

Applications

- A muon decays stochastically with a mean lifetime of $2.2 \mu\text{sec}$ in its rest frame. If a muon is created by a pion decay 30 km above the earth and travels at $v=0.99 c$, how long will it take to reach the earth in the LAB frame? In the muon rest frame?

LAB Frame

- The velocity is $0.99 c = 2.96 \times 10^8 \text{ m/s}$
- The distance is 30 km
- The time is $1.01 \times 10^{-4} \text{ s}$
- $\beta = 0.99$
- $\gamma = 7.0888$

Muon Frame

- Muon velocity is zero
- How far does the muon go?
- How long does it take?

Lorentz Transformation

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ ict_2 \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ ict_1 \end{pmatrix}$$

assuming motion v along the x axis,

$$\gamma = \sqrt{\frac{1}{1-\beta^2}}, \quad \beta = v/c$$

Lorentz Transformation

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 4274.9i \end{pmatrix} = \begin{pmatrix} 7.0888 & 0 & 0 & i(70179) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i(7.0179) & 0 & 0 & 7.0888 \end{pmatrix} \begin{pmatrix} 30000 \\ 0 \\ 0 \\ i(30303) \end{pmatrix}$$

so $ct = 1.43 \times 10^{-5} s = t_{LAB} / \gamma$

Time Dilation