

Problem-Solving Notes 3: Finite Square Well

The finite square well problem has the following potential energy function:
 $U(x)=0$ for $-a < x < a$ and $U(x)=U_0$ for $x < -a$ and $x > a$.

$$-a < x < a :$$

$$p^2 = 2mE = \hbar^2 k^2$$

$$x < -a; \quad x > a$$

$$p^2 = 2m(U_0 - E) = \hbar^2 \kappa^2$$

Combining these equations, yields

$$2mU_0 - \hbar^2 k^2 = \hbar^2 \kappa^2$$

Matching boundary conditions at well edges yields

$k \cot(ka) = -\kappa$; Combining these equations gives

$$\hbar^2 k^2 \cot^2(ka) = 2mU_0 - \hbar^2 k^2$$

$$\hbar^2 k^2 [1 + \cot^2(ka)] - 2mU_0 = 0$$

Because cotangent is a periodic function, this equation clearly has multiple roots. Each root yields a value of k , which can then be related to the energy in the well. The easiest way to solve this equation in Excel is to use the Solver in the Tools menu. Another way is to write a VBA routine that implements Newton's method. In either case, what root you find depends on what starting value you use.

Let $U_0=10$ eV and solve for the three smallest values of k .

Problems for next week: Ch. 36: 24, 25, 27, 28, 33