

Chiral $SU(3)$ symmetry and strangeness

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Abstract

In this paper we review recent progress on the systematic evaluation of the kaon and antikaon spectral functions in dense nuclear matter based on a chiral $SU(3)$ description of the low-energy pion-, kaon- and antikaon-nucleon scattering data.

1. Introduction

A good understanding of the antikaon spectral function in nuclear matter is required for the description of K^- atoms [1, 2] and the subthreshold production of kaons in heavy ion reactions [3]. An exciting consequence of a significantly reduced effective K^- mass could be that kaons condense in the interior of neutron stars [4–6]. The ultimate goal is to relate the in-medium spectral function of kaons to the anticipated chiral symmetry restoration at high baryon density. To unravel quantitative constraints on the kaon spectral functions from subthreshold kaon production data of heavy-ion reactions requires transport model calculations which are performed extensively by various groups [6–10]. The next generation of transport codes which are able to incorporate more consistently particles with finite width are being developed [11–14]. This is of considerable importance when dealing with antikaons which are poorly described by a quasi-particle ansatz [15, 16].

There has been much theoretical effort to access the properties of kaons in nuclear matter [15–22]. An antikaon spectral function with support at energies smaller than the free-space kaon mass was already anticipated in the 1970s by many K-matrix analyses of the antikaon-nucleon scattering process (see e.g. [23]) which predicted considerable attraction in the subthreshold scattering amplitudes. This leads, in conjunction with the low-density theorem [17, 24], to an attractive antikaon spectral function in nuclear matter. Nevertheless, the quantitative evaluation of the antikaon spectral function is still an interesting problem. The challenge is first to establish a solid understanding of the vacuum antikaon-nucleon scattering process, in particular reliable subthreshold antikaon-nucleon scattering amplitudes are required, and secondly, to evaluate the antikaon spectral function in a systematic many-body approach.

The antikaon–nucleon scattering is complicated due to the open inelastic $\pi\Sigma$ and $\pi\Lambda$ channels and the presence of the *s*-wave $\Lambda(1405)$ and *p*-wave $\Sigma(1385)$ resonances just below and the *d*-wave $\Lambda(1520)$ resonance not too far above the antikaon–nucleon threshold. In this paper we review recent progress made within the newly formulated χ -BS(3) approach, for chiral Bethe–Salpeter approach to the $SU(3)$ flavour group [25]. It constitutes a systematic and non-perturbative application of the chiral $SU(3)$ Lagrangian to the meson–baryon scattering problem consistent with covariance, crossing symmetry, large- N_c sum rules of QCD and the chiral counting concept. The low-energy pion–, kaon– and antikaon–nucleon scattering data were reproduced successfully demonstrating that the chiral $SU(3)$ flavour symmetry is a powerful tool for analysing and predicting hadron interactions systematically. The amplitudes obtained in that scheme are particularly well suited for an application to the nuclear kaon dynamics, because it was demonstrated that they are approximately crossing symmetric in the sense that the KN and $\bar{K}N$ amplitudes smoothly match at subthreshold energies. Therefore, we believe that those amplitudes, which are of central importance for the nuclear kaon dynamics, lead to reliable results for the propagation properties of kaons in dense nuclear matter [26].

As was pointed out in [15], the realistic evaluation of the antikaon self-energy in nuclear matter requires a self-consistent scheme. In particular, the feedback effect of an in-medium modified antikaon spectral function on the antikaon–nucleon scattering process was found to be important for the $\Lambda(1405)$ resonance structure in nuclear matter. In this paper we present a selection of results obtained in a novel covariant many-body framework [26]. Self-consistency was implemented in terms of the free-space meson–nucleon scattering amplitudes, where the amplitudes of the χ -BS(3) approach were used. Besides presenting realistic kaon and antikaon spectral functions, we discuss the in-medium structure of the *s*-wave $\Lambda(1405)$ and *p*-wave $\Sigma(1385)$ resonances.

2. Kaon– and antikaon–nucleon scattering

We briefly review the most striking phenomena that arises when applying the chiral $SU(3)$ Lagrangian to the kaon– and antikaon–nucleon interaction processes. At leading chiral order these interactions are supposedly described by the Weinberg–Tomozawa term,

$$\mathcal{L}_{\text{WT}} = \frac{i}{8f^2} (\bar{N}\gamma_\mu N)((\partial^\mu K)^\dagger K - K^\dagger(\partial^\mu K)) + \frac{3i}{8f^2} (\bar{N}\gamma_\mu \vec{\tau} N)((\partial^\mu K)^\dagger \vec{\tau} K - K^\dagger \vec{\tau}(\partial^\mu K)), \quad (1)$$

where the parameter $f \simeq f_\pi \simeq 92.4$ MeV is known from the decay process of charged pions. In contrast to the successful application of the chiral Lagrangian in the flavour $SU(2)$ sector, its application to the strange sector of QCD is flawed by a number of subtleties if the rigorous machinery of chiral perturbation theory is applied. The leading interaction term (1) fails miserably in describing the *s*-wave scattering lengths of both kaons and antikaons off a nucleon. Most stunning is the failure of reproducing the repulsive K^-p scattering length [27]. The chiral Lagrangian predicts an attractive scattering length at leading order instead. This is closely linked to the presence of the $\Lambda(1405)$ resonance in the K^-p scattering amplitude just below the K^-p threshold. Considerable theoretical progress has been made over the last few years by incorporating the dynamics of that $\Lambda(1405)$ resonance into the chiral dynamics. The key point is to change the approximation strategy and expand the interaction kernel rather than the scattering amplitude directly. This amounts to solving some types of coupled-channel scattering equations such as the Lippmann–Schwinger or the Bethe–Salpeter equation. As a consequence the $\Lambda(1405)$ resonance is generated dynamically by coupled-channel effects. A realistic description of the antikaon–nucleon scattering process requires the inclusion of all $SU(3)$ channels $\bar{K}N$, $\pi\Lambda$, $\pi\Sigma$, $\eta\Sigma$, $\eta\Lambda$ and $K\Xi$ together with correction terms predicted by

the chiral $SU(3)$ Lagrangian. The number of free parameters controlling the chiral correction terms can be significantly reduced by insisting on sum rule relations as they arise from QCD in the large- N_c limit. For a detailed description of an up-to-date scheme with a more complete list of references, we refer to a recent work [25]. In that work it is shown that the chiral $SU(3)$ Lagrangian does describe all low-energy pion-, kaon- and antikaon-nucleon scattering data fairly well, once the chiral perturbation theory is applied to the covariant scattering kernel of the Bethe–Salpeter scattering equation.

In this paper we would like to discuss an aspect in more detail, which is particularly important when applying the chiral kaon- and antikaon-nucleon dynamics to kaon and antikaon propagation in dense nuclear matter. Crossing symmetry relates the $\bar{K}N$ and KN scattering amplitudes at subthreshold energies. This manifests itself in the form of the dispersion-integral representation of the antikaon-nucleon scattering amplitude, $T_{\bar{K}N}^{(0)}(\sqrt{s})$, evaluated in the forward direction,

$$T_{\bar{K}N}^{(0)}(\sqrt{s}) - T_{\bar{K}N}^{(0)}(\sqrt{s_0}) = \frac{f_{\bar{K}N\Lambda}^2}{s - m_\Lambda^2} - \frac{f_{\bar{K}N\Lambda}^2}{s_0 - m_\Lambda^2} + \int_{-\infty}^{(m_N - m_K)^2} \frac{ds' s - s_0}{\pi s' - s_0} \frac{\text{Im} T_{\bar{K}N}^{(0)}(\sqrt{s'})}{s' - s - i\epsilon} + \int_{(m_\Sigma + m_\pi)^2}^{+\infty} \frac{ds' s - s_0}{\pi s' - s_0} \frac{\text{Im} T_{\bar{K}N}^{(0)}(\sqrt{s'})}{s' - s - i\epsilon}, \quad (2)$$

where we performed a subtraction at $s = s_0$ to help the convergence of the dispersion integral. For the sake of clarity, here we consider the isospin zero channel only. The scattering amplitude $T_{\bar{K}N}^{(0)}(\sqrt{s})$ shows unitarity cuts not only for $\sqrt{s} > m_\Sigma + m_\pi$, representing for instance the inelastic process $\bar{K}N \rightarrow \pi \Sigma$, but also for $\sqrt{s} < m_N - m_K$ reflecting the elastic KN scattering process.

A problem arises because the kaon- and antikaon-nucleon scattering processes are described by two distinct Bethe–Salpeter equations. If the Bethe–Salpeter interaction kernel of the $\bar{K}N$ sector, which implies a particular scattering amplitude $T_{\bar{K}N}^{(0)}(\sqrt{s})$, is evaluated in perturbation theory, the amplitude $T_{\bar{K}N}^{(0)}(\sqrt{s})$ does not properly describe the unitarity cuts of the KN channel, manifest at the subthreshold energy $\sqrt{s} < m_N - m_K$. Analogously, an evaluation of the kaon-nucleon scattering amplitudes in terms of a perturbative interaction kernel is not reliable far below threshold where the $\bar{K}N$ channel opens. Of course, crossing symmetry is reconciled once the interaction kernels of the KN and $\bar{K}N$ sectors include the appropriate infinite class of Feynman diagrams. It is important to face this problem since the in-medium antikaon spectral function tests the subthreshold $\bar{K}N$ amplitudes,

$$\Pi_{\bar{K}}(\omega, \vec{q} = 0) \simeq \left(\frac{1}{4}T_{\bar{K}N}^{(0)}(\omega + m_N) + \frac{3}{4}T_{\bar{K}N}^{(1)}(\omega + m_N)\right)\rho + \dots, \quad (3)$$

where we recalled the low-density theorem written down for the antikaon polarization function $\Pi_{\bar{K}}(\omega, \vec{q})$ [17, 24]. If an antikaon can propagate in dense nuclear matter with $\omega < m_K$ and say $\vec{q} = 0$ for simplicity, the evaluation of the antikaon polarization function $\Pi_{\bar{K}}(\omega, 0)$ at that energy requires knowledge of the scattering amplitudes $T_{\bar{K}N}^{(1)}(\sqrt{s})$ at a subthreshold energy $\sqrt{s} < m_N + m_K$.

This problem is solved in the chiral coupled-channel framework by supplementing the Bethe–Salpeter equation with an appropriate renormalization programme that leads to the matching of the subthreshold KN and $\bar{K}N$ amplitudes,

$$T_{\bar{K}N}^{(0)}(s) \simeq -\frac{1}{2}T_{\bar{K}N}^{(0)}(2s_0 - s) + \frac{3}{2}T_{\bar{K}N}^{(1)}(2s_0 - s), \\ T_{\bar{K}N}^{(1)}(s) \simeq +\frac{1}{2}T_{\bar{K}N}^{(0)}(2s_0 - s) + \frac{1}{2}T_{\bar{K}N}^{(1)}(2s_0 - s), \quad (4)$$

close to the optimal matching point $s_0 = m_N^2 + m_K^2$. This is a crucial input of the χ -BS(3) approach developed in [25]. Even though the renormalization of the chiral Lagrangian

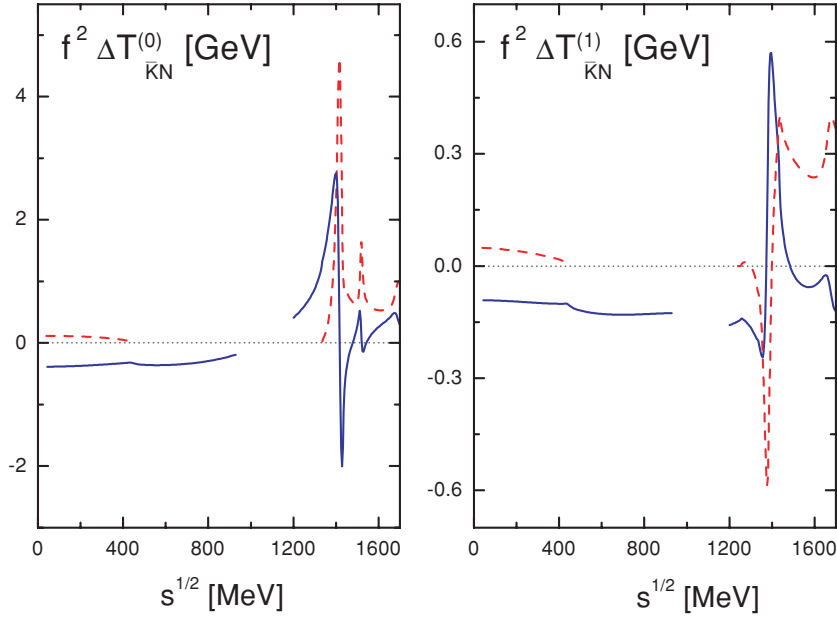


Figure 1. Approximate crossing symmetry of the hyperon-pole subtracted kaon–nucleon forward scattering amplitudes. The lines in the left-hand parts of the figures result from the KN amplitudes. The lines in the right-hand parts of the figures give the $\bar{K}N$ amplitudes.

in perturbation theory is straightforward, this is not the case anymore once the infinite diagram summation implied by the Bethe–Salpeter equation is performed. An additional renormalization condition is needed. In [25] it was strongly argued that this condition is naturally provided by the crossing symmetry constraint. In figure 1 we demonstrate that the hyperon pole-subtracted scattering amplitudes, $\Delta T_{KN}^{(I)}$ and $\Delta T_{\bar{K}N}^{(I)}$, comply with the crossing identities (4) close to $s \simeq s_0 = m_N^2 + m_K^2$ approximately. We are therefore convinced that our subthreshold antikaon–nucleon scattering amplitudes are reliably determined and well suited for an application to the nuclear kaon dynamics.

We close this section on the microscopic input with a presentation of total cross sections relevant for transport model calculation of heavy-ion reactions. We believe that the χ -BS(3) approach is particularly well suited to determine some cross sections not directly accessible in scattering experiments. Typical examples would be the $\pi\Sigma \rightarrow \pi\Sigma, \pi\Lambda$ reactions. Here the quantitative realization of the chiral $SU(3)$ flavour symmetry, including its important symmetry breaking effects, is an extremely useful constraint when deriving cross sections not accessible in the laboratory directly. It is common to consider isospin averaged cross sections [6, 28]

$$\bar{\sigma}(\sqrt{s}) = \frac{1}{N} \sum_I (2I + 1) \sigma_I(\sqrt{s}). \quad (5)$$

The reaction-dependent normalization factor is determined by $N = \sum (2I + 1)$, where the sum extends over isospin channels which contribute in a given reaction. In figure 2 we confront the cross sections of the channels $\bar{K}N, \pi\Sigma$ and $\pi\Lambda$ with typical parameterizations used in transport model calculations. The cross sections in the first column are determined by detailed balance from those of the first row. Uncertainties are present nevertheless, reflecting the large empirical uncertainties of the antikaon–nucleon cross sections close to threshold. The remaining four cross sections in figure 2 are true predictions of the χ -BS(3) approach.

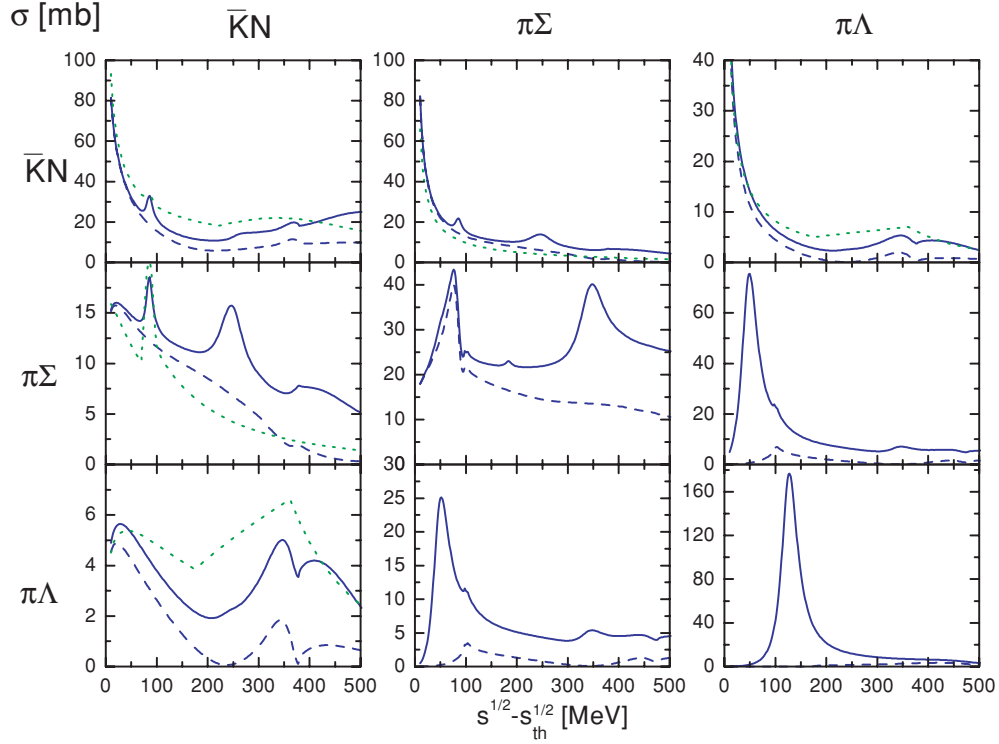


Figure 2. Total cross sections $\bar{K}N \rightarrow \bar{K}N$, $\bar{K}N \rightarrow \pi\Sigma$, $\bar{K}N \rightarrow \pi\Lambda$, etc relevant for subthreshold production of antikaons in heavy-ion reactions. The solid and dashed lines give the results of the χ -BS(3) approach with and without p- and d-wave contributions, respectively. The dotted lines correspond to the parameterizations given in [6].

We should emphasize that we trust our results quantitatively only for $\sqrt{s} < 1600$ MeV. It is remarkable that nevertheless our cross sections agree with the parameterizations in [6] qualitatively up to much higher energies except in the $\bar{K}N \leftrightarrow \pi\Sigma$ reactions where we overshoot those parameterizations somewhat. Besides some significant deviations of our results from [6, 28] at $\sqrt{s} - \sqrt{s_{th}} < 200$ MeV, an energy range where we trust our results quantitatively, we find that most interesting is the sizeable cross section of about 30 mb for the $\pi\Sigma \rightarrow \pi\Sigma$ reaction. Note that here we include the isospin 2 contribution as part of the isospin averaging. As demonstrated by the dotted line in figure 2, which represents the χ -BS(3) approach with s-wave contributions only, the p- and d-wave amplitudes are of considerable importance for the $\pi\Sigma \rightarrow \pi\Sigma$ reaction.

3. Kaon and antikaon propagation in nuclear matter

We begin with a discussion of kaon propagation in dense nuclear matter. The kaon self-energy is evaluated in terms of the real parts of the s- and p-wave kaon–nucleon scattering amplitudes of [25]. The resulting quasi-particle energy $E_K(\vec{q})$ is shown in figure 3 for nuclear saturation density, ρ_0 , and $2\rho_0$. We neglect small rescattering effects proportional to $(T_{KN})^n$ with $n \geq 2$, which introduce a 15% correction term at small kaon momenta only. A nuclear environment leads to an increase of the kaon energy where the effect is reduced as the kaon momentum

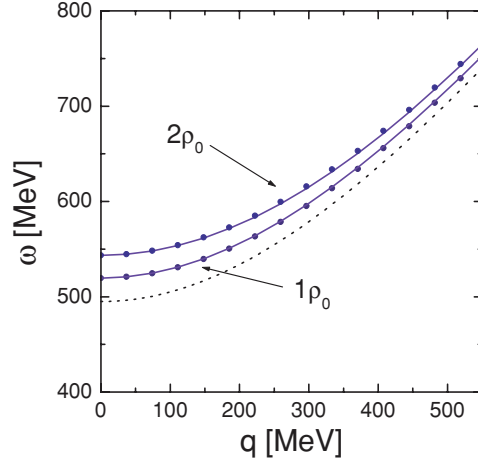


Figure 3. The spectrum of a kaon with energy ω and momentum \vec{q} at nuclear saturation density, ρ_0 , and $2\rho_0$. The solid lines follow from the real part of the KN scattering amplitudes of [25]. The circles represent the parameterization (6) and the dashed line shows the free-space kaon spectrum for comparison.

\vec{q} increases. Our result can be interpreted rather accurately in terms of scalar and vector potentials, parameterizing the kaon polarization function,

$$\Pi_K(\omega, \vec{q}) \simeq \left(1.1m_K - \omega + 0.2 \frac{\vec{q}^2}{m_K} \right) 46.8 \text{ MeV} \frac{\rho}{\rho_0}, \quad (6)$$

for $\omega = E_K(\vec{q})$. We emphasize that the parameterization of the polarization function $\Pi_K(\omega, \vec{q})$ is reliable only at the quasi-particle energy of the kaon, $E_K(\vec{q})$, as implied by (6). In particular, the parameterization (6) does not correctly describe any derivative of the polarization function. The parameterization $\Pi_K(\omega, \vec{q})$ is in striking contradiction to the naive representation,

$$\Pi_K(\omega, 0) = \left(\frac{3\omega}{4f^2} - \frac{\Sigma_{KN}}{f^2} \right) \rho, \quad (7)$$

in terms of the Weinberg–Tomozawa term and the so-called kaon–nucleon sigma term, $\Sigma_{KN} > 0$, frequently seen in the literature. The scalar and vector terms in (6) and (7) have opposite signs. As we argued in the previous section, the chiral Lagrangian does not correctly describe the kaon–nucleon scattering process if evaluated in perturbation theory. The small attractive energy dependence of the self-energy reflects important range terms required to describe the s-wave KN phase shifts. The rather small repulsive momentum dependence follows from the net-repulsion of the p-wave amplitudes. For non-zero momentum the small attractive vector potential in (6) leads to a reduction of the repulsion implied by the s-wave scattering lengths and p-wave scattering volumes. We checked that the parameterization (6) is valid to high accuracy for kaon momenta smaller than $|\vec{q}| < 600$ MeV reproducing quantitatively the kaon quasi-particle energy $E_K(\vec{q})$. This is clearly demonstrated in figure 3 where we compare the spectrum following from the exact polarization operator with that from the parameterization (6) at two different nuclear densities.

We turn to antikaon and hyperon resonance propagation in dense nuclear matter. In figure 4 we present the antikaon spectral function together with the antikaon–nucleon scattering amplitudes of selected channels at various nuclear matter densities, ρ , as evaluated in a self-consistent manner in [26]. The antikaon spectral function exhibits a rich structure

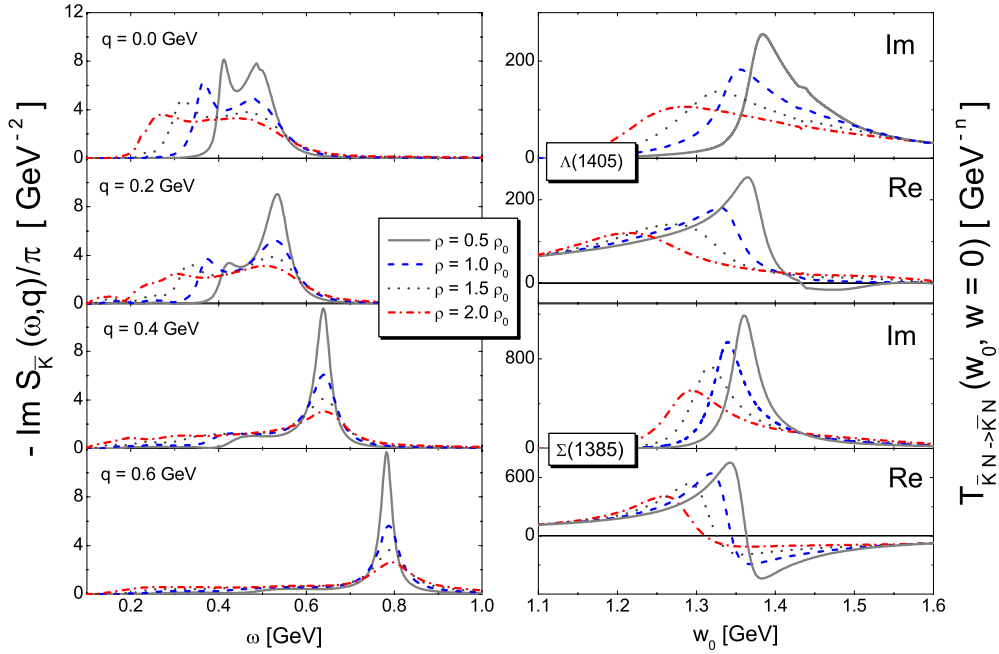


Figure 4. The antikaon spectral function is shown in the left-hand panel as a function of the antikaon energy ω , momentum q and nuclear density with $\rho_0 = 0.17 \text{ fm}^{-3}$. The right-hand panel illustrates the in-medium modification of the $\Lambda(1405)$ and $\Sigma(1385)$ hyperon resonances. The real and imaginary parts of the antikaon–nucleon scattering amplitudes are plotted in the appropriate channels. The hyperon energy and momentum are w_0 and $w = 0$, respectively.

with a pronounced dependence on the antikaon 3-momentum. This reflects the coupling of the $\Lambda(1405)$ and $\Sigma(1385)$ hyperon states to the $\bar{K}N$ channel. Typically, the peaks seen are quite broad and not always of quasi-particle-type. As was emphasized in [15, 26] the realistic evaluation of the antikaon propagation in nuclear matter requires the simultaneous consideration of the hyperon resonance propagation. The most important contributions, the s-wave $\Lambda(1405)$ and p-wave $\Sigma(1385)$ resonances, experience important medium modifications as demonstrated in figure 4. The results at $2\rho_0$ should be considered cautiously because nuclear binding and correlation effects were not yet included in [26].

4. Summary

We reviewed the application of the microscopic χ -BS(3) dynamics developed recently in [25, 26] to kaon, antikaon and hyperon resonance propagation in nuclear matter. Of central importance for the microscopic evaluation of the kaon and antikaon spectral functions in cold nuclear matter are the kaon– and antikaon–nucleon scattering amplitudes, in particular at subthreshold energies. The required amplitudes are well established by the χ -BS(3) approach and show sizeable contributions from p-waves not considered systematically so far [15, 16, 19]. For the antikaon spectral function one finds a pronounced dependence on the 3-momentum of the antikaon reflecting the presence of hyperon–nucleon–hole states. The quantitative evaluation of the antikaon self-energy requires the self-consistent consideration of the in-medium change of the hyperon resonance structures. At nuclear saturation density we reported attractive mass shifts of about 60 MeV, 60 MeV and 100 MeV, respectively for

$\Lambda(1405)$, $\Sigma(1385)$ and $\Lambda(1520)$. The resonance widths increase to about 120 MeV, 70 MeV and 90 MeV. Whereas the kaon spectral function is well approximated by a quasi-particle approach, the antikaon spectral function shows typically a rather wide structure invalidating a simple quasi-particle description.

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