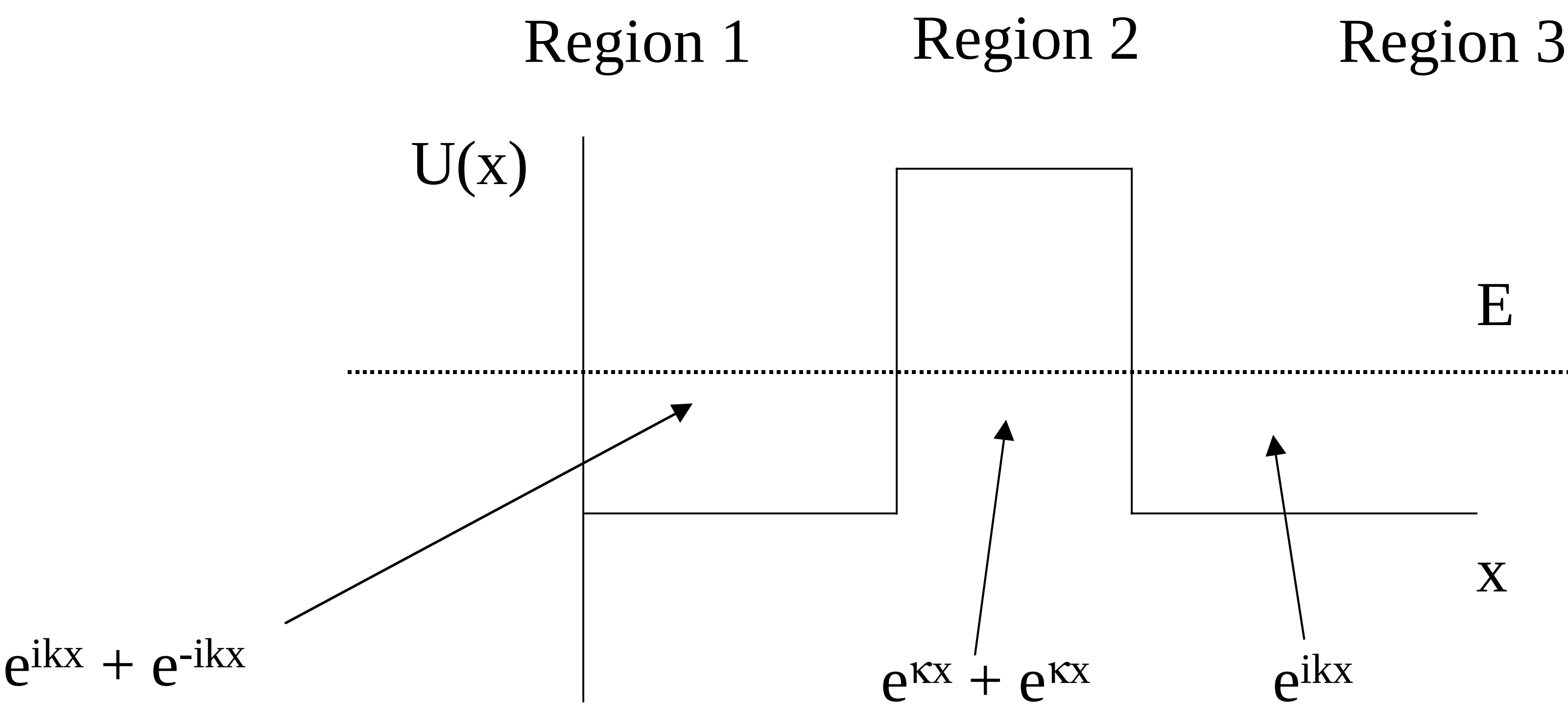
Barrier Penetration



Barrier Penetration Probability

Square Barrier

$$P = |E|^2 = \frac{16k^2\kappa^2}{|(\kappa - ik)^2 - (\kappa + ik)^2 e^{-2\kappa(b-a)}|^2} e^{-2\kappa(b-a)}$$

Arbitrary Barrier

$$P \cong e^{-2\int_a^b \sqrt{2m[U(x)-E]}/\hbar \; dx}$$

Orthogonal Polynomials

- SE differs with different potentials
- Solutions are generally some set of orthogonal polynomials
- Sine and cosine are examples of orthogonal polynomials
- General solutions are sums of basis vectors

Harmonic Oscillator

- $U=(1/2)kx^2$
- Harmonic oscillator is first-order Taylor approximation to any potential around an equilibrium point
- Solutions to SE for harmonic oscillator potential is a series of orthogonal polynomials (Hermite polynomials)

Harmonic Oscillator

Eigenvalues are

$$E = (n + \frac{1}{2})\hbar\omega, \quad \omega = \sqrt{\frac{k}{m}}$$

$$\varphi_1(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-m\omega x^2/(2\hbar)}$$

$$\varphi_2(x) = \sqrt{2}\left(\frac{m\omega}{\hbar\pi}\right)^{1/4}\left(\frac{m\omega}{\hbar}\right) x e^{-m\omega x^2/(2\hbar)}$$

$$\varphi_3(x) = \frac{1}{\sqrt{2}}\left(\frac{m\omega}{\hbar\pi}\right)^{1/4}\left(2\frac{m\omega}{\hbar}x^2 - 1\right) e^{-m\omega x^2/(2\hbar)}$$

SE in Three Dimensions

- Degeneracy occurs when more than one wave function has same energy
- Infinite cube potential is 3-fold degenerate
- Solution to SE in spherical coordinates are spherical harmonics, that are Legendre polynomials multiplied by $e^{\pm im\phi}$