# NEW OLD INFLATION

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We propose a new class of inflationary solutions to the standard cosmological problems (horizon, flatness, monopole,...), based on a modification of old inflation. These models do not require a potential which satisfies the normal inflationary slow-roll conditions. Our universe arises from a single tunneling event as the inflaton leaves the false vacuum. Subsequent dynamics (arising from either the oscillations of the inflaton field or thermal effects) keep a second field trapped in a false minimum, resulting in an evanescent period of inflation (with roughly 50 e-foldings) inside the bubble. This easily allows the bubble to grow sufficiently large to contain our present horizon volume. Reheating is accomplished when the inflaton driving the last stage of inflation rolls down to the true vacuum, and adiabatic density perturbations arise from moduli-dependent Yukawa couplings of the inflaton to matter fields. Our scenario has several robust predictions, including virtual absence of gravity waves, a possible absence of tilt in scalar perturbations, and a higher degree of non-Gaussianity than other models. It also naturally incorporates a solution to the cosmological moduli problem.

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## 1. Introduction

Guth's "Old Inflation" [1] was a very simple idea. The universe is trapped in some false vacuum state, with the false vacuum energy being $V_0$. Such a universe inflates at a rate characterized by the inflationary Hubble constant

$$H_0 = \frac{\sqrt{V_0}}{M_P}\,. \tag{1}$$

The original idea was that a graceful exit could be provided by tunneling from the false vacuum to the true vacuum [2]. Such tunneling creates bubbles of the true vacuum, which could (it was hoped) then percolate and reheat the universe. Unfortunately, it was shown in e.g. [3] that for the desired parameters of the potential, the bubbles would not percolate, and one would end up with an empty universe.

This problem was soon resolved in new inflation [4] and other models. These postulate the existence of a scalar field inflaton $\Phi$, and rely on some fine tuning of the inflaton potential to yield slow-roll inflation with $N_e > 60$ e-foldings. When designing an appropriate potential for $\Phi$, one must solve three problems in addition to obtaining the needed e-foldings: graceful exit (end to inflation), reheating the Standard Model degrees of freedom, and production of density perturbations. Each of these typically adds constraints on the desired form of the inflation potential. Simple potentials like $\lambda\Phi^4$ are compatible with $N_e > 60$ and the required density perturbations only if the dimensionless parameters are tuned rather finely, e.g. $\lambda \sim 10^{-14}$. Solving all of the problems listed above typically involves the introduction of additional fields and couplings, as in hybrid inflation [5]. See [6] for a nice review.

Here, we propose an extension of the old inflation scenario. For reasonable choices of parameters, it preserves the successes of modern inflationary models, without requiring a slow-roll potential. This eliminates one of the major sources of tuning in inflationary potentials. While designing working models of our scenario then involves several additional ingredients and assumptions, we believe this is nevertheless a worthwhile addition to the large array of inflationary models, both because of the absence of slow-roll and because of the striking differences between our predictions and typical inflationary predictions.

Consider a generic system of two cross-coupled scalar fields, $\Phi$ and $\phi$, with a potential that has a meta-stable false vacuum, a stable true vacuum, and number of unstable saddle points. For instance, there could be a false vacuum at $\Phi = \Phi_{false}, \phi = \phi_{false}$, a true vacuum at $\Phi = \Phi_{true}, \phi = \phi_{true}$, and the lowest energy path connecting these vacua could pass through a saddle point at $\Phi = \Phi_{true}, \phi = \phi_{false}$. As we will show, fairly generic looking potentials for approximate moduli in supersymmetric theories will have the required structure. We assume that the universe starts in the false vacuum, as in old inflation. At some point the $\Phi$ field tunnels through the barrier, and the fields roll through the saddle point and relax into the true vacuum. We shall assume that the curvature of the potential is everywhere larger than of order $V/M_P^2$, and hence the system cannot satisfy the standard slow roll conditions. Nevertheless we show that after tunneling the system can lock itself into a period of a non-slow roll inflation, where the oscillations of $\Phi$ about $\Phi_{true}$ stabilize the false vacuum for $\phi$ at $\phi = \phi_{false}$. This is because in suitable potentials of this sort, the $\phi$ field cannot roll to its true vacuum until the amplitude of $\Phi$ oscillations become smaller than the negative curvature at the saddle point. Until this moment, the system is stabilized at the saddle point and inflates. We will show in § 2 that the resulting bubble universe can easily be large enough to accommodate observations with very natural choices of $V(\Phi, \phi)$.[a]

In § 3, we describe two classes of more elaborate generalizations of the simplest model. Our simplest model described above involves fields with properties similar to those of moduli in string theory. It is natural to ask whether there are new, generic inflationary possibilities in cases with several moduli fields. In § 3.1, we show that one can write down reasonable multi-field models with a cascade of stages of evanescent inflation inside the bubble. In § 3.2, we also show how one can arrange for models where the Hubble constant during the first stage of inflation (which is constrained in the simplest model of § 2) could be arbitrarily large.

The natural choices of scales in § 2 and § 3 would not give rise to suitable density perturbations by the standard mechanism (since we are not using slow-roll inflation, the inflaton mass during the relevant stage of inflation is much larger than the associated Hubble constant, so there are no fluctuations). However, as discovered recently in [7, 8], the existence of moduli-dependent Yukawa couplings gives a new mechanism for generating density perturbations. In § 4 we show that this mechanism can easily be incorporated in our models, giving rise to the required $\frac{\delta\rho}{\rho} \sim 10^{-5}$. In § 5, we discuss the predictions for the spectrum of density perturbations and gravity waves in our kind of scenario.

[a] We shall refer to the size of the bubble, but the observer inside the bubble naturally sees an open FRW universe. When we discuss the bubble size, more precisely we are discussing the radius of curvature inside the bubble. We will conform to calling this the size throughout the paper, so this should not cause any confusion.

In § 6, we show that our scenario naturally accommodates a solution to the cosmological moduli problem. Finally, in § 7, we summarize the observational signatures of this class of models. They are rather distinctive. We predict an absence of tensor perturbations and gravity waves. On the other hand, we argue following [7] that our model predicts significantly more non-Gaussianity than most existing models. Finally, because we have no period of slow-roll inflation, our model is consistent with zero tilt in the spectrum (though there are variants which would produce non-zero tilt as well).

It is worth emphasizing that while we discuss many issues in the present paper, the basic idea is quite simple. Given the new mechanism for generating density perturbations in [7,8], there is no longer a reason to constrain the inflationary scenario by requiring that the inflaton generate density perturbations (this is similar to the way that incorporating an additional field in hybrid models changes the way one views the issues of exit from inflation and reheating [5]). It is then possible to develop scenarios without any period of slow-roll inflation at all, and with scale(s) of inflation much lower than in conventional models. Although the stage of evanescent inflation inside the bubble lasts for less than 60 e-foldings, the resulting scenario can solve the horizon and flatness problems because of the long period of inflation in the original false vacuum (which can persist eternally, bubbling off different universes every now and then).

## 2. Two Field Models

### 2.1. *The basic scenario*

Our scenario begins with false vacuum decay via bubble nucleation. When the bubble materializes, part of the false vacuum energy density is released in some other form of energy density inside the bubble (e.g. oscillating scalar fields, or radiation). The key point of our scenario is the following observation. In the presence of two or more scalar fields (easily motivated in string theory or supersymmetric field theory constructions) the released energy density can have a rather profound effect. It can (temporarily) lock the system in an evanescent false vacuum state, and thus drive subsequent stage(s) of inflation, even if the scalar potential *does not* satisfy any slow-roll requirement. In other words, some unstable points of the scalar potential are converted into local minima in the presence of non-zero energy density (such as an oscillating scalar field). The nice point about such an inflationary mechanism is that it has a natural graceful exit, due to the fact that the stabilizing energy density gets diluted in the course of inflation, and sooner or later the the negative curvature of the potential takes over. Hence, the number of e-foldings is necessarily limited in such a scenario. We will show however that the expansion factor can be more than enough to accommodate the visible part of the universe within a single bubble.

Note that through both stages of evolution, the scalar fields are trapped in false vacua and have masses much larger than the inflationary Hubble parameters. So at no point do our inflatons have to satisfy the standard slow-roll requirements.

### 2.2. *Building blocks, "locked inflation"*

The essential building block of our scenario is what we shall refer to as "locked inflation." This can take place in systems with generic potentials that at no point satisfy the standard slow-roll conditions. However, for some natural initial conditions, the system gets locked at unstable saddle points of the potential which are stabilized by oscillations of a scalar field, resulting in a limited period of inflation with essentially constant Hubble parameter. The stabilizing oscillations are diminished by the inflation until their amplitude is no longer sufficient to stabilize the system at the saddle point. At this juncture, the system relaxes to its true vacuum, ending inflation. Such a situation is typical for many generic potentials and can easily be arranged for the moduli potentials one expects in supersymmetric field theories and/or string compactifications.

We shall now discuss a simple example of "locked inflation." Consider two fields $\Phi$ and $\phi$ with the following generic potential

$$V(\Phi,\phi) \,=\, m_\Phi^2\Phi^2 \,+\, \lambda\,\Phi^2\phi^2 \,+\, \frac{\alpha}{2}\,(\phi^2 \,-\, M_*^2)^2\,. \tag{2}$$

For definiteness, we shall take $\alpha \,\sim\, M^4/M_P^4,\;\; m_\Phi^2 \sim M^4/M_P^2,\; M_* \sim M_P$, where $M$ is some intermediate scale of order the supersymmetry breaking scale. $\lambda$ is a dimensionless coupling. The above potential is a typical potential for two moduli fields that parameterize supersymmetric flat directions, after the vacuum degeneracy is lifted by supersymmetry breaking effects. In the limit $M \to 0$, the directions become flat (the two branches of the moduli space touch at $\phi = \Phi = 0$ if $\lambda \neq 0$). We keep $\lambda$ as a free parameter, which may or may not be suppressed by the supersymmetry breaking scale (e.g., depending whether it comes from the Kähler or the superpotential). Although our construction is motivated by string theory moduli potentials, it is easy to envision generalizations to potentials characteristic of non-moduli fields.

The above potential has a true vacuum at $\phi \,=\, M_*$ and $\Phi \,=\, 0$, and an unstable saddle point at $\Phi = \phi = 0$. As long as we limit ourselves to sub-Planckian expectation values for the fields (which is the regime in which we shall work), at no point does our potential satisfy the standard slow roll requirements, and so naively inflation in this system is impossible. However, this expectation is false.

To see this let us imagine that in a homogeneous patch of size $> 1/H_* \simeq M_P/(\sqrt{\alpha}M_*^2) \sim M_P/M^2$, the two fields assume the initial values

$$\Phi_{in}^2 >> \alpha\,\frac{M_*^2}{\lambda} = \frac{m_\phi^2}{\lambda} \tag{3}$$

and $\phi = 0$. We shall show below that such an initial condition is naturally prepared by an initial tunneling process in a slightly more elaborate potential, but let us first discuss the resulting dynamics inside the patch, which proceeds as follows. $\Phi$ rolls toward zero, but overshoots and performs oscillations about $\Phi = 0$. The oscillation frequency is $m_\Phi \sim M^2/M_P$, and the time $\Phi$ spends in the neighborhood of the saddle point (within the interval $\Delta\Phi < \frac{m_\phi}{\sqrt{\lambda}}$) is $\Delta t \sim \frac{m_\phi}{m_\Phi\sqrt{\lambda}\Phi_{amplitude}}$. Until this time is shorter than the inverse tachyonic mass of the $\phi$-field ($m_\phi \sim M^2/M_P$) evaluated at the the saddle point, $\phi$ has no time to roll away from the saddle. Instead, it gets trapped in a false vacuum state with an effective mass$^2$

$$m_{eff}^2 \sim \lambda\,\langle\Phi^2\rangle - \alpha\,M_*^2\,, \tag{4}$$

where $\lambda\,\langle\Phi^2\rangle$ is the value averaged over the oscillation period. Thus, $\phi$ will get destabilized only after the second term takes over.

Until that moment, the energy density inside the bubble universe is composed out of the following two sources: 1) energy density of the oscillating scalar field

$$\rho_\Phi \simeq \frac{1}{2}\,(\dot{\Phi}^2 + m_\Phi^2\Phi^2)\,, \tag{5}$$

and 2) the false vacuum potential energy

$$V_0 = V(\Phi = \phi = 0) \sim M^4\,. \tag{6}$$

Note that for $\Phi_{in} \sim M_P$, the oscillation energy can be comparable to or even bigger than the false vacuum energy, but it quickly gets redshifted and the false vacuum energy takes over.

Once the false vacuum energy takes over, the original homogeneous patch starts inflating with Hubble parameter

$$H_*^2 \sim \frac{\alpha M_*^4}{M_P^2}\,. \tag{7}$$

Notice that we do not require any slow-roll regime, and in particular both of our fields have masses of order $H_*$ or heavier. The oscillations of $\Phi$ are governed by the equation

$$\ddot{\Phi} + 3H\dot{\Phi} + m_\Phi^2\Phi = 0 \tag{8}$$

according to which the amplitude of $\Phi$ diminishes as

$$\langle\Phi\rangle \simeq \Phi_{in}\,\mathrm{e}^{-\frac{3}{2}N}, \tag{9}$$

where $N$ is the number of e-foldings since the start of the "locked" period of inflation. The false vacuum will get destabilized and inflation will end only after the amplitude of $\Phi$ is reduced to the value for which $m_{eff}^2$ in equation (4) becomes negative. Therefore, the total number of e-foldings is given by

$$N \simeq \frac{1}{3}\ln\left(\frac{\lambda\Phi_{in}^2}{m_\phi^2}\right). \tag{10}$$

To estimate the expansion factor we can plug in some representative numbers. For instance, taking $\Phi_{in} \sim M_P$, $M \sim$ TeV, and $\lambda \sim 1$, we get $N \simeq 50$ or so. Note that these numbers enable us to reheat the universe to $T_R \sim TeV$ temperatures (after $\phi$ is liberated and the false vacuum energy is released in radiation). It follows that the size today of the original patch will be

$$R_{today} \sim \frac{1}{H_*}\,\mathrm{e}^N\,\frac{T_R}{T_{today}} \sim 10^{37}\mathrm{cm} \tag{11}$$

which is much larger than the present Hubble size.

Instead of the quartic cross-coupling, we might have chosen to stabilize $\phi$ by a higher order interaction of the form

$$\phi^2\,\frac{\Phi^n}{M_P^{n-2}}\,. \tag{12}$$

Then the size of the bubble today, expressed in terms of the inflationary scale $M$, would be

$$R_{today} \sim \frac{1}{M}\,(M_P/M)^{8/3n}\,\frac{M_P}{T_{today}}\,. \tag{13}$$

For instance, for $n = 4$ this would be sufficiently bigger than the present Hubble size for $M \sim 10$ GeV.

We should emphasize that although $N \leq 50$, there is no need to worry about the conflict with the usual lore that $\geq 60$ e-foldings of inflation are required. The horizon and flatness problems in our scenario are partially solved by the fact that preceding the "locked" phase of inflation, there is a stage of inflation in the original false vacuum (the decay from which sets the stage for the locked inflation). For this reason, even values of $N$ substantially less than 50 would be acceptable. $N$ must simply be large enough to inflate the bubble to a size large compared to $1/H_{today}$. Roughly speaking, our final stage of inflation must solve a smaller residual piece of the usual flatness problem, by inflating away the negative curvature of the bubble FRW universe until it is insignificant.

Finally, we would like to mention several caveats about our analysis of the model above.[b] Firstly, we have assumed the dynamics of the locked inflationary period is well described by the oscillating $\Phi$ solution. This is indeed true in the most natural regime for us, where $m_\Phi > \frac{3}{2}H_*$ but is not too much larger. For much larger $m_\Phi$, say $m_\Phi > 10H_*$, one must consider the effects of parametric resonance, explored in many papers on preheating in similar models [9]. The upshot in many cases is that the number of e-foldings will be roughly what we found here, but in such cases the description in terms of oscillating $\Phi$ fields becomes inaccurate after a few oscillations, and the locking occurs in a stranger way due to fluctuations of $\phi$. This is not important for $m_\Phi \leq 10H_*$ because the fluctuations in $\phi$ redshift away due to inflation faster than they grow due to the resonance. Secondly, one must worry about the spike in density perturbations produced when $\Phi$ has stopped oscillating and $\phi$ is about to condense and is experiencing large fluctuations. This spike shows up at very small scales, but could lead to excessive black hole production [10, 11]. To avoid this one should simply choose parameters so $m_\phi$ is greater than $H_*$ by a reasonable amount, or $M_*$ is suitably less than $M_P$. Thirdly, the potential (2) exhibits two vacua for the $\phi$ field, with a spontaneously broken $Z_2$ symmetry. For a fully realistic model, one must modify it to avoid overproduction of topological defects (in this case, domain walls). This is a standard issue with such potentials, and the fixes used in other cases can also be used here. For instance, one can promote $\phi$ into a representation of a larger symmetry group (e.g. make it an $SU(2)$-doublet) that does not permit topological defects. Alternatively, one can add a symmetry breaking term to the potential – this should not change our analysis at all, since the $\phi$ field plays almost no roll in the inflationary dynamics.

[b] We thank L. Hui, L. Kofman and A. Linde for discussions on these points.

### 2.3. *The complete model*

Now we are ready to discuss the complete model that would prepare the desired initial conditions for the oscillating $\Phi$ field. We shall argue that these initial conditions can naturally follow if $\Phi$ tunnels from a false vacuum state that was driving a preceding stage of "old"-type vacuum inflation. To achieve this we have to supplement the potential for $\Phi$ by weak self-interactions that would prepare another false vacuum for large values $\sim M_P$. Note that because $\Phi$ is a very weakly self-coupled modulus, all of the previously discussed dynamics is completely unaffected by the new self-couplings. They only modify the story before tunneling, and can be ignored after.

Consider, then, two fields $\Phi$ and $\phi$ with the following generic potential

$$V(\Phi,\phi) \,=\, \alpha_0\Phi^2(\Phi - M_0)^2 \,+ m^2\Phi^2 \,+ \,+ \lambda\,\Phi^2\phi^2 \,+\, \alpha\,(\phi^2 \,-\, M_*^2)^2\,. \quad (14)$$

Here, the new parameters are $m^2 \sim \frac{M^4}{M_P^2},\ \alpha_0 \sim \frac{M^4}{M_P^4},\ M_0 \sim M_P$.

The self-interaction potential of $\Phi$ adds a new false vacuum state $\Phi \simeq M_0, \phi = 0$ to the already existing extrema of § 2.2, which are more or less unaffected by the new terms.

As in the "old inflation" scenario the universe starts in the false vacuum and ends in the true one. Just as in old inflation, the first inflationary stage ends via bubble nucleation.[c] From this point on our scenario differs dramatically from old inflation. Notice that in the false vacuum, the mass of the $\phi$ field is typically very large ($\sim \lambda M_P$) as compared to the inflationary Hubble scale

$$H_0^2 \,\simeq\, M^4/M_P^2 \quad (15)$$

and so $\phi$ can be integrated out. Then to first approximation, the presence of $\phi$ cannot not affect the tunneling dynamics, and $\Phi$ tunnels towards $\Phi = 0$, as if $\phi$ never existed. The initial size of the nucleated bubble is an important issue, which we shall discuss separately in a moment. For a moment we shall assume that the size is $\sim 1/H_0 \,\sim\, 1/H_*$ (but as we shall see in § 2.4 this is not an important assumption). In order to understand the bubble dynamics after nucleation, note that typically $\Phi$ will not materialize at $\Phi \,=\, 0$ but at some initial value $\Phi = \Phi_{in} \,\sim\, M_P$. As the bubble evolves, $\Phi$ will roll towards $\Phi = 0$ and perform oscillations about that point, and the rest of the story proceeds as in § 2.2.

[c] Given the shape of our potential, the tunneling is best thought of as a Hawking-Moss instanton [12]. The most reasonable description of such instantons is given by the stochastic approach to inflation, as described in [6].

As in the previous case, $\phi$ will be trapped in an evanescent false vacuum state and will drive inflation with essentially constant Hubble rate $H_*$, and a number of e-foldings given by (10). The inflation will redshift away the $\Phi$ oscillations, eventually destabilizing the $\phi$ field. It then rolls to it's true vacuum and reheats the universe. The final size of the bubble at today's temperature is given by (11).

### 2.4. *Size doesn't matter*

In this section we shall discuss the issue of the initial bubble size. It was important in the previous section to take the initial size of the bubble large enough ($\sim 1/H_*$), so that once the potential energy of the $\phi$ field dominates the system could inflate. What happens if instead bubbles get materialized at a much smaller size, which would be generic for certain potentials?

Before proceeding, we want to stress that there is a standard way of dealing with this, by choosing the potential parameters in such a way that the most probable bubbles have size $\sim 1/H_*$. However, in our case there is no need to do this. We shall now try to argue that in the two-field model we have just discussed, the bubble size does not really matter, in the sense that even much smaller initial bubbles will actually inflate and achieve a size well approximated by (11).

To see this let us follow the dynamics of a bubble of size $<< 1/H_*$ (that is, curvature $>> H_*^2$). The bubble observer sees an open FRW universe, and its dynamics is governed by the Friedmann equation

$$(\frac{\dot{a}}{a})^2 - \frac{1}{a^2} = \frac{8\pi}{3} G\rho\,, \tag{16}$$

where $\rho = \rho_\Phi + V_0$ and thus the right hand side is $\sim H_*^2$. Therefore, for the small bubbles, for which the curvature radius is $<< 1/H_*$, the curvature term in the Friedmann equation dominates over the energy density $\rho$ and the universe expands quickly, reaching size $1/H_*$ in a time of order $1/H_*$.

What happens to the amplitude of the $\Phi$ field during the curvature-dominated expansion? It follows from equation (16) that during the curvature dominated expansion, we have $H^2 \simeq 1/a^2 >> H_*^2$. Hence, it is obvious from equation (8) that the amplitude will stay constant until the curvature of the bubble becomes comparable to $m_\Phi^2$, since for $H >> m_\Phi^2$ the oscillations are strongly over-damped (the friction dominates) and $\Phi$ is essentially frozen at its initial value. $\Phi$ will start oscillations only after the curvature drops to $\sim m_\Phi^2$, which, since in our model $m_\Phi \sim H_*$, means that field will start oscillations only after the bubble reaches size $1/H_*$. Hence, irrespective of the initial size of the bubble, by the time it grows to the size $1/H_*$, $\Phi$ will have essentially the same initial amplitude. Thus, the smaller bubbles will achieve roughly the same final size that we derived by assuming they materialized at size $1/H_*$ and began inflating immediately.

### 2.5. *Thermal case*

Let us briefly discuss what would happen in the regime in which $\Phi$-field could quickly decay (e.g., through parametric resonance effects) into quanta that thermalize at temperature a $T_{in}$. If the $\Phi$-quanta come into thermal equilibrium at temperature $T_{in}$, they will create a thermal mass$^2$ for the $\phi$ field

$$\sim \lambda \phi^2 T^2 \tag{17}$$

which can also stabilize $\phi$ at an evanescent false vacuum, and drive thermal inflation [13]. However the scenario based entirely on thermal stabilization has the following disadvantage. The resulting number of e-foldings is smaller than in the case of locking by $\Phi$ oscillations. Since the stabilizing temperature redshifts as $T \propto 1/a \propto \mathrm{e}^{-N}$, the number of e-foldings is given by

$$N \simeq \frac{1}{2} \ln\left(\frac{\lambda T^2}{m_\phi^2}\right). \tag{18}$$

However, unlike the initial value of the scalar field $\Phi$, the initial value of the temperature at the onset of thermal inflation ($T_{in}$) is limited by $T_{in} < \sqrt{H_* M_P} \ll M_P$. In addition, taking into account the fact that natural value for the curvature in the unstable direction of the zero temperature potential is $\sim H_*^2$ (no fine tuning), the resulting number of e-foldings is limited to

$$N \simeq \frac{1}{2} \ln\left(\frac{M_P}{H_*}\right) \tag{19}$$

and is significantly smaller, than in the oscillation-stabilization case.

Although this scenario is less flexible than that of § 2.2, it is still rather interesting, for the following reason. Suppose that after thermal inflation, the bubble reheats to a (maximal possible) temperature $\sim \sqrt{H_* M_P}$, and undergoes normal FRW evolution until reaching today's temperature $T_{today}$. The resulting size is then given by the following expression

$$R_{today} \sim R_{in} \left(\frac{\sqrt{H_* M_P}}{H_*}\right) \left(\frac{\sqrt{H_* M_P}}{T_{today}}\right) \sim R_{in} \frac{M_P}{T_{today}}. \tag{20}$$

We would like to require at the very least that

$$R_{today} >\sim 10^{42} (GeV)^{-1} \,. \tag{21}$$

Assuming $R_{in} \sim 1/H_*$ and plugging in the numbers, we see that with $H_* \sim 10^{-13} GeV$ (the minimal value consistent with TeV reheating inside the bubble), we are consistent with (but close to saturating) the inequality (21). We then get the prediction that with the requirement that reheating to $\sim$ TeV should be possible, our two field thermal models predict a bubble radius within a few orders of magnitude of the current horizon size![d] Of course if one requires only the bare minimum of reheating to temperatures $\sim$ GeV (for consistency with baryon creation), one can make two field thermal models with considerably larger values of $R_{today}$.

## 3. Generalizations

In this section, we describe two generalizations of the basic scenario in § 2. The first involves a cascade with several stages of locked inflation, while the second explains how to construct models where the initial false vacuum has very high Hubble parameter.

### 3.1. *Cascades*

So far we have illustrated our scenario using simple two field examples. In realistic string theory models, because there can be a large number of cross-coupled fields, one might expect several stages of evanescent "locked" inflation. We envisage this dynamics as follows. The string landscape is rather complicated (see e.g. [14–16]) and presumably has many false vacua and saddle points. We assume that the inflation that gave rise to an observable part of the universe was initiated in one of these false vacua, and the first stage of inflation was terminated by tunneling. Unless parameters are severely fine tuned, in the system of several moduli fields, tunneling from the false vacuum never proceeds directly into the true one. Instead, after tunneling the system has to roll down to the true minimum through a cascade of saddle points. Because the original rolling direction could rather generically be expected to be orthogonal to the direction of eventual exit from the saddle point, the system necessarily oscillates about each of these, perhaps locking itself into a temporary inflationary state. For instance, in our two field example of § 2, the rolling direction ($\Phi$) was orthogonal to the unstable mode of

[d] L. Susskind has seriously considered the possibility that there may be signatures arising from a bubble wall just outside of our present horizon, for other reasons.

the saddle point ($\phi$), and the system was forced to oscillate about the saddle before finding the exit. After exiting from the first saddle point, the system may well get trapped in another one, and so on, resulting in a cascade of inflationary stages. Of course, some of the exits may well be accompanied by energy release into radiation, reducing the number of e-foldings of inflation in subsequent stages.

In order to estimate the *maximal* expected size of the bubble after $n$ stages of such locked inflation, let us make the following simplifying assumptions. Assume that after exiting the first stage of false vacuum inflation, which may have an arbitrarily large Hubble (see the next section), the typical curvature of the moduli potential is of order

$$V'' \sim V/M_P^2 \sim H^2 \,. \tag{22}$$

Notice that this is generically true for moduli fields (this is related to the supergravity eta problem), and it is precisely the condition one usually tries to *avoid*, via fine tuning, in order to find slow-roll inflation. We do not need to perform such a tune in our scenario. The scale factor after the $i$-th stage of saddle point inflation grows by a factor

$$\mathrm{e}^{N_i} \simeq \left(\frac{\lambda_i \Phi_{in}^{(i)2}}{|V_i''|}\right)^{\frac{1}{3}} \tag{23}$$

where: 1) $\Phi_{in}^{(i)}$ is the initial amplitude of the oscillating field which blocks the exit from the saddle point by giving a positive mass$^2$ to the unstable mode of the the saddle point; 2) $\lambda_i$ is the coupling between the unstable and oscillating modes; and 3) $V''$ is the negative mass$^2$ of the unstable mode. The size of the bubble today is then given by the following expression

$$R_{today} \sim R_0 \prod \left(\frac{\lambda_i \Phi_{in}^{(i)2}}{|V_i''|}\right)^{\frac{1}{3}} \frac{T_R}{T_{today}} \,, \tag{24}$$

where $R_0$ is the bubble size at the onset of the first saddle point inflation and is equal to the inverse Hubble parameter at that time, and $T_R$ is the final reheating temperature. To estimate the maximal bubble size, let us take for the maximal initial amplitudes $\lambda_i \Phi_{in}^{(i)} \sim M_P$, and for the minimal destabilizing curvature $V_i'' \sim H_*^{(i)}$ (no slow-roll fine tuning). Assuming maximally efficient reheating, the temperature after the last stage of inflation is given by $\sim \sqrt{H_*^{last} M_P}$. Then (24) reduces to

$$R_{today}^{max} \sim \frac{1}{H_*^{(1)}} \prod \left( \frac{M_P}{H_*^{(i)}} \right)^{\frac{1}{3}} \frac{\sqrt{H_*^{last} M_P}}{T_{today}} . \tag{25}$$

If we further assume that the order of magnitude of the potential throughout the $n$ stages stays the same (which can be a good approximation if all of the stages are driven by light moduli fields with scales in their potentials set by SUSY breaking as in § 2), that is $H_*^i \sim H_*$, this expression simplifies to

$$R_{today}^{max} \sim \left( \frac{M_P}{H_*} \right)^{\frac{2n}{3} + \frac{1}{2}} \frac{1}{T_{today}} . \tag{26}$$

Requiring that this size is at least couple of orders of magnitude bigger that the present horizon, we can estimate the maximal values for $H_*$, for a given $n$. For instance, for $n = 2$, $H_*$ can be around $\sim 100$ GeV.

### 3.2. *Large initial Hubble*

There are simple variations of our scenario that allow one to have arbitrarily large initial Hubble parameter $H_0$, without affecting the initial size of the inflating bubble or the rest of our scenario. This is important in thinking about the initial conditions for inflation. One would ideally like a scenario which begins very close to Planckian energy density, since one might expect the universe to only survive roughly a Planck time unless it begins inflation immediately after it is born at the Planck energy density. Let us present one such generalization (obviously, there are many others).

It is well known that in realistic superstring theories the supersymmetry-breaking soft masses and self-couplings of the moduli fields are not constant but rather are "spurions", functions of (super)fields whose auxiliary components ($F$-terms) break supersymmetry spontaneously. One has to keep this in mind when studying the cosmological evolution of the moduli fields. For instance, the soft mass and self couplings of the oscillating scalar field $\Phi$ considered in our examples, can originate from interactions in the Kähler potential of the form

$$\frac{\Theta^* \Theta}{M_P^2} \left( \Phi^* \Phi + \frac{(\Phi^* \Phi)^2}{M_P^2} + ... \right) , \tag{27}$$

where $\Theta$ is superfield with non-zero $F$-term $F_\Phi^* F_\Phi \sim M^4$. In our previous analysis we have regarded these VEVs as frozen. This was justified since our inflation was proceeding at a scale not exceeding $M^4$, at which effectively the $\Theta$ field can be integrated out. However, if we are interested in inflation at much higher scales, the dynamics of the $\Theta$-field has to be taken into the account. In particular, $\Theta$ can have a false vacuum state at a much larger energy scale, in which its $F$-term has much higher value. Consequently, the soft masses of the moduli would be much larger. Then the first stage of our inflation could be driven in this false vacuum, with a very high Hubble.

To take this possibility into account, we will consider the simplest prototype model in which we shall promote the mass of the oscillating $\Phi$ field into a *spurion*, a super-heavy field that tunnels from the high energy density false vacuum to the lower one. As above let us call this new heavy field $\Theta$. The potential then can be written as

$$V(\theta,\Phi,\phi) = \tilde{M}^2\,\Theta^2 + a\,\tilde{M}\Theta^3 + b\,\Theta^4 + c\,\Theta^2(\Phi - M_0)^2 \\ + m_\Phi^2\Phi^2 + \lambda\,\Phi^2\phi^2 + \alpha\,(\phi^2 - M_*^2)^2, \qquad (28)$$

where $\tilde{M}$ is some large mass scale and $a, b, c \sim 1$. For simplicity only the essential couplings are displayed here, and one could add many other generic couplings that would to the job (for instance, all possible cross couplings like $\Theta^2\phi^2$, $\Theta^4\Phi^2$ etc.). $\Theta$ can be thought of as a spurion field that has a false vacuum with supersymmetry broken at the scale $\tilde{M}$. In that vacuum all the other fields get huge soft masses, and are trapped in false vacuum states. After the $\Theta$ tunneling the scale of supersymmetry-breaking diminishes, and the light fields are liberated, resulting into our picture.

Indeed, the above potential has: 1) a false vacuum at $\Theta \sim \tilde{M}$, $\Phi = M_0$, $\phi = 0$ with energy $V_0 \sim \tilde{M}^4$; 2) a true vacuum at $\Theta = \Phi = 0$, $\phi = M_*$ with zero energy; and 3) a saddle point $\Theta = \Phi = \phi = 0$, with energy $V_* = \alpha M_*^4 \sim H_*^2 M_P^2$. We assume that inflation starts in the false vacuum state, with corresponding Hubble $H_0 \sim \tilde{M}^2/M_P$ that can be arbitrarily large. The "old" inflation ends when the heavy $\Theta$ field tunnels to $\Theta = 0$ in a bubble of size $R_{in}$. The mass of the $\Phi$ field (which was stuck at $\Phi_{in} = M_0$ in the false vacuum) drops to $m_\Phi \sim H_*$ inside the bubble. This mass tries to push $\Phi$ it toward the saddle point. However, $\Phi$ is prevented from rolling by the expansion rate inside the bubble, until the curvature drops to $\sim 1/H_*$. At this point, $\Phi$ rolls and starts to oscillate around $\Phi = 0$. Then, the locked inflation sets in and our scenario of § 2 follows.

## 4. Density Perturbations

### 4.1. *Basic mechanism*

Because we do not invoke slow-roll inflation, the inflaton field never has $m^2 << H^2$ during either inflationary phase. This means that it will not generate sufficient density perturbations (actually given our exceptionally low scale of inflation it could not do so anyway), and we must find a different explanation for the observed $\frac{\delta\rho}{\rho}$. Luckily, there has been a recent suggestion [7, 8] of a new, natural mechanism to generate such perturbations.

We introduce another scalar field $\chi$, with the properties typical of a modulus field in string theory. That is, $\chi$ is a very weakly self-interacting modulus, that contributes almost no energy. The sole role of $\chi$ is to control the decay rate of the inflaton driving the final stage of inflation (which we will call $\phi$ as in § 2) into Standard Model particles. All the above features are accommodated by the following prototype potential (one can include many additional couplings without affecting our mechanism, but we shall keep things as simple as possible)

$$V(\phi, \chi) = V(\phi) + \mu^2\chi^2\,. \tag{29}$$

Here $V(\phi)$ can be any inflationary potential which satisfies the requirements described in § 2.

As for the parameter $\mu^2$, it should be understood as an *effective* mass of the $\chi$ field on a given inflationary background. Because of the possible cross-couplings between $\chi$ and the inflaton fields, $\mu^2$ in general will be a function of the inflaton VEVs, and thus may change in time. One necessary requirement is that $\mu$ must be at least an order of magnitude or so smaller than the inflationary Hubble parameter $H$, in order to accommodate the mechanism of reference [7]. $H$ should be understood to mean the Hubble parameter at the inflationary stage during which perturbations are imprinted in $\chi$. As we shall see in § 5, it is only important in our scenario that $\chi$ be light during the very last stage of inflation. Let us first however briefly discuss the mechanism which generates perturbations.

We assume that $\phi$ can decay into some Standard Model fermions (call them $q$) through the following dimension-five operator

$$\phi\,\frac{\chi}{M_0}\,qq\,, \tag{30}$$

where $M_0$ is the mass scale of the new physics which was integrated out to yield the dimension five operator (30). The crucial point is that the effective decay rate of $\phi$ is set by $\chi$ through

$$\Gamma_{\phi\to qq} \sim \left(\frac{\chi}{M_0}\right)^2 M_\phi\,, \tag{31}$$

where $M_\phi$ is the mass of the $\phi$ particles at the true minimum, $\phi_{true}$. This enables one to convert all the de Sitter fluctuations imprinted in $\chi$ into the observed density perturbations, during the $\phi$-decay.

More specifically, the mechanism of [7] works as follows. During reheating, the perturbations imprinted in the $\chi$ field get translated into variations of the effective Yukawa coupling controlling $\phi$ decay to Standard Model particles. This yields the density perturbations. Intuitively, this is because the spatial variation in the Yukawa coupling yields a spatially varying reheat temperature. The resulting density perturbations are

$$\frac{\delta\rho}{\rho} = -\frac{2}{3}\frac{\delta\Gamma}{\Gamma} = -\frac{4}{3}\frac{\delta\chi}{\chi}. \tag{32}$$

### 4.2. *Perturbations from dimension four operators*

Here, we present a slight modification of the mechanism of § 4.1 which may be more efficient in some cases, since reheating can be controlled by dimension 4 operators.

Let us imagine that instead (or in addition) of a dimension 5 operator controlling the $\phi$ decay, we have instead the following generic couplings to some $\tilde{q}$-scalars

$$V_{\phi\to\tilde{q}\tilde{q}} = -c\phi\chi\tilde{q}^2 + m^2\tilde{q}^2 + d\tilde{q}^4 + M^2\frac{\phi^2}{M_P^2}\tilde{q}^2 + f\frac{\chi}{M_P}\tilde{q}^4 + g\tilde{q}\frac{\chi}{M_P}qq + \ldots. \tag{33}$$

Here $m^2, M^2$ are a masses of order $(TeV)^2$, $c, d, f, g$ are dimensionless, and we assume that the quartic coupling is negative ($c > 0$). The essence of the above potential is that masses and couplings (self-couplings as well as couplings to other $q$-species), of $\tilde{q}$ scalars are controlled by $\chi$. Interestingly, the natural candidate for a $\tilde{q}$-scalar is the electroweak Higgs of the Standard Model ($q$-particles can represent quarks and leptons).

Reheating and density perturbations then work as follows. During both stages of inflation, de Sitter fluctuations are imprinted on the $\chi$ field. During inflation $\phi = 0$ and $\tilde{q} = 0$ (because of the positive mass term). After inflation, $\phi$ rolls away from its false vacuum and gets a large VEV. As a result, $\tilde{q}$ gets a negative contribution to it's mass, and can be destabilized provided we choose the couplings so that at some point on the trajectory

$$-c\phi\chi + m^2 < 0. \tag{34}$$

We also choose $d > 0$ so that $\tilde{q}$ has a stable vacuum.

Since the VEV of $\chi$ is larger than the inflationary Hubble scale $H_* \sim (TeV)^2/M_P$, and since $\phi$ rolls towards $M_P$, with natural choices of couplings $\tilde{q}$ will be destabilized. At this point, the system can start rapid oscillations about the true minimum for the $\tilde{q}$ field, with reheating to a temperature of order $T_R \sim TeV$. Due to the $\chi$ fluctuations, however, the reheating process will not be uniform. Depending on how large $\delta\chi$ is, the non-uniformity could be imprinted either through dimension 5, or dimension 4 operators. If $\delta\chi$ is sufficiently large (see 5.4) there is no need to use dimension 4 operators (in such a case we can set $c = 0$), and density perturbations will be generated as discussed in the previous section. That is, for large $\delta\chi$ the rate by which $\tilde{q}$-particles self-thermalize and thermalize with the other species, will fluctuate significantly, and this fluctuation will be the main source for density perturbations, which will be given by (32).

However, for small $\delta\chi$, dimension 5 operators are irrelevant, and perturbations should be imprinted through dimension 4 operators. This is easy to understand: since $\chi$ varies over space, the condition (34) will be satisfied at different times (different values of the scale factor) in different places. This leads to a situation where the reheating process can be delayed at some points relative to others, and yields adiabatic density perturbations: $\frac{\delta\rho}{\rho} \sim \frac{\phi\delta\chi}{m^2}$, and $T_R \sim m$.

## 5. The Spectrum

### 5.1. *Scalar perturbations*

We shall now analyze the spectrum of fluctuations. In the approximation of rapid reheating, this spectrum is essentially defined by perturbations imprinted in $\chi$, and we have to make sure that it is sufficiently scale-invariant. In the case of standard inflation, flatness of the spectrum is guaranteed by an almost constant value of the Hubble parameter. In our two-stage scenario, the Hubble parameter jumps and one could worry that the two different inflationary Hubble parameters may lead to observable deviations from flatness in the observed spectrum. This is not the case, for the following simple reason: Any perturbations generated during the first stage of inflation end up well outside of the present horizon, and hence do not spoil scale invariance of the perturbations generated during the second stage.

To see this, imagine the bubble nucleates at a generic size $\sim 1/H_0$, and recall that $H_0 > H_*$. As in § 2.4, the bubble observer sees a curvature dominated universe until the bubble reaches size $1/H_*$. During this period, the waves of a given initial wavelength expand like the scale factor $a$. The smallest wavelength perturbations from the first phase enter the bubble with wavelength $\sim 1/H_0$ (shorter wavelengths are strongly suppressed). They are therefore redshifted to size $\sim 1/H_*$ by the time the second stage of inflation begins. It follows that by the end of the second period of inflation and the subsequent reheating, they have been redshifted to a size

$\sim (1/H_*)e^N(T_R/T_{today})$. Therefore, they are at wavelengths larger than our current horizon size, by equation (11).

### 5.2. *Explicit numbers*

We can now choose some representative numbers, to see how things work. Reheating happens when the Hubble parameter becomes equal to the decay rate

$$H \ = \ \Gamma \ . \tag{35}$$

Assuming instant reheating, and using the formula (31), one finds

$$T_R \ \sim \ \sqrt{\Gamma M_P} \ \sim \ \chi\Big(\frac{M_P M_*}{M_0^2}\Big)^{1/2} \ , \tag{36}$$

where $M_*$ should be understood as the mass of the oscillating field that decays into the radiation, which may not necessarily coincide with an inflaton field. For instance, in the model of § 4.2, $M_*$ is the mass of a $\tilde{q}$ particle. We would like to have $T_R \geq TeV$.

It follows from these equations that if we demand that the observed perturbations are generated during the bubble inflation stage, meaning that $\delta\chi \sim H_*$, while insisting on dimension-5 dominated reheating, the $M_0$ scale must be taken way below $M_P$. For $H_* \sim 10^{-13}\ GeV$, we see that we should choose $\chi \sim 10^{-8}\ GeV$ to give the required density perturbations. Finally, from (36), we see that one needs $M_0 \sim GeV$ to get $T_R \sim TeV$. In this respect for the scenarios in which perturbations are imprinted during our low-scale bubble inflation stage, it is more natural to use dimension 4 operators for translating $\delta\chi$ into the density perturbations as in § 4.2. This mechanism does not require any small dimensionless couplings.

In the case of more than one stage (a cascade) of bubble inflation, the numbers are much more flexible. For instance, consider the case with two stages of such inflation with Hubble $H_* \ \sim$ 100 GeV - TeV, discussed at the end of § 3.1. For dimension-five reheating we have $\chi \sim 10^8$ GeV and $M_0 \sim 10^8$ GeV for $T_R \sim 10^{11}$ GeV, and $M_0 \sim 10^{16}$ GeV for $T_R \sim$ TeV.

Finally let us comment that the light field $\chi$ required by our scenario can naturally originate, e.g., from a "hidden" sector to which supersymmetry breaking is transmitted via gravitational interactions. Alternatively, $\chi$ may be a pseudo-Goldstone particle of some spontaneously broken symmetry. In either case, keeping the mass of such a field below the Hubble parameter is easier than in the case of the slow-roll inflaton field, since at no stage is the $\chi$ energy density required to dominate the universe. In the other words $\chi$ is just an another light field, and has many fewer "responsibilities" than the standard inflaton, which is required to play the role of the clock that stops inflation by exiting slow-roll.

### 5.3. *Gravitational waves*

Gravitational waves basically cannot be generated during our stage of bubble inflation, due to the low Hubble constant. So the only gravitational waves that could be potentially observed are the ones generated at the first inflationary stage (in the cases with large $H_0$). However, these are generically redshifted outside of our present horizon, as in § 5.1. We therefore have a universal prediction that tensor perturbations will not be observed in our scenario.

## 6. Cosmological Moduli Problem

In this section we shall discuss possible implications of our scenario for the cosmological moduli problem. In models where there are additional moduli fields which receive masses in the dangerous range for the cosmological moduli problem [17], a second stage of inflation, with a tiny Hubble constant, could dilute them sufficiently to address the problem (for the case of thermal inflation this was suggested in [13]).

Let $Z$ be a canonically normalized modulus field, and we shall assume that *today's* zero-temperature minimum is at $Z = 0$. The small oscillations of $Z$ about such a minimum are then governed by the usual equation

$$\ddot{Z}_k + 3\,H\,\dot{Z}_k + m_Z^2\,Z \,=\, 0\,. \tag{37}$$

Due to the extremely small self-couplings of $Z$ (typically suppressed by powers of $M_Z^2/M_P^2$) the anharmonic corrections to $Z$ are only important for $Z \sim M_P$, and thus, can be neglected for small oscillations.

The source of the problem is that in the early universe $Z$ is expected to have an initial value $Z \sim M_P$. With this amplitude $Z$ would start late oscillations about the true minimum, and because of the small mass and decay rate, would either overclose the universe, or decay and destroy the successful predictions of big bang nucleosynthesis.

To be more rigorous, we have to take into account the fact that in the early universe the moduli receive an additional contribution to their masses of the form [18, 19]

$$\Delta\, m_Z^2 \,\sim\, \frac{\rho}{M_P^2} \,\sim\, H^2\,, \tag{38}$$

where $\rho$ is the energy density at the given time. Such corrections to the masses are induced not only during inflation but also during reheating, and can be seen by taking the thermal average of the matter kinetic terms (to which $Z$ is coupled) [19], or more rigorously by computing thermal diagrams with external $Z$-legs [20]. Thus, the friction term in (37) never dominates, and moduli will perform damped oscillations about their temporary minima, with amplitudes that fall as $\sim e^{-\frac{3}{2}Ht}$. Hence, if for some reason the early and late minima coincide, the moduli problem can be solved by any inflation, irrespective of the scale [18, 19].

The problem, however, is that in general the early and the late minima *do not* coincide, because at high densities moduli get corrections not only to mass$^2$ terms, but also acquire a tadpole

$$Z\,\frac{\rho}{M_P}\,\sim\,Z\,H^2\,M_P \tag{39}$$

which inevitably displaces $Z$ from its zero temperature minimum. The displacement can be estimated as

$$\Delta Z\,\sim\,\frac{H^2}{m_Z^2\,+\,\Delta\,m^2}\,M_P\,. \tag{40}$$

Below we shall only be interested in the case when the mass-shift generated during the second inflation is smaller than the zero-temperature mass $m_Z^2\,>\,\Delta\,m^2$ (in the opposite limit the problem cannot be solved by inflation anyway). In such a case the resulting energy density difference, due to displacement of $Z$, is roughly

$$\rho_{in}\,\sim\,\frac{H^2}{m_Z^2}\,H^2\,M_P^2\,. \tag{41}$$

Note that any pre-existing energy density will be quickly diluted by the second stage of inflation, during which $Z$ will settle to its temporary minimum at $Z\,=\,\Delta\,Z$.

After reheating and the subsequent cooling of the universe, $Z$ starts oscillations about the new equilibrium point which very quickly approaches the true minimum. So for our analysis we can assume that right after reheating $Z$ starts performing oscillations about its true minimum, with initial energy density given by (41). After this point the energy density stored in the modulus field redshifts as $T^3$, and today (neglecting decay) would be given by

$$\rho_{today}\,\sim\,\rho_{in}\frac{T_{today}^3}{T_R^3}\,. \tag{42}$$

For consistency with observations, $\rho_{today}$ must be smaller than the critical energy density of the universe $\rho_c$. Using equation (41) this requirement gives the condition

$$\rho_c > \frac{T^3_{today}}{T^3_R} \frac{H^2}{m^2_Z} H^2 M^2_P \,, \tag{43}$$

where in our case we shall take $H = H_*$, and $T_R \sim \sqrt{H_* M_P}$. Equation (43) then indicates that for $H_* \sim \text{TeV}^2/M_P$ and $T_R \sim$TeV, the moduli problem can be solved for an arbitrary modulus with mass $m_Z >$ KeV or so. By pushing $H_*$ to its absolute possible lower bound $\sim \text{GeV}^2/M_P$ (compatible with $T_R$ being just above the baryon mass), we can push the solvable range for $m_Z$ all the way to $10^{-3}$ eV.

An important assumption here was that the $Z$ mass-shift generated by the second stage of inflation is $< m^2_Z$. This is quite plausible for low $H_*$ inflation, and less so for inflation at the more conventional high scales. This is why our last stage of low-scale inflation can naturally alleviate the moduli problem (as described for thermal inflation in [13]).

## 7. Predictions

Here, we enumerate several fairly robust predictions of our scenario.
1. The scale of inflation $H_*$ is very small compared to most other proposals for inflation. It is therefore clear that gravitational waves will be reduced to unobservable levels, in generic realizations of our scenario.
2. In the generic models the tilt can be arbitrarily small, and is given by

$$n - 1 \sim \frac{\mu^2}{H^2_*} \,. \tag{44}$$

In this equation $\mu$ is the $\chi$ mass during the second stage of inflation, $\mu << H_*$. In particular, a practically flat spectrum is easily achievable in our picture, in contrast with the standard slow-roll case, and can be a smoking gun for our scenario.
3. Our scenario predicts larger non-Gaussianities than standard inflation. It should be stressed that both the possibility of an exactly flat spectrum (44), as well as large non-Gaussianities, are characteristic features of density perturbations generated through the decay-rate-fluctuation-mechanism of references [7,8]. In particular for non-Gaussianities this mechanism gives [7]

$$f_{nl} = -5/2 \,. \tag{45}$$

However, when implemented in the standard inflationary scenario, with a large scale, the non-Gaussianities as well as the flatness of the spectrum can be washed out by the additional fluctuations coming from the usual source. In our case, since no such contributions are available, both (44) and (45) are elevated into predictions.

It would be interesting to see whether our scenario naturally emerges from some class of microscopic theories. Recently, it has become increasingly clear that string theory has a "discretuum" of closely spaced metastable vacua (see e.g. [14–16] for discussions of this in various contexts). The prospects for embedding slow-roll brane inflation into this discretuum were recently explored in [21] and references therein. It seems at first glance that our scenario would be a natural fit for this picture of the string theory vacuum landscape, since the discretuum naturally has many vacua with bubbles of vacuum decay interpolating between them.

The discretuum idea also suggests a natural solution to the problem of initial conditions for inflation. Imagine there are many metastable de Sitter vacua with a variety of cosmological constants. A portion of the universe originally gets trapped in a vacuum with $H$ very close to $M_P$, which then (eternally) inflates and occasionally seeds bubbles of phases with lower Hubble. These in turn populate further regions of the landscape through vacuum decay. In this way, one can easily envision a cascade where regions of lower and lower $H$ get populated. The penultimate step in the chain which leads to our bubble, is the creation of a phase of Hubble constant $H_0$. Given this phase, the initial conditions for our inflationary scenario are explained.

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