1.) Problem 1.4 in Rybicki & Lightman (radiation pressure)

2.) Consider a star surrounded by a homogeneous gas cloud and observed by an observer at large distance D. The star has a surface temperature $T^*$ and the circumstellar gas temperature has the value, $T_g < T^*$. Assume the star emits like a true black body, so surface brightness $I_\nu$ is $B_\nu$, and that the circumstellar gas is in LTE. To simply the math assume that both the star and the cloud are cylinders with the star at the center of the cloud. The cylinder axes align with the line of sight. The gas cloud has radius $r_c > r^*$, where $r^*$ is the stellar radius and each has an axial length twice its circular radius. Suppose the gas emits a single, narrow spectral line (opacity vanishes outside a narrow band) and that the optical depth through the gas cloud from the observer to the surface of the star is $\tau$ inside this spectral line. We wish to understand the spectrum seen by a distant observer. The observer is distant, so that all sight lines through the source are parallel. The observer cannot spatially resolve the combined star and cloud (They always measure everything that reaches them from the cylinder enclosing the cloud.)

a) For arbitrary optical depth $\tau$, compute the spectral flux seen by the observer in the continuum adjacent to the spectral line and in the line when observing the combined cloud and star. Don’t forget to include all relevant lines of sight. Note that for this geometry you will have to define more than one optical depth.

b) Determine under what conditions the spectral line appears in emission and when it appears in absorption. Then if $\tau >> 1$ and $B_\nu (T^*)/B_\nu (T_g) > 1$ show that the condition for an apparent emission line is approximately $r_c/r^* > (B_\nu (T^*)/B_\nu (T_g))^{1/2}$. Then show if $\tau << 1$ (as well as optical depths along other lines of sight) and $B_\nu (T^*)/B_\nu (T_g) >> 1$ that the condition for an emission line requires approximately $r_c/r^* > ((1/2) B_\nu (T^*)/B_\nu (T_g))^{1/2}$, when $r_c >> r^*$. Note that these conditions are not quite the same.

3.) In class we used a simple geometric argument to show that the transverse, wave electric field in a vacuum produced by a nonrelativistic accelerated charge, $q$, is given by the expression

$$E(\vec{r}) = \frac{\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \hat{a}) q}{c^2 r},$$

where $a$ is the acceleration and $\mathbf{r}$ is the vector from the charge.

a) Show that the Poynting flux, $\vec{S}$, associated with this electric field is given by

$$\vec{S} = \frac{1}{4 \pi c^3} \frac{a^2 q^2}{r^2} \sin^2(\theta) \hat{\mathbf{r}},$$

where $\cos(\theta) = \hat{\mathbf{a}} \cdot \hat{\mathbf{r}}$.

b) Then, using this result show that the total power radiated away from this charge in response to the acceleration, $a$, is given by

$$P = \frac{2}{3} \frac{a^2 q^2}{c^3}.$$

This result is known as Larmor’s formula, and it gives us the starting point to compute the power radiated by any classical system of charges.