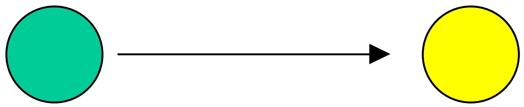
Invariants

- Two important kinds of invariants for special relativity problems
- Lorentz matrix always has an identity sub-matrix within it—2 components invariant
- Lorentz scalars are invariant. You can always form a Lorentz scalar with the square of a 4-vector

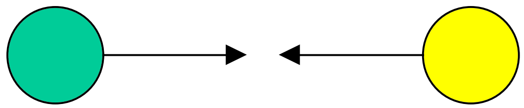
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Two Particle Scattering

- LAB Frame



- CM Frame



Two Particle Scattering

- Each particle has a four-momentum vector, which is different in the LAB frame and the CM frame
- The norm of the four-momentum is Lorentz-invariant and is proportional to the rest mass of the particle

$$m^2 = -\frac{\mathbf{p} \cdot \mathbf{p}}{c^2}$$

Two Particle Scattering

LAB

$$\begin{pmatrix} p_{x1} \\ 0 \\ 0 \\ iE_1/c \end{pmatrix} \quad m_1^2c^4 = E_1^2 - c^2p_{x1}^2$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ iE_2/c \end{pmatrix} \quad m_2^2c^4 = E_2^2$$

Two Particle Scattering

CM

$$\begin{pmatrix} p^*_x \\ 0 \\ 0 \\ iE^*_1/c \end{pmatrix}$$

$$\begin{pmatrix} -p^*_x \\ 0 \\ 0 \\ iE^*_2/c \end{pmatrix}$$

Two Particle Scattering

- Now you can add the two particle four-momentum vectors in both the LAB and the CM and the norms of these summed vectors are also invariant

LAB Frame

$$\begin{pmatrix} p_{x1} \\ 0 \\ 0 \\ \frac{i}{c}E_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{i}{c}m_2c^2 \end{pmatrix} = \begin{pmatrix} p_{x1} \\ 0 \\ 0 \\ \frac{i}{c}(E_1 + m_2c^2) \end{pmatrix} \quad Now \;\; square$$

$$p_{x1}^2 - \frac{1}{c^2}(E_1^2 + m_2^2c^4 + 2E_1m_2c^2)$$

$$-\frac{1}{c^2}(m_1^2c^4 + m_2^2c^4 + 2E_1m_2c^2)$$

CM Frame

$$\begin{pmatrix} p_x^* \\ 0 \\ 0 \\ \frac{i}{c}E_1^* \end{pmatrix} + \begin{pmatrix} -p_x^* \\ 0 \\ 0 \\ \frac{i}{c}E_2^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{i}{c}(E_1^* + E_2^*) \end{pmatrix} \quad Now \;\; square$$

$$s = total \;\; cm \;\; energy \;\; squared$$

$$s_{\min} = c^2 (m_1 + m_2)^2$$

Discovery of the anti-proton

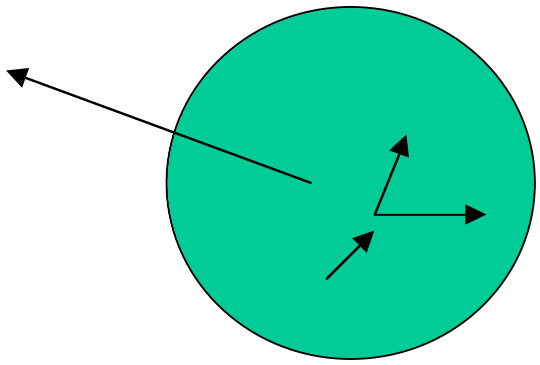
- The Bevatron at UC Berkeley was built explicitly for the purpose of creating anti-protons through the reaction

$$p + p \rightarrow p + p + p + \bar{p}$$

- What is the minimum beam proton energy required for this process?

Monte Carlo Calculations

- Monte Carlo calculations are simulations that can calculate results that are difficult or impossible to do analytically
- One of the first applications was the calculation of the critical mass for U^{235}



Functions in Excel

- Public Function cube (ByRef x As Double) As Double
- Cube=x*x*x
- End Function

Random Walk

```
Public Function randomstep() As Double
Dim step As Double
Dim i As Integer
step = 0
For i = 1 To 10
x = Rnd( )
If (x < 0.5) Then
 step = step + 1
 Else
 step = step - 1
 End If
 Next i
 randomstep = step
End Function
```

Stochastic Distributions

```
Public Function exprand() As Double
Dim x As Double
Dim y As Double
For i = 1 To 1000
x = 10 * Rnd()
If (Rnd() <= Exp(-x)) Then
exprand = x
GoTo a
End If
Next i
exprand = -1
a: End Function
```