} = 1.1$ brings the strong coupling into excellent agreement with experiment).

The result is very sensitive to the form of the Wilson line for two reasons. For $n = 2$ the $m = 0$ "would-be"zero mode state is made heavier than the string scale and its contribution is strongly suppressed. In this case we should use Eq. (18) rather than Eq. (19) leading to the enhancement shown in the second column of the Table. Moreover for $n = 2$ and $M_sR = 2$ a winding mode is made lighter than the string scale and its effect is no longer negligible. Naively including its effect using the field theory estimate gives the results in brackets in the Table. In this case both the unification scale and the strong coupling are in excellent agreement with the measured values.

Although this calculation has been done in a field theory context with no specific string compactification the result is very encouraging for the weakly coupled heterotic string. It would certainly be very interesting to perform the full string theory calculation for a realistic heterotic string compactification.

5. A profile of the supersymmetric extension of the Standard Model

The requirement that the unification prediction obtained using the spectrum of Standard Model gauge nonsinglet states of the Minimal Supersymmetric version of the Standard Model should not be spoiled yields a remarkably clear picture of the low energy phenomenology to be expected if the structure emerges from a string theory. However, as we now discuss, this does not imply that the full structure of the MSSM be recovered.

The first implication is that the scale of unification, compactification and the associated string scale must be very large, close to the Planck scale. As a result one does not expect to produce at low energies any low energy string states or Kaluza–Klein states and so must be content with less direct string effects such as the small corrections to the gauge couplings and the unification scale. These are corrections to the renormalizable terms of the Standard model and are not suppressed by inverse powers of the string scale. However some processes are so rare that even corrections suppressed by inverse powers of the string scale may be significant.

5.1. *Nucleon decay*

The most sensitive of these is nucleon decay. In a supersymmetric generalization of the Standard Model there are new sources of nucleon decay through processes involving squarks and sleptons. In particular, in addition to the usual terms giving mass to quarks and leptons, one may have the additional gauge invariant terms [28] of the form given in Eq. (30).

$$L^{Yuk}_{R_P violating} = \underset{\Delta L = 1}{\lambda_{ijk}\left[l_i\ l_j\ E^c_k\right]_F} + \underset{\Delta L = 1}{\lambda'_{ijk}\left[l_i\ q_j\ d^c_k\right]_F} + \underset{\Delta B = -1}{\lambda''_{ijk}\left[u^c_i\ d^c_j\ d^c_k\right]_F} \,. \tag{30}$$

These have the property that they violate R-parity, allowing single production of supersymmetric states and allowing the decay of the lightest supersymmetric particle. However, they also have property that they violate Baryon number and/or Lepton number and all the terms of Eq. (30) can not be simultaneously present if proton decay is not to proceed at an unacceptably fast rate. The first point to note is that if the terms proportional to λ' and λ'' are simultaneously present the nucleon is unstable through a tree level graph with a down squark exchanged. Since the amplitude is proportional to $(\lambda'\lambda'')/m^2_{\tilde{d}}$, and the squark mass is at most in the TeV range, the decay rate is quite unacceptably fast. The cure is to forbid these new terms by a discrete symmetry. In the MSSM this is done by a Z_2 discrete symmetry known as matter parity under which the quark and lepton superfields appearing in the superpotential change sign while the Higgs superfields are left invariant. Thus the last three terms of Eq. (30) change sign under this symmetry and are forbidden while the terms giving rise to quark and lepton masses having only two matter fields are invariant and allowed [29, 30]. Clearly this forbids all the terms of Eq. (30) and leaves just the couplings of the MSSM. Thus the matter parity leads to the $R-$parity of the MSSM given by Eq. (31),

$$R_p \equiv (-1)^{3B+L+2S}\,. \tag{31}$$

However, there are more possibilities to stabilize the proton than to forbid all the terms of Eq. (30) [28]. Provided the terms proportional to λ and λ' are not simultaneously present the graph generating proton decay will be absent. It is possible to eliminate one or other of these operators by symmetries other than matter parity provided one allows for the possibility that quarks and leptons transform differently. Although this is not possible if the theory is embedded in SU(5), in which the quarks and leptons transform under discrete symmetries in the same way, it is possible in other GUT's and also in string unification, which need not be embedded in a GUT. Indeed a study of discrete Z_N symmetries shows that it is easy to obtain any of the following, all of which inhibit nucleon decay [31]

- Matter parity. $\lambda = \lambda' = \lambda'' = 0$; $\Delta B = \Delta L = 0$
- Lepton "parity". $\lambda = \lambda' = 0$; $\Delta B \neq 0, \Delta L = 0$
- Baryon "parity". $\lambda'' = 0$; $\Delta B = 0, \Delta L \neq 0$

It is clear there are many possibilities; indeed allowing for flavor dependence there are 45 different operators possible in Eq. (30). Obviously it would be useful to limit the possible new terms and various constraints have been considered. In the context of an underlying string theory the possible Z_N symmetries are constrained by the condition that they should be discrete gauge symmetries and satisfy the discrete anomaly free condition. It has also been argued from purely phenomenological considerations that discrete symmetries should be discrete gauge symmetries if they are to avoid large violation through gravitational effects [32]. A study of all possible discrete symmetries for $N < 4$ shows that there are only two discrete anomaly free symmetries, the $R-$parity of the MSSM and a new Z_3 Baryon parity. These have vastly different implications for nature of supersymmetric phenomenology so it is clearly of importance to consider whether one or other is to be preferred.

One distinguishing feature is nucleon decay for it can also proceed via higher dimension operators. The original diagram suggested in SU(5) generating proton decay involves vector boson exchange leading to the amplitude $\sim \frac{1}{M_x^2} qq q\ell$ and the decay rate $\Gamma_{p\to\pi^0 e^+} \propto \frac{m_p^5}{M_X^4}$. Since the amplitude is due to the exchange of a new vector boson X_μ the amplitude is dimension 6 and is suppressed by two powers of the X-boson mass. Allowing for couplings of order one, the current bound on the proton lifetime leads to the bound on the X- boson mass, $\mathrm{M}_X > 1.5.10^{15}$ GeV. In supersymmetry there are also new contributions beyond the dimension 4 terms discussed above which appear through the dimension 5 operators, $[QQQL]_F$, $[\overline{U}\ \overline{U}\ \overline{DE}]_F$. In these it is the exchange of a colored Higgsino triplet that leads to proton decay and as a result the amplitude is only suppressed by one power of the heavy fermion mass (a dimension 5 operator). The bound on this mass scale is some 8 . 10^{23} GeV if the couplings involved are of order one. Thus if this scale is associated with the Grand Unified scale, then there must clearly be some suppression due to small couplings. For example, in a supersymmetric Grand Unified theory, the graph involves Yukawa couplings to the light quarks which are so small that the proton decay rate prediction is acceptable. In this case the decay mode for this second class of diagram is largely into strange quarks because of the larger Yukawa coupling to heavy quarks, leading to the dominant decay modes [33]:

$$\begin{array}{l} p \to K^\dagger \overline{\nu}_\mu,\ \pi^+\nu,\ K^0\mu^+ \\ \qquad\quad 1 \quad\ : 0.5 \quad : 0.007 \\ n \to K^0\overline{\nu}_\mu,\ \pi^0\overline{\nu}_\mu \\ \qquad\quad 1.8 \quad : 0.24 \end{array}$$

The overall rate is model dependent. In a realistic SO(10) model discussed in [34] it is very close to the present limit. In other models such as flipped SU(5) [35] the dimension 5 operators are suppressed and the dimension 6 operators are dominant, leading to the prediction that the proton should decay into $\pi^0 e^+$as in the original non-supersymmetric SU(5). Again the rate may be expected to be close to the upper bound.

What is the expectation if the underlying theory is a string theory? Since the dimension 5 operators with $O(1)$ couplings require a scale of $\mathrm{O}(10^{24}$ GeV) to keep the rate within experimental bounds one may be worried that corrections at the Planck scale will lead to unacceptably fast processes; for example black hole evaporation or wormhole production [36]. Since there is no reason to expect a small coupling in these processes they may give an amplitude which is too large. Similarly in a superstring theory couplings that are allowed by the symmetries are typically of order one and again in this case one would obtain unacceptably fast proton decay from the exchange of heavy string states.

Given this it is clear that there are two possible ways to avoid the problem of rapid nucleon decay in a string theory. If the underlying symmetries allow dimension 5 nucleon decay operators we should ensure that the Yukawa couplings involved in the nucleon decay process are indeed small and this can be guaranteed through a new family symmetry respected by the underlying string physics [37]. This is the strategy that must be adopted in the case of the MSSM with the Z_2 matter parity. The second possibility is that the symmetries forbid the theory should have a symmetry which kills the dimension 5 operators once and for all. In this case one is left with the dimension 6 operators as the dominant source of nucleon decay. A very simple realization of this is given by the Z_3 Baryon parity which, while allowing lepton number violating processes at renormalizable order, forbids both dimension 4 and dimension 5 baryon number violating operators.

So far as the implications for low energy phenomenology, the need to inhibit proton decay leads to two distinct possibilities which are characterized by the differences between models based on Z_2 matter parity and Z_3 baryon parity. In the case of matter parity all the new SUSY states are all odd so they appear quadratically in the Lagrangian with the important phenomenological implication that the super-partners of Standard Model states can only be produced in pairs and that the lightest supersymmetric particle (LSP) will be absolutely stable and indeed a candidate for dark matter. Both baryon and lepton number violation occurs through dimension 5 operators suppressed by only a single inverse power of the unification mass giving strange particle final states for nucleon decay.

In the alternative case of Z_3 baryon parity, nucleon decay is first given by dimension 6 operators. In the flavored case that the string theory does not have a stage of unification below the compactification scale such nucleon decay processes will be too slow to measure. On the other hand the LSP is now unstable leading to many more possibilities for low energy SUSY phenomenology [38] because the LSP may now be charged as it does not contribute to dark matter. For example in a large region of the parameter space the stau, the supersymmetric partner of the tau lepton, is the LSP. This means the usual signature for supersymmetric particle production, namely large missing transverse energy carried by neutral weakly interacting LSPs which escape detection, is absent as essentially all the energy associated with superpartner decay ends up in charged matter. it will be important to search for such processes at the LHC.

Of course one of the most interesting aspects of the MSSM is that it does produce a dark matter candidate and that for a range of the parameter space the dark matter abundance is consistent with observation. Models with Z_3 baryon parity alone do not have a dark. matter candidate and so in them the source of dark matter is uncertain. In fact it is easy to keep the advantages of both Matter parity and Baryon parity if the string theory possesses an underlying $Z_2 \times Z_3$ discrete gauge symmetry. In this case the leading proton decay amplitude is dimension 6 but the LSP is stable as in the MSSM and the LSP is a dark matter candidate. Moreover there are no dimension 4 lepton number violating terms, again as in the MSSM. The direct SUSY signals of this model will be the same as the MSSM, the only phenomenological difference is the absence of nucleon decay mediated by dimension 5 operators. Given the magnitude of the unification scale it is unlikely nucleon decay from dimension 6 operators will be visible in this scheme.

5.2. *Supersymmetry breaking*

One of the essential ingredients leading to a successful prediction of gauge coupling unification is the presence of new supersymmetric partners of the Standard Model states. Their spectrum is determined by soft supersymmetry breaking terms and this largely determines the supersymmetric signals to be expected at the LHC. Remarkably the condition that the precision prediction for the gauge couplings should not be spoiled severely limits the possible supersymmetry breaking mechanism. The reason is that one cannot add even complete $SU(5)$ multiplets of states to the theory without changing the prediction for the gauge unification. This means that the messenger sector, which communicates supersymmetry breaking to the Standard Model states, should not involve gauge non-singlet states, even in complete multiplets. To quantify this we note that the effects of multiplets belonging to vectorlike $5 + \overline{5}$ representations or $10 + \overline{10}$ representations is determined by n where $n = (\#5 + \#\overline{5} + 3\#10 + 3\#\overline{10})/2$. For moderate n, $n \lesssim 7$, the main effect is to increase α_s, taking it further from the experimental value. The effect is dependent on the mass of the new states but goes up to $\alpha_s = 0.137$ in the limit the unified coupling (which is made larger by the new states) is just in the perturbative domain [39]. The value of the unification scale is increased but this effect is largely cancelled by the increase in the value of the unified coupling which in turn increases the string prediction for the unification scale. For larger n the increase in the strong coupling can be a little less but typically the unified coupling enters the non-perturbative domain making the prediction for the unification scale problematic [40]. Overall the net effect of the new states is to worsen the fit to the unification predictions and to reduce the accuracy with which this prediction can be made.

Thus to preserve the accuracy of the predictions for the gauge coupling and the unification scale one must have supersymmetry breaking messenger mechanisms which do not involve gauge non-singlet fields. This suggests gravity mediation. One possible mechanism is anomaly mediation. However in this case one must be careful to solve the lepton tachyon problem by a mechanism that does not introduce gauge nonsinglet fields too [41]. To date the most promising suggestion is through the addition of the anomalous part of a $D-$term associated with a new $U_X(1)$ symmetry (e.g. $X = B - L$). If the underlying theory is a string theory this $U_X(1)$ should be gauged. To avoid the decoupling of the $D-$term the symmetry cannot be broken at a high scale. To avoid conflict with the non-observation of a new gauge symmetry one then has to make the gauge coupling, g_X, anomalously small. As a result one can only get the required magnitude of the anomalous part of the $D-$term through a tuning of the anomalous term ξ such that $g_X\xi$ is the supersymmetry breaking scale. Since there is no reason for this choice we consider this mechanism to be unlikely, disfavor anomaly mediation as the only effect. This then leaves the usual gravity mediated mechanism. In this case the usual assumption is that the gravity is flavor blind and at the Planck scale the gaugino masses are all equal to one common scale, $M_{1/2}$, and squark, slepton and Higgs masses are equal to another common scale, M_0 (These are both expected to be of order the gravitino mass $m_{3/2} = \Lambda^2/M_{Planck}$ where Λ is the supersymmetry breaking scale). A study of soft scalar mass terms in simple orbifold heterotic string compactifications yielded the form

$$m_i^2 = m_{3/2}^2(1 + n_i \cos^2\theta)\,, \tag{32}$$

where n_i are the modular weights of the scalar fields, $m_{3/2}$ is the gravitino mass, and the goldstino angle θ parameterizes the contribution of the $F-$terms of the dilaton and radius moduli fields S and T (assuming for simplicity a common radius) to SUSY breaking. An acceptable vacuum structure is inconsistent with the dilaton dominance limit ($\theta = \pi/2$) [42] and so one must consider the contribution of the T field. However as can be seen Eq. (32) this need not be family independent so the usual flavor blind supergravity assumption does not necessarily apply in string compactifications. In this case another constraint may be necessary, the most plausible being that there is a non-Abelian family symmetry forcing the flavor blind structure. This leads to a more general parameterization of the SUSY spectrum than is assumed in the SUGRA case but one in which the flavor changing neutral current effects are still within experimental limits. The extended parameter space this opens up can lead to interesting new phenomenological signals for supersymmetry [43].

5.3. *Flavor changing processes and the structure of quark and lepton mass matrices*

One of the striking features of the properties of matter states is the hierarchical pattern of quark masses and mixing angles and the hierarchical pattern of the charged lepton masses. In the Standard Model this structure is put in by hand but in a more unified theory one hopes the structure will emerge naturally. There have been various suggestions for generating this structure. One possibility is that there is an underlying family symmetry which, when unbroken, allows only the third family to acquire a mass. The symmetry is then spontaneously broken by one, or more, familon fields θ which carry family quantum numbers and generate the Yukawa couplings and associated mass matrix structure through higher dimension operators of the form coming from the superpotential W given by

$$W = \left(\frac{\theta}{M}\right)^{\alpha_{ij}} H_a Q_{Li} q^c_{Rj} \,. \tag{33}$$

Another suggestion is that the hierarchy is generated through a spatial separation of the matter and Higgs fields in one or more of the compactified dimensions [44, 45],

$$Y_{ijk} = h_{qu} \sum_{\vec{n} \pm \in H_2^D(\mathcal{M}, \cup_a \Pi_a, ijk)} d_{\vec{n}} \; e^{-\frac{A_{ijk}(\vec{n})}{2\pi\alpha'}} e^{-2\pi i \phi_{ijk}(\vec{n})}. \tag{34}$$

The meaning of the various terms is defined in [45] but the most important contribution comes from the exponentiation of $A_{ijk}(\vec{n})$, which is the target-area of the "triangle" with vertices at the location in the higher dimensional space of the three participating fields. The Yukawa couplings will then have an exponential suppression as the separation between the fields becomes large, corresponding to the need to stretch the string between participating states.

In both these cases the requirement we discussed in the previous section that supersymmetry breaking should be mediated by gravity leads to the conclusion that the structure of Yukawa couplings is significantly restricted. The reason is that Yukawa couplings giving rise to quark and lepton mixing between different families also gives rise to flavor changing neutral currents (FCNC) and (flavor conserving) CP violating effects which are known to be strongly suppressed. In supersymmetric theories these effects can be significantly enhanced because the squarks and sleptons also contribute to FCNC.

For the case the Yukawa structure comes from a broken family symmetry a very significant source of such effects comes from the fact that the θ field(s) acquire non-vanishing $F-$terms, $F_\theta = \beta m_{3/2} < \theta >$. A study of various models shows that the expectation is that $\beta = O(1)$ [46, 47], although in models with numerous intermediate scales of symmetry breaking a suppression is possible. The $F-$term then induces trilinear soft supersymmetry breaking terms

$$A_{ij}\hat{Y}^{ij} = F^\eta \hat{K}_\eta Y^{ij} + \alpha_{ij}\ \frac{e^{K/2}}{M}\left(\frac{\theta}{M}\right)^{\alpha_{ij}-1}\ \beta m_{3/2}\theta \tag{35}$$

with $\hat{K}_\eta = \partial\hat{K}/\partial\eta$ where K is the Kähler potential and $Y^{ij} = e^{K/2}(\theta/M)^{\alpha_{ij}}$ are the Yukawa couplings. From Eq. (35) we see that, in any model which explains the hierarchy in the Yukawa textures through non-renormalizable operators, the trilinear couplings are necessarily nonuniversal. Moreover, due to the factor α_{ij} these trilinear terms are not diagonalized at the same time as the Yukawa couplings and hence the fermion masses are diagonalized. They thus give rise to FCNC. Moreover, even in the most conservative case where all soft SUSY breaking parameters and μ are real, we know that the Yukawa matrices contain phases $\mathcal{O}(1)$. If the trilinear terms are nonuniversal, these phases are not completely removed from the diagonal elements of Y^A in the SCKM basis and hence can give rise to large EDMs [47, 48].

The most significant bound on the Yukawa couplings is provided by the mercury Electric Dipole Moment (EDM) bound. In particular it imposes a significant constraint on the Yukawa coupling matrix elements below the diagonal in the $(3, 1)$ and $(3, 2)$ positions. This is particularly interesting because these are the terms responsible for right handed mixing which are very poorly constrained in non-supersymmetric theories due to the absence of right-handed weak currents. Moreover in $SU(5)$ based models the (left handed) neutrino mixing angles are related to the down quark right handed mixing angles.. Demanding that the terms of Eq. (35) do not violate the experimental bounds on the mercury EDM constrains the $(3, 1)$ and $(3, 2)$ elements to be $\leq O((\theta/M)^3)$ and $\leq O(((\theta/M)^2)$ respectively and means they should be no larger than the $(1, 3)$ and $(2, 3)$ elements It is also in conflict with $SU(5)$ based models for large neutrino mixing angles being relates to large down quark right-handed mixing angles.

The trilinear couplings implied by Eq. (35) also contribute significantly to quark and lepton flavor violation. With the Yukawa coupling necessary to generate the fermion mass structure the $b \to s\gamma$ rate should be close to the present bound. Even more interesting is the lepton flavor violation process $\mu \to e\gamma$. If there is an underlying relation between charged lepton and down quark mass matrices we expect off diagonal elements in the charged lepton Yukawa couplings which will lead to lepton flavor violation. To generate the correct muon and electron mass we follow Georgi and Jarlskog's suggestion and put a relative factor of 3 in the $(2, 2)$ entry. We also put a factor of 3 in the $(2, 3)$ and $(3, 2)$ entries as is required by non-Abelian models which seek to explain the near equality in the down quark mass matrix of the $(2, 2)$ and $(2, 3)$ elements. With this to keep $\mu \to e\gamma$ at the level of current experimental bounds requires a slepton mass greater than 320 GeV. At this level $\mu \to e\gamma$ should be seen by the proposed experiments in the near future.

Although we have concentrated on the case there is an underlying family symmetry organizing the fermion mass hierarchy, similar conclusions apply in the case that the fermion hierarchy is generated through a spatial separation of the matter and Higgs fields in one or more of the compactified dimensions. The reason is that, as discussed above, the T field is expected to acquire a significant F term. Due to the target area dependence the matrix of Yukawa couplings of Eq. (34) has different T dependence in different matrix elements and this induces trilinear soft supersymmetry breaking terms which are not diagonalized when the masses are diagonalized and are similar in magnitude to those just discussed. The general conclusion is that unification schemes involving low scale SUSY, which we have argued are essential to the success of the gauge coupling unification prediction, leads to FCNC and CP violating process just at the present experimental limits.

6. Summary

Gauge coupling unification provides the only quantitative indication of physics beyond the Standard Model suggesting an underlying unified theory. The precision is such that requiring that it not be spoiled by heavy threshold effects severely limits the nature of the underlying theory. In the case of a string theory it requires that the inverse of the compactification radius be very large and close to the string or Planck scale to avoid power law running in the evolution of the couplings. This effectively rules out all models with large new space dimensions.

The fact that the gauge unification scale lies tantalizingly close to the Planck scale provides some evidence that this unification extends to include gravity. The original prediction of the unification scale in the weakly coupled heterotic string failed by an order of magnitude. However the prediction did not include the threshold effects associated with the massive string states and the unification scale, being related the argument of the logarithm in the gauge coupling evolution, is much more sensitive to such threshold effects than the prediction for the gauge coupling. Although the precise calculation of the correction awaits the construction of a fully realistic string theory, it is possible to estimate the magnitude of the threshold effects using an effective field theory approach. Remarkably this indicates that, if the breaking of the underlying unified gauge symmetry is via discrete Wilson lines, the correction to be expected systematically reduces the string prediction for the unification scale. This can readily bring the prediction into good agreement with the value needed for gauge coupling unification without spoiling the precision of the relation between the gauge couplings.

It is also possible determine the nature of the supersymmetric extension of the Standard Model (SSM) from the condition it should come from a string theory and also maintain the accuracy of the gauge coupling prediction. The condition that nucleon decay should be within experimental bounds suggests the MSSM needs to be modified. One possibility is that there is a new family symmetry which suppresses nucleon decay. In this case the dominant nucleon decay mode is into strange particles. A second possibility is that the dimension five nucleon decay is forbidden by a new symmetry such as baryon parity. In this case nucleon decay will be too slow to observe. However baryon parity allows the LSP to decay so supersymmetric phenomenology can be quite different from the MSSM; for example a charge lepton may be the LSP so missing momenta signals do not apply. It is also possible that both R-parity and Baryon parity are symmetries of the SSM in which case the phenomenology will return to that of the MSSM but proton decay will be unobservable.

Gravity mediated supersymmetry breaking is favored by the requirement that the precision prediction for gauge couplings be preserved. However the requirement that this breaking be flavor blind is questionable in the context of the underlying string theory. A non-Abelian family symmetry can force the flavor blind structure and leads to a more general parameterization of the SUSY spectrum than is assumed in the SUGRA case but one in which the flavor changing neutral current effects are still within experimental limits.

Finally $\mu \to e\gamma$ and dipole electric moments are likely to be very close to the present bounds. Keeping them under control in a theory with SUGRA mediated supersymmetry breaking strongly constrains the Yukawa structure and suggests the existence of a family symmetry. The constrained Yukawa structure limits the magnitude of the right handed mixing angles and is incompatible with explanations for large neutrino mixing relating it to the down quark right handed mixing.

It is encouraging that future experiments at the LHC and experiments looking for rare processes will be able to detect such signals which provide some further, albeit indirect, evidence for unification. This will only be the tip of the iceberg for the measurement of the detailed supersymmetric spectrum and properties will undoubtably shed further light on the nature of the underlying theory.

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