ONCE MORE ON WEAK RADIATIVE DECAYS

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We consider weak radiative decays of hyperons. It is shown that there exists an exact unitary lower bound for the decay probability, for example, $\mathrm{BR}(\Xi^- \to \Sigma^- \gamma) \geq 1.0 \cdot 10^{-4}$. We show that the real part of the amplitude is singular in the chiral limit, i.e. it contains terms $\sim \ln m_q$, where m_q is the current quark mass. The coefficient of the logarithmic terms is fixed uniquely. In the case of the decays $\Xi^- \to \Sigma^- \gamma$ and $\Omega^- \to \Xi^- \gamma$ the nonsingular (model-dependent) terms are relatively small, and it is possible to obtain a reasonably accurate estimate for the real part of the amplitudes $\Xi^- \to \Sigma^- \gamma$ and $\Omega^- \to \Xi^- \gamma$. Taking the real part into account, $\mathrm{BR}(\Xi^- \to \Sigma^- \gamma) \approx 1.7 \cdot 10^{-4}$.

1. Introduction and Formulation of the Problem

In the last ten years weak radiative decays of hyperons have repeatedly been discussed in the literature [1, 2]. So far there exists no adequate theoretical description of these processes. This is one of the reasons which stimulate theorists to return continually to this problem. Another reason for so much steady attention is the hope of obtaining nontrivial information on the structure of the weak interaction and on the structure of hadrons (hyperons).

In this note we will show that for the decays

$$\Xi^- \to \Sigma^- \gamma \qquad \text{and} \qquad \Omega^- \to \Xi^- \gamma \,, \tag{1}$$

there exist reliable and fairly accurate theoretical predictions. With some reservations one can say that there exists a theory of the decays in framework of which the amplitudes (1) can be expressed in terms of quantities which are either empirically or theoretically well known. In fairness one has to note that the analysis of the decays $\Xi^- \to \Sigma^- \gamma$ and $\Omega^- \to \Xi^- \gamma$ adds practically nothing new to what is already known about the weak interactions.

Before formulating the predictions and giving their derivation, we recall the general situation with respect to weak radiative decays. A short historical digression will serve as an introduction and will also contain a series of critical remarks on the literature.

The amplitude of the weak radiative decay has the form

$$M\big(B_i \to B_f \gamma\big) = -ie \int d^4x\, e^{iqx} \big\langle B_f | T\{H_W(0), J_\mu^{\rm em}(x)\} | B_i \big\rangle \varepsilon_\mu\,, \qquad (2)$$

where $J_\mu^{\rm em} = \frac{2}{3}\,\bar{u}\gamma_\mu u - \frac{1}{3}\,\bar{d}\gamma_\mu d + \ldots$ is the electromagnetic current, B_i and B_f are the initial and final baryon states, q_μ and ε_μ are the momentum and polarization of the emitted photon, and finally H_W is the Hamiltonian of the weak interaction.

There are two contributions to the amplitude (2) whose nature is essentially different. The first of these is connected with the emission of the photon at short distances $x \sim 1/M_W$ (M_W is the mass of the W boson) and can be expressed in terms of the local quark operator

$$T = i\,\bar{s}_R \sigma_{\mu\nu} d_L F_{\mu\nu}\,, \qquad (3)$$

which describes the transition $s \to d\gamma$ (the notation is self-evident). The coefficient of this operator is determined by the graphs of Fig. 1 and can be calculated reliably in the framework of QCD.

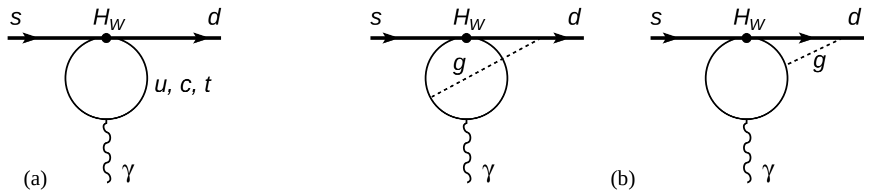

Figure 1. One- and two-loop diagrams which determine the local transition $s \to d\gamma$. The solid lines denote quarks, the wavy lines photons, and the dashed lines gluons.

Let us forget for a moment about gluon exchanges and concentrate on Fig. 1a. A structure of the type (3) can only arise from the region of virtual momenta $p^2 \sim M_W^2$. It is easy to convince oneself of this if one takes into account: a) gauge invariance with respect to the photon; b) the fact that the W boson interacts only with left-handed currents. It is obvious that the unitarity of the Kobayashi-Maskawa matrix leads to a cancellation of the u, c, and t contributions in leading orders, i.e. to order $G_F e/\pi^2$, where $G_F = 10^{-5} M_P^{-2}$ and $e^2/4\pi = 1/137$. This results in an extra power suppression of the type $(m_c^2 - m_u^2)/m_W^2$, and the graph of Fig. 1a is negligibly small. Taking gluons into account (Fig. 1b) removes the power suppression, since the factors $(m_c^2 - m_u^2)/m_W^2$ are replaced by $\ln(m_c^2/m_u^2)$ (for more details see Ref. [2], where an accurate calculation of the coefficient of the operator T has been performed in the leading logarithmic approximation in the four-quark model). Of course, there remains a very strong numerical suppression, since the important diagrams have two or more loops. A generalization of the results of Ref. [2] to the modern Kobayashi-Maskawa six-quark model does not exist in the literature, as far as we know. The contribution of the t quark, however, cannot be dominant, since there exist rather strong bounds on the corresponding mixing angles. The estimate presented in the Appendix shows that it is not larger than the c and u contributions. Thus, the result of Ref. [2] gives a good idea of the order of magnitude of the effect; the local $s \to d\gamma$ transition is

$$M(s \to d\gamma) \simeq i\, \frac{e\, G_F}{16\pi^2}\, 0.4\, m_s\, \bar{s}_R\, \sigma_{\mu\nu} d_L F_{\mu\nu}\ . \tag{4}$$

It is natural to assume that the matrix element of the operator T for baryon states is of order of unity, for instance,

$$\left\langle p, \gamma | T | \Sigma^+ \right\rangle \sim \frac{1}{2}\, \bar{u}_p \sigma_{\mu\nu} (1+\gamma_5) u_\Sigma F_{\mu\nu}\ . \tag{5}$$

Combining (4) and (5), we get for the contribution of the local transition $s \to d\gamma$ to the decay $\Sigma^+ \to p\gamma$

$$\mathrm{BR}\big(\Sigma^+ \to p\gamma\big)\big|_{s\to d\gamma} \sim 2 \cdot 10^{-6}\,, \tag{6}$$

where the current s quark mass m_s in formula (4) is about 150 MeV. We recall that experimentally

$$\mathrm{BR}\big(\Sigma^+ \to p\gamma\big)\big|_{\mathrm{exp}} = 1.2 \cdot 10^{-3}\,. \tag{7}$$

Even if for some unknown reason the estimate (5) has been underestimated by an order of magnitude (which is extremely unlikely), the contribution of the local transition $s \to d\gamma$ is still an order of magnitude less than we see experimentally.

Thus, purely theoretical arguments show that the mechanism of Fig. 1 gives a negligibly small contribution to the amplitude of weak radiative decays. One can also give an independent phenomenological argument. Indeed, let us assume for a moment (although this is absolutely inconceivable) that the local $s \to d\gamma$ transition gives the whole amplitude of the decay $\Sigma^+ \to p\gamma$ measured experimentally. Since the coefficient in front of the operator T is fixed (see Eq. (4)), this means that we fix the matrix element $\langle p\gamma | T | \Sigma^+ \rangle$.

By means of $SU(3)$ symmetry the latter can be connected to the matrix elements of other decays, for instance $\langle \Sigma^-\gamma|T|\Xi^-\rangle$. A detailed analysis has been carried out in Ref. [3], where the following prediction has been obtained:

$$\mathrm{BR}\big(\Xi^- \to \Sigma^-\gamma\big) \sim 10^{-2}\,,$$

which exceeds by an order of magnitude the experimental upper bound

$$\mathrm{BR}\big(\Xi^- \to \Sigma^-\gamma\big)_{\mathrm{exp}} < 1.2 \cdot 10^{-3}\,. \tag{8}$$

Thus, the mechanism of Fig. 1 cannot play an important role in weak radiative decays. This conclusion is unambiguous, and consequently assuming dominance of photon emission at short distances does not even remotely correspond to the real situation.

We will turn to another mechanism which therefore should play the main role in weak radiative decays. Let us consider photon emission at large distances, i.e. $x \sim R_c$ in the amplitude (2), where R_c is the confinement radius. In this case the contribution from weak and electromagnetic interactions can be separated. The amplitude (2) can be presented in the form of a sum over intermediate hadronic states

$$\begin{aligned} M\big(B_i \to B_f\gamma\big) = -ie\varepsilon_\mu \int d^4x\, e^{iqx} \Big(&\sum_X \theta(\text{-}x_0)\langle B_f|H_W(0)|X\rangle\langle X|J_\mu^{\mathrm{em}}(x)|B_i\rangle \\ &+ \sum_X \theta(x_0)\langle B_f|J_\mu^{\mathrm{em}}(x)|X\rangle\langle X|H_W(0)|B_i\rangle \Big)\,. \end{aligned} \tag{9}$$

Usually one considers the pole approximation, in which only one-baryon intermediate states are kept in (2). The corresponding diagrams are given in Fig. 2 . This approximation is apparently adequate for rough estimates of

Figure 2. Pole diagrams. The solid lines denote baryons.

decays like $\Sigma^+ \to p\gamma$, but is completely unacceptable for the decays (1) ; see below.[a] Moreover, even in the favorable case of amplitudes like $\Sigma^+ \to p\gamma$ one

[a] In a recent paper [4], the decays (1) are considered in the pole approximation. The probabilities predicted are much smaller than ours.

cannot calculate with an accuracy better than a factor of 2, which gives an uncertainty of a factor of 4 in the decay probability.

In this paper we wish to draw attention to two facts. First, one can determine exactly a lower bound for the probability of weak radiative decays from the imaginary part of the amplitude.[b] We will show that in the case of the decays (1) this lower bound is of the same order of magnitude as the total probability of the decay, and amounts to

$$\begin{aligned} &\mathrm{BR}\big(\Xi^- \to \Sigma^-\gamma\big)_{\substack{\text{unitary}\\ \text{limit}}} > 1\cdot 10^{-4}\,, \\ &\mathrm{BR}\big(\Omega^- \to \Xi^-\gamma\big)_{\substack{\text{unitary}\\ \text{limit}}} > 0.8\cdot 10^{-5}\,. \end{aligned} \tag{10}$$

Second, for the decays (1) one can estimate fairly reliably the real part of the amplitude, on the basis of an exact low-energy theorem, which states that in the chiral limit two-particle baryon-pion states dominate in the amplitude (9). The contribution of the latter contains a logarithmic singularity in the pion mass, and the coefficient in front of $\ln\mu^2$ is fixed uniquely (μ is the pion mass). So far, these two aspects have not been discussed in the literature, at least not on the level required by the experimental investigation of the decays (1). For the decay $\Xi^-\to\Sigma^-\gamma$ we have obtained the following value of the relative decay probability: $\mathrm{BR}(\Xi^-\to\Sigma^-\gamma)\approx 1.7\cdot 10^{-4}$. To avoid misunderstanding we emphasize that these two points are not specific for the decays (1). The amplitudes of all weak radiative transitions have an imaginary part, determined by unitarity, as well as a factor $\ln\mu^2$ in the real part. However, in most cases these contributions are relatively small and the real part, which corresponds to Fig. 2 and does not contain $\ln\mu^2$, dominates in the amplitudes. The decays (1) are distinguished by the fact that for them the diagrams of Fig. 2 are forbidden (more accurately, strongly suppressed), which is due to the specific form of H_W. On these decays loop diagrams, which are usually very small against the background of the tree diagrams of Fig. 2, play the major role.

The layout of the paper is as follows. In Section 2 we discuss the amplitude of radiative decay and calculate its imaginary part in the case $\Xi^-\to\Sigma^-\gamma$. In Section 3 we calculate the real part, which dominates in the chiral limit, and we discuss the estimate of the relative probability of the decay $\Xi^-\to\Sigma^-\gamma$. In Sections 4 and 5 we estimate the contribution of the mechanism suggested

[b] The existence of real intermediate states which generate imaginary parts of the amplitudes has apparently first been noted in Ref. [5], which is devoted to the decay $\Sigma^+\to p\gamma$. In the same paper a unitary bound for the decay $\Xi^-\to\Sigma^-\gamma$ is quoted which practically coincides with (10). Unfortunately, we learned about this only after our paper was sent to the Publisher.

to the relative probability of the decays $\Omega^- \to \Xi^- \gamma$ and $\Sigma^+ \to p\gamma$, and we discuss the pole contributions, which dominate in the latter case and are absent in the cases $\Xi^- \to \Sigma^- \gamma$ and $\Omega^- \to \Xi^- \gamma$. In the Appendix we give an estimate of the t quark contribution to the local transition $s \to d\gamma$.

2. Calculation of the Imaginary Part of the $\Xi^- \to \Sigma^- \gamma$ Amplitude

The matrix element of a weak radiative hyperon decay is usually parametrized [6] in the form

$$M\big(B_i \to B_f \gamma\big) = G_F \sqrt{4\pi\alpha}\, \Delta\, \bar{B}_f (a\gamma_5 + b)\sigma_{\mu\nu} q_\nu B_i\, \varepsilon_\mu \,, \tag{11}$$

where q_v and ε_μ are the momentum and polarization of the photon, and $\Delta = 140\,\mathrm{MeV}$, which coincides numerically with the pion mass μ. We note however that we will consider μ as a variable parameter, and in particular $\mu \to 0$ in the chiral limit. The parameter Δ is a fixed number introduced for convenience. Generally speaking, the amplitudes a and b have real as well as imaginary parts, which do not interfere in the decay probability. Therefore, the imaginary parts of the amplitudes determine the theoretical lower bound of the decay width. By virtue of unitarity the imaginary part of the amplitude is determined by the contribution of real intermediate states. In the case of the decay $\Xi^- \to \Sigma^- \gamma$ there is only one real intermediate state $\Lambda\pi^-$ in first order in the weak and the electromagnetic interactions. Thus, the imaginary part is determined by the diagram of Fig. 3. The amplitude of the

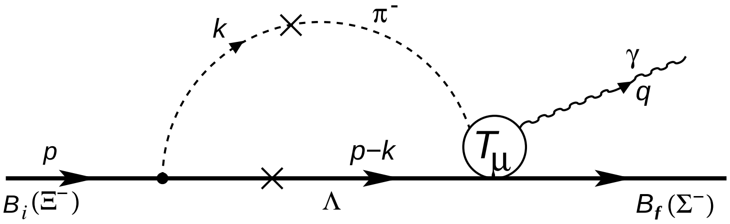

Figure 3. The diagram which determines the imaginary part of the amplitude of weak radiative decay. The dashed line denotes a pion.

weak decay $\Xi^- \to \Lambda\pi^-$ is known from experiment. It is usually presented [6] in the form

$$M\big(\Xi^- \to \Lambda\pi^-\big) = -iG_F \Delta^2 \bar{\Lambda}\big(A + B\gamma_5\big)\Xi \,, \tag{12}$$

where

$$A = 2.04 \pm 0.01 \,, \qquad B = -7.49 \pm 0.28 \,, \tag{13}$$

and we denote bispinors by the same symbols as particles. The right-hand block in Fig. 3 is the amplitude T_μ for photoproduction of the pion on the hyperon. Since the values of the pion momentum k and the photon energy q_0 in the c.m.s. are small ($k = 139\,\mathrm{MeV}$ and $q_0 = 118\,\mathrm{MeV}$), the amplitude T_μ is accurately fixed in the framework of PCAC. More accurately, this amplitude

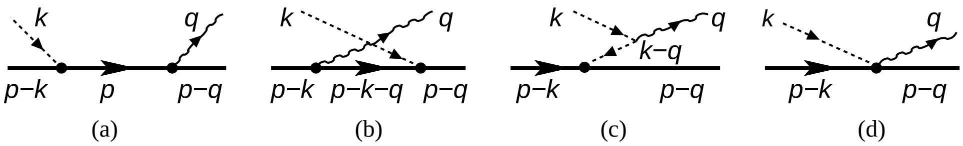

Figure 4. The diagrams which determine in leading order the amplitude of pion photoproduction on a baryon.

can be found theoretically up to terms linear in the momentum q_μ [7]. In this approximation T_μ is determined by the pole and contact diagrams of Fig. 4 and $T_\mu = T_{\mu_1} + \ldots + T_{\mu_4}$. The explicit expressions for the amplitudes $T_{\mu_1}, \ldots, T_{\mu_4}$ have the following form:

$$
\begin{aligned}
T_{\mu_1} &= -i\sqrt{4\pi\alpha}\, f\,\bar{\Sigma}\left(-\gamma_\mu + \frac{\varkappa_\Sigma}{2M_\Sigma}\,\sigma_{\mu\nu}q_\nu\right)\frac{1}{\hat{p}-M_\Sigma}\,\hat{k}\gamma_5\Lambda\,,\\
T_{\mu_2} &= -i\sqrt{4\pi\alpha}\, f\,\bar{\Sigma}\,\frac{\hat{k}\gamma_5}{\hat{p}-\hat{k}-\hat{q}-M_\Lambda}\,\frac{\varkappa_\Lambda}{2M_\Lambda}\,\sigma_{\mu\nu}q_\nu\Lambda\,,\\
T_{\mu_3} &= i\sqrt{4\pi\alpha}\, f\,\bar{\Sigma}\,\frac{(2k-q)_\mu}{(k-q)^2-\mu^2}\left(\hat{k}-\hat{q}\right)\gamma_5\Lambda\,,\\
T_{\mu_4} &= -i\sqrt{4\pi\alpha}\, f\,\bar{\Sigma}\gamma_\mu\gamma_5\Lambda\,,
\end{aligned}
\tag{14}
$$

where the momenta p, k, and q are arranged in Fig. 4 and $\varkappa_\Sigma$ and $\varkappa_\Lambda$ are the anomalous magnetic moments of the Σ^- and Λ hyperons, respectively ($\varkappa_\Sigma = -0.48 \pm 0.37$ and $\varkappa_\Lambda = -0.67 \pm 0.06$). The PCAC coupling constant is

$$
f = g_A^{\Lambda\Sigma}/f_\pi\,,
\tag{15}
$$

where $f_\pi = 133\,\mathrm{MeV}$ and $g_A^{\Lambda\Sigma}$ is the axial constant of the transition $\Sigma^- \to \Lambda e^- \nu$. We note that, since we use the PCAC technique, the $\Sigma\Lambda\pi$ vertex has the axial-vector form, and the corresponding constant $g_A^{\Lambda\Sigma}/f_\pi$ is connected with the standard pseudoscalar constant $g_{\Sigma\Lambda\pi}$ via the Goldberger-Treiman relation. We note that exactly the same approximation has been verified empirically in an analogous process, pion photoproduction on a nucleon, where it has an accuracy of the order of 10% in the amplitude. This accuracy is in general characteristics for the PCAC technique. One can show (see Ref. [7], Sec. 4.1.1) that the terms linear and of higher orders in q_μ which we have not kept, give contributions of order $(k/M)^2 \sim 10^{-2}$ to the imaginary part of $M(\Xi^- \to \Sigma^- \gamma)$, where M is the hyperon mass.[c] These contributions are negligibly small. Thus, the diagram of Fig. 3 is completely known. It is essential to note the following. The contributions of the amplitudes T_{μ_1} and T_{μ_2} which describe photon emission by Σ^- and Λ hyperons (see Figs. 4a,b ; 5a,b) is suppressed relative to the contribution of the amplitudes T_{μ_3} and T_{μ_4} (see Figs. 4c,d ; 5c,d) by k/M and $(k/M)^2$ in the charged and magnetic parts, respectively. But since we have neglected terms of the order of q_μ in the pho-

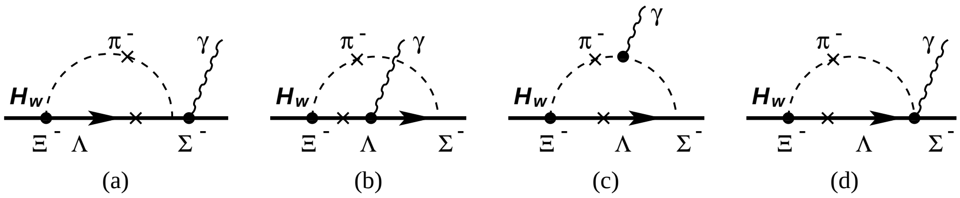

Figure 5. The diagrams which determine in leading order the imaginary part of weak radiative decay in the case $\Xi^- \to \Sigma^m \gamma$.

toproduction amplitude, which correspond to corrections of order $(k/M)^2$, it is not legitimate to keep the terms with anomalous magnetic moments. Thus, within our approximation one can take $T_{\mu_2} = 0$, and keep only the charged part in T_{μ_1} .

Using the standard formula for the imaginary part

$$\begin{aligned} \operatorname{Im} M\big(\Xi^- \to \Sigma^- \gamma\big) = \frac{1}{2} \int \frac{d^4 k}{(2\pi)^2}\, \delta\big(k^2 - \mu^2\big)\, \delta\big((p-k)^2 - M_\Lambda^2\big) \\ \times \bar{\Sigma}\, \varepsilon_\mu T_\mu \big(\hat{p} - \hat{k} + M_\Lambda\big) M\big(\Xi^- \to \Lambda \pi^-\big) \Xi\,, \end{aligned} \tag{16}$$

where p is the momentum of the Ξ^- hyperon, we get

$$\begin{aligned} \operatorname{Im} a &= \frac{\Delta f A}{8\pi} \left(\frac{M_\Xi + M_\Sigma}{2 M_\Xi}\right) \frac{|\mathbf{k}|}{M_\Xi} \Big[- \frac{M_\Xi + M_\Lambda - 2\varepsilon}{q_0} + \frac{|\mathbf{k}|(M_\Lambda + M_\Sigma)}{2 q_0^2}\, \gamma \Big] \\ &= -5.7 \cdot 10^{-2}\,, \end{aligned} \tag{17}$$

$$\operatorname{Im} b = - \frac{\Delta f B}{8\pi} \frac{|\mathbf{k}|}{M_\Xi} \frac{|\mathbf{k}|}{2 q_0}\, \gamma = 6.2 \cdot 10^{-3}\,, \tag{18}$$

[c] More accurately, to Ima, where the invariant amplitude a is defined in Eq.11.

in which $\varepsilon = \sqrt{\mathbf{k}^2 + \mu^2}$ is the energy of the pion in the c.m.s. and

$$\gamma = \left(1 - \frac{\mu^2}{\varepsilon k} \ln \frac{\varepsilon + |\mathbf{k}|}{\mu}\right) \frac{\varepsilon}{|\mathbf{k}|} \approx 0.53\,.$$

To obtain these numbers we substituted

$$g_A^{\Sigma\Lambda} = 0.6 \quad \text{and} \quad f = g_A^{\Sigma\Lambda}/f_\pi \simeq 4.6\,\mathrm{GeV}^{-1}\,,$$

which follows directly from the data on the decay $\Sigma^- \to \Lambda e^- \nu$ [8]. A near value $g_A = 0.65$ is obtained by using $SU(3)$ symmetry and the well known value of the nucleon axial coupling constant.

We emphasize that the amplitude Ima of zeroth order in the momentum ($|\mathbf{k}|/q_0 \sim 1$) dominates. The fact that the amplitude does not vanish at threshold leads also to terms logarithmic in μ in the real part. We will consider these terms in the next section. The amplitude Imb is parametrically smaller since it tends to zero for $|\mathbf{k}| \to 0$.

Using the standard formula for the decay width,

$$\begin{aligned} \Gamma\big(\Xi^- \to \Sigma^- \gamma\big) &= 4\alpha G_F^2 \Delta^2 \big(|a|^2 + |b|^2\big) \left(\frac{M_\Xi^2 - M_\Sigma^2}{2M_\Xi}\right)^3 \\ &= 1.84 \cdot 10^8 \big(|a|^2 + |b|^2\big) \sec^{-1}\,, \end{aligned} \tag{19}$$

we get automatically that

$$\mathrm{BR}\big(\Xi^- \to \Sigma^- \gamma\big) = 3.04 \cdot 10^{-2} \big(|a|^2 + |b|^2\big)\,. \tag{20}$$

Since we know the imaginary parts of the amplitudes a and b [see (17) and (18)], we get the exact lower bound

$$\mathrm{BR}\big(\Xi^- \to \Sigma^- \gamma\big) \geq 1.0 \cdot 10^{-4}\,. \tag{21}$$

This unitary bound is more than an order of magnitude higher than the estimate of the relative decay probability due to the transition $s \to d\gamma$. This demonstrates once more the negligibly small contribution of this mechanism to the amplitude of weak radiative decay.

3. Calculation of the Real Part of the $\Xi^- \to \Sigma^- \gamma$ Amplitude

In this section we will show that the amplitude of weak radiative decay has a logarithmic singularity $\ln \mu^2$, and we will determine the coefficient in front of the logarithm by means of a low-energy theorem.

The fact that the imaginary part is constant at threshold (see Sec. 2) suggests a logarithmic singularity in the real part of the amplitude. Indeed, let us write down a dispersion representation for the amplitude $M(\Xi^-\to\Sigma^-\gamma)$ in the square p^2 of the momentum of the Ξ^- hyperon,

$$M\big(\Xi^-\to\Sigma^-\gamma\big)=\frac{1}{2\pi}\int_{p^2_{\rm thr}}^{\infty}\frac{{\rm Im}\,M(p^2)\,dp^2}{p^2-M_\Xi^2-i\varepsilon}\ . \tag{22}$$

Since the imaginary part contains terms which are constant at threshold (Im$M\to$*const* for $p^2\to p^2_{\rm thr}$), the dispersion integral (22) has an infrared logarithmic divergence in the chiral limit ($\mu=0$, $p^2_{\rm thr}=M_\Xi^2=M^2$). In the real world this divergence will be cut off by the pion mass or by the mass difference $(M_\Xi-M_\Lambda)$. Thus, if in the chiral limit the amplitude of weak radiative decays contains $\ln(M^2/0)$ in the real part, then in the the real world it has the form

$${\rm Re}\,M=\alpha\big(\ln(M^2/\delta^2)+\beta\big)\,, \tag{23}$$

as one can easily convince oneself. Here, δ=max$\{\mu,\Delta M\}$, ΔM is the mass difference of the initial baryon and the baryon in the intermediate state, and α and β are constants which are nonsingular in the chiral limit. [The coefficient α is determined uniquely, and β is model-dependent. It is important that in the decays (1) the coefficient $|\beta|\lesssim\pi$. In the decay $\Sigma^+\to p\gamma$ the value of β is about $\pi^2\gg\ln(M^2/\delta^2)$, see below.]

In the case of the decay $\Xi^-\to\Sigma^-\gamma$

$$\Delta M=M_\Xi-M_\Lambda\simeq 206\,{\rm MeV}\,.$$

This number is of the same order of magnitude as the pion mass. Therefore, after calculating the coefficient α we will assume that it is multiplied by $\ln(M^2/\mu^2)$. The difference between $\ln(M^2/\mu^2)$ and $\ln\big(M^2/(\Delta M)^2\big)$ comes down to a definite constant term β which is anyway not fixed theoretically. We note that the logarithmic term occurs only in the P-odd amplitude a of weak radiative decay. Similar P-odd logarithmic terms have been discussed recently in connection with the dipole moment of the neutron [9].

In calculating the amplitude by means of the dispersion integral (22) one has to take account of the physical as well as the nonphysical imaginary parts which arise from all cuts of the diagrams of Fig. 5 with any two-particle intermediate states. To avoid unnecessary work it is more convenient to determine the coefficient of the logarithmic term by calculating directly the uncut Feynman diagrams of Fig. 5 in the infrared region. All one has to do is to extract from these diagrams the part which (in the chiral limit) is proportional to $\int d^4k/k^4$, where k is the momentum of the virtual pion. Of course, one has to take into account all possible two-particle states which contain a pion. In the case of $\Xi^- \to \Sigma^-\gamma$, there are three such states: $\Lambda\pi^-$, $\Sigma^0\pi^-$ and $\Xi^0\pi^-$, while in the latter case the weak transition occurs at the end.

It is easy to convince oneself that the logarithmic term occurs only in the diagram with photon emission from the pion. This can be seen from the fact that the diagrams with contact emission of the photon and with emission from the hyperon converge into the infrared region as $\int d^4k/k^3$. Thus, the logarithmic part of the amplitude of the decay $\Xi^- \to \Sigma^-\gamma$ is completely determined by the diagrams of Fig. 6, while the P-odd amplitude B of the weak transition (corresponding to the P-even amplitude b of weak radiative decay) does not lead to a logarithmic term. The P-even amplitudes A of

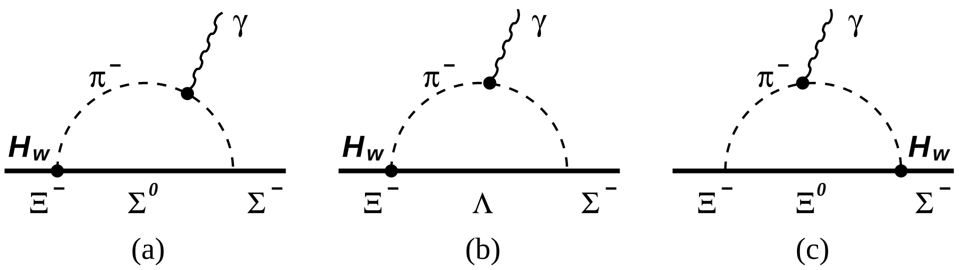

Figure 6. The diagrams which lead to a logarithmic singularity in the real part of the amplitude of the weak radiative decay $\Xi^-\to \Sigma^-\gamma$.

the weak transitions $\Xi^- \to \Sigma^0\pi^-$ and $\Xi^0 \to \Sigma^-\pi^+$ (corresponding to the P-odd amplitude a of weak radiative decay) and the axial constants $g_A^{\Sigma^-\Sigma^0}$ and $g_A^{\Xi^0\Xi^-}$) can by means of $SU(3)$ symmetry be connected with $g_A^{\Sigma^-\Lambda} = 0.6$ and known P-even amplitudes A of weak nonleptonic decays [10], which leads to the following values of these quantities:

$$\begin{aligned} g_A^{\Sigma^-\Sigma^0} &= g_A^{\Sigma\Lambda}, & g_A^{\Xi^0\Xi^-} &= 0.54 g_A^{\Sigma\Lambda}, \\ A\big(\Xi^-\to \Sigma^0\pi^-\big) &= 0.51\,, & A\big(\Xi^0\to \Sigma^-\pi^+\big) &= -0.06\,. \end{aligned} \tag{24}$$

Since the contribution of each diagram to the real part of the amplitude is proportional to the product $g\cdot A$, it is easy to see that the dominant state is $\Lambda\pi^-$, which, as the reader will remember, is also completely responsible for the imaginary part of the amplitude. From (24) it follows that the total contribution is 1.3 times the $\Lambda\pi^-$ contribution.

We note that $A\big(\Xi^0\to \Sigma^-\pi^+\big) = 0$ if the $\Delta T = 1/2$ rule works. Therefore the graph of Fig. 6c is very small. The calculation of the diagrams of Fig. 6 is not difficult and the expression for the real part of the amplitude has the form

$$\begin{aligned}\mathrm{Re}\, M\big(\Xi^{-}\!\to \Sigma^{-}\gamma\big) &= 1.3\,\mathrm{Re}\, M\big(\Xi^{-}\!\to \Sigma^{-}\gamma\big)_{\Lambda\pi^{-}} \\ &= -1.3\,\frac{G_F\Delta^2 f\sqrt{4\pi\alpha}}{8\pi^2}\,\ln\frac{M}{\mu}\,A\big(\Xi^{-}\!\to\Lambda\pi^{-}\big)\Sigma\sigma_{\mu\nu}\gamma_5 q_\nu \Xi\varepsilon_\mu\,.\end{aligned} \tag{25}$$

Thus

$$\mathrm{Re}\, a = -1.3\,\frac{\Delta f}{8\pi^2}\,\ln\frac{M}{\mu}\,A\big(\Xi^{-}\!\to\Lambda\pi^{-}\big) = 0.85\,\mathrm{Im} a\,,$$

where we have assumed that the upper cutoff of the logarithmic is equal to M_Ξ. Then, according to (20), the contribution of (25) to $\mathrm{BR}\big(\Xi^{-}\!\to\Sigma^{-}\gamma\big)$ amounts to

$$\mathrm{BR}\big(\Xi^{-}\!\to\Sigma^{-}\gamma\big)_{\text{log part}} \simeq 0.7\cdot 10^{-4}\,. \tag{26}$$

If we assume that the contribution of the nonlogarithmic terms is small (we will give arguments for this in the next section), then we expect for the relative probability of the decay $\Xi^{-}\!\to\Sigma^{-}\gamma$, according to (21) and (26),

$$\mathrm{BR}\big(\Xi^{-}\!\to\Sigma^{-}\gamma\big) \simeq 1.7\cdot 10^{-4}\,, \tag{27}$$

which makes this decay easily accessible for experimental observation in present-day hyperon beams.

4. The Decay $\Omega^{-}\!\to\Xi^{-}\gamma$

From a fundamental point of view the theoretical analysis of this process does not differ from the analysis in the case $\Xi^{-}\!\to\Sigma^{-}\gamma$. All assertions on the imaginary part formulated above for Ξ^{-} are automatically applicable to the decay of Ω^{-}. As far as the real part is concerned, there are no logarithmic terms, which is due to the fact that Ξ^{-} and Ω^{-} belong to different multiplets (see below). Moreover, there are very important differences in the technical details, which finally lead to a much lower prediction for $\mathrm{BR}(\Omega^{-}\to\Xi^{-}\gamma)$. First of all, the spin of Ω^{-} is equal to 3/2 and, consequently, for a description one needs in general four invariant amplitudes instead of two:

$$\begin{aligned}M\big(\Omega^{-}\!\to\Xi^{-}\gamma\big) = \sqrt{4\pi\alpha}\,G_F\big(&\alpha_1\bar\Xi\sigma_{\alpha\beta}\gamma_5\Omega_\mu q_\nu F_{\alpha\beta} + \tilde\Delta\alpha_2\bar\Xi\gamma_\alpha\gamma_5\Omega_\mu F_{\alpha\mu} \\ &+ \beta_1\bar\Xi\sigma_{\alpha\beta}\Omega_\mu q_\mu F_{\alpha\beta} + \tilde\Delta\beta_2\bar\Xi\gamma_\alpha\Omega_\mu F_{\alpha\beta}\big)\,.\end{aligned} \tag{28}$$

Here, $\tilde\Delta = 314\,\mathrm{MeV}$ is a numerical constant introduced for convenience (it coincides with the energy of the photon in the c.m.s.); α_1, α_2, β_1, and β_2 are dimensionless constants. The first two (α_1 and α_2) correspond to P-waves, and β_1 and β_2 to S- and D-waves. We note that in the literature it is usually tacitly implied that α_2 and β_2 are equal to zero. In fact, as we see below, the unitary bound is essentially determined by the amplitude α_2.

The imaginary part is generated by the following intermediate states:

$$\Xi^0\pi^-, \quad \Xi^-\pi^0, \quad \Lambda K^-, \quad \Xi^*(1530)\pi\,.$$

One readily convinces oneself that $\Xi^0\pi^-$ gives the dominant contribution. Indeed, let us recall what happens in the decay $\Xi^-\to\Sigma^-\gamma$. The left-hand block in the diagram of Fig. 3 contains an S-wave which is converted into the P-wave amplitude of the decay $\Xi^-\to\Sigma^-\gamma$. The smallness of the phase space ($\sim k/M_\Xi$), corresponding to the $\Lambda\pi^-$ intermediate state is characteristic for P-wave amplitudes compared to S-waves. There is no additional suppression, and this is due to the pole character of the diagrams of Fig. 4. Formally, a similar situation occurs in Ω^- decay. The weak block contains now a P-wave and a D-wave, and it is clear that the main contribution to the imaginary part comes from the P-wave, which in this case is converted into the P-wave amplitudes α_1 and α_2 of the radiative decay. It is clear that for the pole enhancement to occur (see above) the pion (K meson) has to be ultrarelativistic, since soft particles do not easily emit photons. In the decay $\Omega^-\to\Lambda K^-$ the K^- meson is soft, $v_k\approx 0.39$, but the pion velocity in $\Omega^-\to\Xi^0\pi^-$ is close to 1, $v_\pi\approx 0.9$. One can easily see that the ΛK^- contribution is, roughly speaking, suppressed by a factor v_K^2/v_π^2, i.e. by one order of magnitude.

For the decay $\Omega\to\Xi^*(1530)\pi$, although it is also S-wave, the pion velocity is very small, $\sim 10^{-2}$, so that the corresponding contribution to the imaginary part is negligible. In the intermediate state $\Xi^-\pi^0$ the photon can only be emitted by the Ξ^- hyperon, whose velocity is also small.

We see that the main contribution is due only to $\Xi^0\pi^-$. The decay $\Omega^-\to\Xi^0\pi^-$ is parametrized as

$$M\big(\Omega^-\to\Xi^0\pi^-\big) = G_F\tilde\Delta\,\bar\Xi\big(A+B\gamma_5\big)\Omega_\mu k_\mu\,. \tag{29}$$

If it is described by the mechanism of Ref. [11], then certainly

$$B/A\sim m_s/M\ll 1\,. \tag{30}$$

Even if we assume that $B\sim A$ (and this seems inconceivable), the B contribution to $\Gamma\big(\Omega^-\to\Xi^0\pi^-\big)$ is still negligibly small, of the order of 1% of the A contribution. Thus, the constant A is uniquely determined from the experimental data: the Ω^- lifetime and $\mathrm{BR}\big(\Omega^-\to\Xi^0\pi^-\big)$. Concretely,

$$|A|^2 = \frac{24\pi}{|\mathbf{k}|^3}\left(\frac{M_\Omega}{M_\Omega+M_\Xi}\right)^2\frac{\Gamma_{\rm tot}\mathrm{BR}\,(\Omega^-\to\Xi^0\pi^-)}{G_F^2\,\tilde\Delta^2} = 0.14\,. \tag{31}$$

On the other hand, one can also neglect the B contribution to $\mathrm{Im}M\big(\Omega^-\!\to\Xi^-\gamma\big)$, since also here it is suppressed relative to the A contribution and not larger than one percent. Performing standard calculations, we get

$$\begin{aligned}
\mathrm{Im}\alpha_1 &= -\frac{M_\Xi|\mathbf{k}|^2 f\tilde{\Delta}A}{4\pi(M_\Omega^2-M_\Xi^2)(M_\Omega-M_\Xi)}\left[\frac{1}{2}\Big(1-\frac{2\varepsilon}{q_0}\Big)\gamma+\frac{1}{3}\,\frac{|\mathbf{k}|\,(3M_\Omega+M_\Xi)}{M_\Omega^2-M_\Xi^2}\right]\\
&= -1.27\cdot 10^{-3}\,,\\
\mathrm{Im}\alpha_2 &= -\frac{M_\Xi k^2 fA}{2\pi(M_\Omega^2-M_\Xi^2)}\left[\frac{1}{2}\Big(1-\frac{\varepsilon}{q_0}\Big)\gamma+\frac{1}{3}\,\frac{|\mathbf{k}|}{M_\Omega-M_\Xi}\right]=-4.2\cdot 10^{-3}\,,
\end{aligned}\tag{32}$$

$$\gamma=\frac{\varepsilon}{k}\left(1-\frac{\mu^2}{k\varepsilon}\ln\frac{\varepsilon+k}{\mu}\right),$$

where the constant $g_A^{\Xi^-\Xi^0}\!=f_\pi\cdot f$ is obtained by means of $SU(3)$ symmetry and is equal to $g_A^{\Xi^-\Xi^0}\!= 0.35$. In turn, these estimates give

$$\mathrm{BR}_{\substack{\text{unitary}\\ \text{limit}}}\big(\Omega^-\!\to\Xi^-\gamma\big)\geq 0.8\cdot 10^{-5}\,. \tag{33}$$

Taking into account the real part of the amplitude we can expect a relative probability at the level $(1\div 1.5)\cdot 10^{-5}$. Thus, the prediction is an order of magnitude less than in the case $\Xi^-\to\Sigma^-\gamma$. (We note that the ΛK^- intermediate state gives the unitary bound $\approx 1\cdot 10^{-6}$.)

5. The Decay $\Sigma^+\to p\gamma$

The decay $\Sigma^+\!\to p\gamma$ is the only weak radiative decay which is observed experimentally (see Ref. [6]). We will estimate the contribution of the imaginary part and of the logarithmic terms in the real part to the width of this decay. The imaginary part is determined by the intermediate states $n\pi^+$ and $p\pi^0$ corresponding to the diagrams of Fig. 7. The logarithmic terms in the real part of the amplitude arise because of the diagrams of Fig. 8. We will first examine the imaginary part. Since the amplitudes with photon emission by the hyperons are suppressed relative to the amplitudes with photon emission by the pion or with contact emission (see Fig. 2), the imaginary part will be determined mainly by the intermediate state $n\pi^+$, since in the case $p\pi^0$ the photon is only emitted by the proton. The corresponding diagrams are completely analogous to the diagrams of Fig. 5 for the decay $\Xi^-\!\to\Sigma^-\gamma$ which we have considered earlier. Using formulas (17) and (18) with the obvious substitutions of masses and pion-baryon coupling constants, it is easy to show that the corresponding unitary bound of the relative decay probability is of

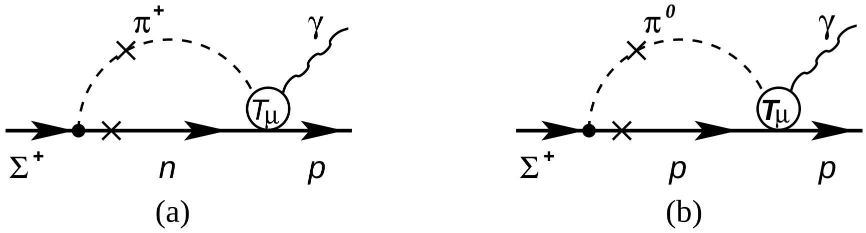

Figure 7. The imaginary part of the amplitude of the decay $\Sigma^+ \to p\gamma$ resulting from the two intermediate states $n\pi^+$ and $p\pi^0$.

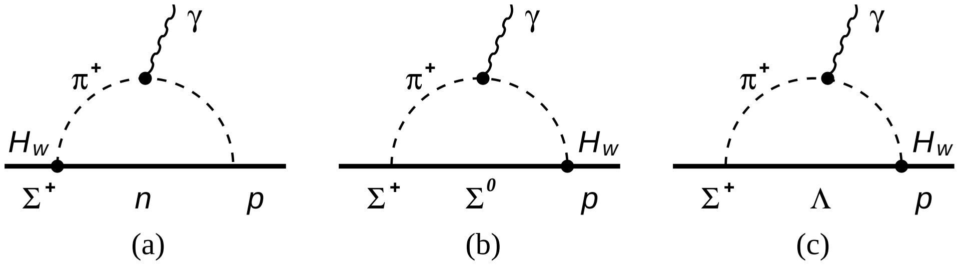

Figure 8. The diagrams which lead to a logarithmic singularity in the real part of the amplitude in the decay $\Sigma^+ \to p\gamma$.

the order of $3\cdot 10^{-5}$. In this case, because of the fact that the P-even amplitude $A\big(\Sigma^+\to n\pi^+\big) = 0.06$ and the P-odd amplitude $B\big(\Sigma^+\to n\pi^+\big) = 19.07$, the unitary bound is completely determined by the amplitude B. To estimate the real part of the amplitude, whose logarithmic singularity is only determined by the amplitude A of the weak transition, we use $SU(3)$ symmetry to determine the P-even amplitudes A and the corresponding pion-baryon coupling constants [6]. In this case, the contributions of the intermediate states $\Sigma^0\pi^+$, $n\pi^+$, and $\Lambda\pi^+$ strongly cancel each other in the sum (apparently accidentally), which has, for instance, been noted by Skovpen' [1]. As a result we get for the real part of the amplitude the following estimate

$$\mathrm{Re}M_{\log}\big(\Sigma^+\to p\gamma\big) \simeq 0.14\,\mathrm{Re}M_{\log}\big(\Xi^-\to\Sigma^-\gamma\big)\,. \tag{34}$$

Knowing that the contribution of $\mathrm{Re}M_{\log}\big(\Xi^-\to\Sigma^-\gamma\big)$ to the relative probability of the decay $\Xi^-\to\Sigma^-\gamma$ is $\sim 10^{-4}$, and the ratio of the total widths is $\Gamma(\Sigma^+\to p\gamma)/\Gamma(\Xi^-\to\Sigma^-\gamma) = 2$, it is easy to estimate the contribution of $\mathrm{Re}M_{\log}(\Sigma^+\to p\gamma)$ to the relative probability of the decay $\Sigma^+\to p\gamma$. It amounts to $\sim 10^{-6}$, which is negligibly small, even compared to the unitary bound, whose contribution is equal to $3\cdot 10^{-5}$, not to mention the experimental value $\sim 1.2\cdot 10^{-3}$. What is going on? The point is that there are tree pole diagrams, depicted in Fig. 2. In contrast to the pole diagrams we have so far dealt with, they are not suppressed by loop factors line $1/\pi^2$, i.e. roughly speaking, their contribution should be a factor π^2 larger. The pole model has been discussed repeatedly in the literature [1], but as we have already noted it can only claim an order of magnitude accuracy. However, another aspect which is more important for us here is the absence of pole contributions in the decays (1) . Why is the mechanism of Fig. 2 not important in these decays?

The point is that the weak block of the hyperon-hyperon transition is determined by the matrix element $\langle B'|H_W|B\rangle$, where

$$H_W = \sqrt{2}G_F \sin\theta\cos\theta \sum_{i=1}^{6} c_i O_i$$

is the effective Hamiltonian of the weak interaction with $\Delta S = 1$, the O_i are the standard local four-quark operators introduced in Ref. [11], and the c_i are numerical coefficients. It is well known [11] that the operator

$$O_1 = \bar{s}_L\gamma_\mu d_L \bar{u}_L\gamma_\mu u_L - \bar{s}_L\gamma_\mu u_L \bar{u}_L\gamma_\mu d_L$$

appears with the largest coefficient $c_1 = -2.5$, while the coefficients of the other operators O_2 , . . . , O_6 are an order of magnitude smaller. Since neither Ξ^- nor Σ^- contains valence u quarks, Ξ^- cannot transform into Σ^- by means of the operator O_1 , but Ξ^- can transform into Σ^0 or Λ^0 with emission of a π^- meson. Thus, the leading operator in H_W in the case of the decays (1) works only in a loop diagram, while in the decay $\Sigma^+\to p\gamma$ it works already in a tree pole diagram. In other words, in the case of the decay $\Xi^-\to\Sigma^-\gamma$ large tree diagrams of Fig. 2 can only be realized either by means of the quark sea in the Ξ^- hyperon, or through nonleading operators, for instance O_5 in H_W . In both cases one expects a suppression of one order of magnitude in the amplitude, i.e. the corresponding ratio is $\lesssim 10^{-5}$ [4]. Thus, the mechanism of Fig. 3 dominates. On the contrary, in decays like $\Sigma^+\to p\gamma$ the dominant mechanism is that of Fig. 2, which induces a real amplitude. This amplitude is, roughly speaking, a factor

$$\left(\frac{\Delta\varkappa_p}{M_\Sigma - M_p}\frac{f_\pi}{\sqrt{2}M_p}\right) \Big/ \left(\frac{\Delta f}{4\pi^2}\ln\frac{M}{\mu}\right) \approx 4 \qquad (35)$$

larger than the estimate (25) for $\mathrm{Re}M\left(\Xi^-\to\Sigma^-\gamma\right)$. We see that everything is self-consistent since, combining (26) and (35), we get $\mathrm{BR}\left(\Sigma^+\to p\gamma\right) \sim 10^{-3}$.

6. Concluding Remarks

The dominance of the mechanism considered here in the decay $\Xi^- \to \Sigma^-\gamma$ and $\Omega^- \to \Xi^-\gamma$ is determined by the quark structure of the operator O_1 in the effective weak Hamiltonian H_W. It is clear that this mechanism dominates in the amplitudes of weak radiative decays only of negatively charged hyperons, since they consist of down s and d quarks, and in the pole approximation the operator O_1 cannot generate a direct transition of the hyperons. In this case, large differences are possible between the relative decay widths in the various decays. Thus, we have found that

$$\mathrm{BR}\big(\Omega^- \to \Xi^-\gamma\big)/\mathrm{BR}\big(\Xi^- \to \Sigma^-\gamma\big) \sim 10^{-1}\,.$$

Experimental investigation of the decays $\Xi^- \to \Sigma^-\gamma$ and $\Omega^- \to \Xi^-\gamma$ would be of great interest, since a sharp excess of $\mathrm{BR}\big(\Xi^- \to \Sigma^-\gamma\big)$ compared to $\mathrm{BR}_{\substack{\text{unitary}\\ \text{limit}}}\big(\Xi^- \to \Sigma^-\gamma\big)$ would mean that there is some mysterious enhancement in the real part, either due to non-dominant operators of the weak Hamiltonian, or because of the t quark contribution to the $s \to d\gamma$ transition. Our unitary bound (21) and the estimate (27) of the relative probability of the decay $\Xi^- \to \Sigma^-\gamma$ indicate that this decay can be observed in present-day intense hyperon beams, for instance at the Serpukhov accelerator.

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Appendix. Estimate of the *t* quark Contribution to the $s \to d\gamma$ Amplitude

The coefficient of the operator $T = i\bar{s}_R\sigma_{\mu\nu}d_L F_{\mu\nu}$ has been calculated in Ref. [2] in the leading log approximation in the four-quark model. In first order in α_s it has the following form:

$$t = -\frac{\alpha_S e G_F\sqrt{2}}{12\pi^3}\,\sin\theta_C\cos\theta_C\, m_s \ln\frac{m_c^2}{m^2}\,, \tag{A.1}$$

where $m_s \simeq 150\,\mathrm{MeV}$ is the mass of the current s quark, $m \simeq 300\,\mathrm{MeV}$, and $m_c \sim 1.5\,\mathrm{GeV}$ is the mass of the c quark. It is easy to see that the contribution of one of the up quarks (u, c, t) to the diagram of the Fig. 1b is

$$x_{u,c,t}\,\frac{\alpha_S e G_F\sqrt{2}}{12\pi^3}\, m_s \ln\frac{m^2_{u,\,c,\,t}}{M_W^2}\,, \tag{A.2}$$

where the coefficient $x_{u,c,t}$ is determined by the mixing of a given quark with s and d quarks. In the four-quark model

$$x_u = \sin\theta_C \cos\theta_C\,, \qquad x_c = -\sin\theta_C \cos\theta_C\,,$$

which leads to the expression (A.1). In the six-quark model using the explicit form of the Kobayashi-Maskawa matrix elements [10], it is easy to show that

$$\begin{aligned} x_u &= c_1 s_1 c_3\,, \\ x_c &= -s_1 c_2 (c_1 c_2 c_3 - e^{i\delta} s_2 s_3)\,, \\ x_t &= -s_1 s_2 (c_1 s_2 c_3 + e^{i\delta} c_2 s_3)\,. \end{aligned} \tag{A.3}$$

Using (A.2) and (A.3) we get the following expression for the coefficient t:

$$t = -\frac{\alpha_S e G_F \sqrt{2}}{12\pi^3}\, m_s c_1 s_1 \left[c_3 \ln\frac{m_c^2}{m^2} + \frac{s_2}{c_1} \ln\frac{m_t^2}{m_c^2} \left(c_1 s_2 c_3 + s_3 c_2 e^{i\delta} \right) \right], \tag{A.4}$$

where we have replaced the mass of the u quark by the dynamical light-quark mass $m \simeq 300\,\text{MeV}$ [2]. Assuming $s_2, s_3 \leq 0.5$ we can estimate the relative t quark contribution which is $< 0.31 \ln(m_t/m_c)$. Even for $m_t \simeq 100\,\text{GeV}$, which is very unlikely, this does not exceed 1.2. Thus, for reasonable values of the t quark mass, its contribution to the amplitude of the local $s \to d\gamma$ transition does not change the amplitude by an order of magnitude and thus cannot ensure the dominance of this transition in weak radiative decays.

References

1. M. K. Gaillard, Nuovo Cim. **6A**, 559 (1971);
   G. R. Farrar, Phys. Rev. D **4**, 212 (1971);
   L. Gavroglu and H. P. W. Gottleib, Nucl. Phys. B **79**, 168 (1974);
   M. A. Ahmed and G. G. Ross. Phys. Lett. B **59**, 293 (1975);
   N. Vasanti, Phys Rev. D **13** 1889 (1976);
   F. Gilman and M. B. Wise, Phys. Rev. D **19**, 976 (1979);
   P. N. Pandit, R. C. Verma, and M. P. Khanma, Progr. Theor. Phys. **64**, 2158 (1980);
   Yu. I. Skovpen', Yad. Fiz. **34**, 1392 (1981) [Sov. J. Nucl. Phys **34**, 773 (1981)];
   A. N. Kamal and R. C. Verma, Phys. Rev. D **26**, 190 (1982);
   Lo Ghong-Huah, Phys Rev. D **26**, 199 (1982).
2. M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Rev. D **18**, 2583 (1978) [Erratum-ibid. D19, 2815 (1979)].
3. M. B. Wise, *Strong effects in weak nonleptonic decays*, PhD Thesis, SLAC-R-0227 (1980) [SLAC-0227, UMI-80-24758].
4. J. O. Eeg, *The penguin diagram and the process* $\Omega \to \Xi\gamma$, Intitute of Physics, Oslo University, OSLO-82-26 (1982).
5. V. I. Zakharov and A. B. Kaidalov, Yad. Fiz. **5**, 369 (1967) [Sov. J. Nucl. Phys. **5**, (1967)].
6. Particle Data Group, Phys. Lett. B **111**, 1 (1982).

7. A. I. Vainshtein and V. I. Zakharov, Usp. Fiz. Nauk **100**, 225 (1970) [Sov. Phys. USP **13**, 73 (1970)].
8. A. Garcia and P. Kalinowski, Phys. Lett. B **110**, 498 (1982).
9. R. J. Grewther, P. DiVecchia, G. Veneziano and E. Witten, Phys. Lett. B **88**, 123 (1979);
A. R. Zhitnitskii and I. B. Khriplovich, Yad. Fiz. **34**, 167 (1981) [Sov. J. Nucl. Phys. **34**, 95 (1981)].
10. L. B. Okun', *Leptony i kvarki*, M. Nauka (1981);
Leptons and quarks, North Holland (1982).
11. A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Zh. Exp. Teor. Fiz. **72**, 1275 (1977) [JETP **45**, 670 (1977)].