Infinite Square Well

Normalization condition to determine B

$$\int_{-a}^{a} B^2 \cos^2 kx = B^2 a = 1$$

$$B = \frac{1}{\sqrt{a}}$$

$$\varphi = \frac{1}{\sqrt{a}} \cos \frac{2mE}{\hbar^2} x$$

Particle most likely found in middle of well

Newton vs QM

- Classical (Newton)
- $T(x) = E - U(x)$
- $E \geq U(x)$
- $p = (2mT)^{1/2} = [2m(E - U(x))]^{1/2}$
- Square well: all E possible, p constant, particle anywhere in well

- Quantum Mechanics
- Solve SE for wave function
- E quantized, most probable location in middle of well
- $E < U(x)$ allowed

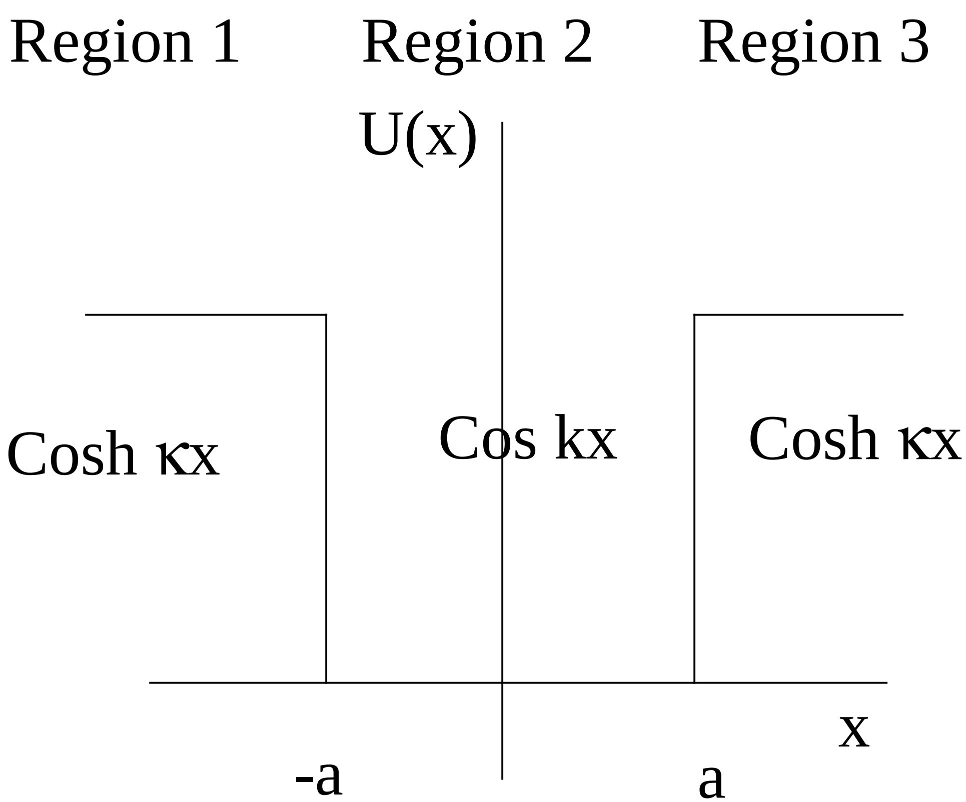
Negative Kinetic Energy

- Kinetic energy $T(x) = [2m(E\text{-}U(x))]^{1/2}$
- In classical physics, $E \geq U(x)$
- In QM, E can be less than U(x). In this case, p is imaginary. Since, $p = (hk)/(2\pi)$, k is also imaginary. If k is imaginary, $e^{\pm ikx}$ becomes $e^{\pm \kappa x}$ and cos kx becomes cosh κx

Finite Square Well

cosh κx has 2 terms—one is an increasing exponential and the other is a decreasing exponential

Only decreasing exponential term is allowed



Finite Square Well

$$\text{Region } 1: \ \varphi = Ae^{\kappa x}$$

$$\text{Region } 2: \ \varphi = B\cos kx$$

$$\text{Region } 3: \ \varphi = Ce^{-\kappa x}$$

$$x = -a: \ \ Ae^{-\kappa a} = B\cos ka$$

$$x = a: \ \ B\cos ka = Ce^{-\kappa a}$$

$$A = C$$

Finite Square Well

$$x = -a: \ -\kappa A e^{-\kappa a} = -ikB\sin k(-a) = ikB\sin ka$$

$$x = a: \ \text{same equation}$$

Combine these conditions:

$$k\cot ka = -\kappa$$

Transcendental equation with no algebraic solution

Solve by Newton's method