1.) For an isotropic dielectric medium the dispersion relation we derived in class becomes 
\[ m^2 = k^2 c^2 / \omega^2 = \varepsilon, \]
where, \( \varepsilon = 1 + 4\pi i \sigma / \omega \) is the dielectric constant and \( m = m_r + i m_i \) is the index of refraction. Consider propagation in the z direction of such a wave with intensity \( I = (c/4\pi)E_0^2 \) at \( z = 0 \). (This last relation assumes \( |m| \approx 1 \).) Let \( \omega \) be real, so that \( k \) is generally complex. Also let the conductivity, \( \sigma = \sigma_r + i \sigma_i \), be complex. Show then that the absorption coefficient in the equation of radiation transfer is then \( \alpha = (4\pi/c)(\sigma_r / m_r) \), assuming \( |m_i| \ll |m_r| \). Explain physically why this result depends on the real part of the conductivity, not the imaginary part. (Remember, if \( E \) is complex, then \( |E|^2 = E \cdot E^* \).)

2.) In class we derived the dispersion properties of a transverse electromagnetic wave propagating parallel to a static magnetic field in a diffuse plasma. The two normal modes are circularly polarized and propagate according to 
\[ (\omega / k_1^2 / \omega^2 \pm (1/2)(\omega^2 / \omega) \pm (1/2)(\omega^2 / \omega^3) \sigma_b. \]

a) If a plane polarized wave packet can be described by coherent contributions from right and left handed components, \( E_z = (1/\sqrt{2}) (E_x + i E_y) \), show that the Q and U Stokes parameters can be written \( Q = (c/4\pi)2 \text{Re}(E_x E_y^*) \) and \( U = (c/4\pi)2 \text{Im}(E_x E_y^*) \). (Note in this case that \( V = 0 \). Why?)

b) Then letting \( \Delta k = k_r - k_0 \), show that as the wave packet propagates through our magnetized plasma \( dQ / dz = -\Delta k U \) and \( dU / dz = \Delta k Q \). Show from this that the polarization plane of the wave packet rotates through an angle \( \chi = (1/2)\Delta k z \) if the wave propagates a distance \( z \) and express this in terms of the plasma density, \( n \), the magnetic field strength, \( B \) and the wavelength, \( \lambda = 2\pi / k \). (Note: A convenient computational device here is to define \( P = Q + iU \) and then to write a transfer equation for \( dP/dz \).)

3.) Consider an electromagnetic wave propagating through a medium described by two orthogonal linearly polarized normal modes. The normal wave polarization (E-field) directions (1, 2) are \( E_x \) and \( E_y \). The medium is birefringent with phase velocities \( v_{1,2} = \omega / k_{1,2} \) and the wave propagates in the z direction. Consider a wave packet composed of equal amplitudes in \( E_x \) and \( E_y = E_0 \), but with an arbitrary phase difference, \( \Delta \phi = \phi_1 - \phi_2 \), between the two components.

a) Write down expressions for the 4 Stokes parameters in terms of \( E_0 \), \( \Delta k z = (k_1 - k_2) z \) and \( \Delta \phi \).

b) Show that as the wave packet propagates through the medium its polarization alternates between linear and circular as \( \Delta k z \) changes through \( \pi/2 \). What polarization angles are represented? Which signs of circular polarizations appear?

c) Describe how you could use this behavior to design an instrument to detect circular polarization of incident radiation if you had in hand a device that could measure linear polarization and only linear polarization? (Hint: You need one other component in the optical path in addition to the polarization detector.)