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Chiral dynamics and pionic 1s states of Pb and Sn isotopes*

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Recent accurate data on $1s$ states of π^- bound to Pb [1] and Sn [2] isotopes have set new standards and constraints for the detailed analysis of s-wave pion-nucleon interactions. This topic has a long history [3] culminating in various attempts to understand the notorious "missing repulsion" in the π-nucleus interaction: the standard ansatz for the (energy independent) s-wave pion-nucleus optical potential, given in terms of the empirical threshold πN amplitudes times densities $\rho_{p,n}$ and supplemented by sizable double-scattering corrections, still misses the observed repulsive interaction by a large amount. This problem has traditionally been circumvented on purely phenomenological grounds by introducing an extraordinarily large repulsive real part (ReB_0) in the ρ^2 terms of the π-nucleus potential. The arbitrariness of this procedure is of course unsatisfactory.

In recent papers [4,5] we have re-investigated this issue from the point of view of the distinct explicit energy dependence of the pion-nuclear polarization operator [4] in a calculation based on systematic in-medium chiral perturbation theory [5,6]. Ref. [4] has also clarified the relationship to a working hypothesis launched previously [7,8]: that the extra repulsion needed in the s-wave pion-nucleus optical potential at least partially reflects the tendency toward chiral symmetry restoration in a dense medium. To leading order, this information is encoded in the in-medium reduction of the pion decay constant f_π, which, by its inverse square, drives the isospin-odd pion-nucleon amplitudes close to threshold. The aim of the this note is to present an updated summary of the situation and to compare with the new Sn data [2]. A detailed assessment of the overall systematics covering the complete pionic atoms data base has recently been given in ref. [9], using optical potential phenomenology.

The starting point is the energy- and momentum-dependent polarization operator (the pion self-energy) $\Pi(\omega, \vec{q}; \rho_p, \rho_n)$. In the limit of very low proton and neutron densities, $\rho_{p,n}$, the pion self-energy reduces to $\Pi = -(T^+\,\rho + T^-\,\delta\rho)$ with $\rho = \rho_p + \rho_n$ and $\delta\rho = \rho_p - \rho_n$, where $T^\pm$ are the isospin-even and isospin-odd off-shell πN amplitudes. In the long-wavelength limit ($\vec{q} \to 0$), chiral symmetry (the Weinberg-Tomozawa theorem) implies $T^-(\omega) = \omega/(2\,f_\pi^2) + \mathcal{O}(\omega^3)$. Together with the observed approximate vanishing of the isospin-even threshold amplitude $T^+(\omega = m_\pi)$, it is clear that $1s$ states of pions bound to heavy, neutron rich nuclei are a particularly sensitive source of information for in-medium

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chiral dynamics.

At the same time, it has long been known that term of non-leading order in density (double scattering (Pauli) corrections of order $\rho^{4/3}$, absorption effects of order ρ^2 etc.) are important. The aim must, therefore, be to arrive at a consistent expansion of the pion self-energy in powers of the Fermi momentum k_F together with the chiral low-energy expansion in ω, $|\vec{q}|$ and m_π. In-medium chiral effective field theory provides a framework for this approach. We apply it here systematically up to two-loop order, following ref. [5]. Double scattering corrections are fully incorporated at this order. Absorption effects and corresponding dispersive corrections appear at the three-loop level and through short-distance dynamics parameterized by πNN contact terms, not explicitly calculable within the effective low-energy theory. The imaginary parts associated with these terms are well constrained by the systematics of observed widths of pionic atom levels throughout the periodic table. (We use $\mathrm{Im}B_0 = -0.063 m_\pi^4$ in the s-wave absorption term, $\Delta\Pi_S^{abs} = -8\pi\,(1+m_\pi/2M)\,B_0\,\rho_p\,(\rho_p+\rho_n)$, and the canonical parameterization of p-wave parts, in accordance with refs. [3,9]). The real part of B_0 is still the primary source of theoretical uncertainty. In practice, our strategy is to start from $\mathrm{Re}B_0 = 0$ (as suggested also by the detailed analysis of the pion-deuteron scattering length) and then discuss the possible error band induced by varying B_0 within reasonable limits [4].

We proceed by using the local density approximation (with gradient expansion for p-wave interactions, $\vec{q}^{\,2}\,F(\rho) \to \vec{\nabla}\,F(\rho(\vec{r}))\,\vec{\nabla}$) and solve the Klein-Gordon equation

$$\left[\left(\omega - V_c(\vec{r})\right)^2 + \vec{\nabla}^2 - m_\pi^2 - \Pi\left(\omega - V_c(\vec{r});\rho_p(\vec{r}),\rho_n(\vec{r})\right)\right]\phi(\vec{r}) = 0\,. \qquad (1)$$

Note that the explicit energy dependence of Π requires that the Coulomb potential $V_c(\vec{r})$ must be introduced in the canonical gauge invariant way wherever the pion energy ω appears. This is an important feature that has generally been disregarded in previous analysis.

With input specified in details in ref. [4], we have solved eq. (1) with the explicitly energy dependent pion self-energy, obtained in two-loop in-medium chiral perturbation theory for the s-wave part, adding the time-honored phenomenological p-wave piece. The results for the binding energies and widths of $1s$ and $2p$ states in pionic ^{205}Pb are shown in Fig. 1 (triangles). Also shown for comparison is the outcome of a calculations using a "standard" phenomenological (energy independent) s-wave optical potential,

$$\Pi_S = -T_{\rm eff}^{+}\,\rho - T_0^{-}\,\delta\rho + \Delta\Pi_S^{abs}\,, \qquad (2)$$

with $T_{\rm eff}^{+} = T_0^{+} - \frac{3k_F(\vec{r})}{8\pi^2}\left[(T_0^{+})^2 + 2\,(T_0^{-})^2\right]$ and the amplitudes $T_0^{\pm} \equiv T^{\pm}(\omega = m_\pi)$ taken fixed at their threshold values. This approach fails and shows the "missing repulsion" syndrome, leading to a substantial overestimate of the widths. Evidently, a mechanism is needed to reduce the overlap of the bound pion wave functions with the nuclear density distributions, The explicit energy dependence in $T^{\pm}$ provides such a mechanism: the replacement $\omega \to \omega - V_c(\vec{r}) > m_\pi$ increases the repulsion in T^- and disbalances the "accidental" cancellation between the πN sigma term σ_N and the range term proportional to ω^2 in T^+, such that $T^+(\omega - V_c) < 0$ (repulsive).

Uncertainties in $\mathrm{Re}B_0$, in the radius and shape of the neutron density distribution, and in the input for the sigma term σ_N have been analysed in ref. [4]. Their combined effect falls within the experimental errors in Fig. 1.

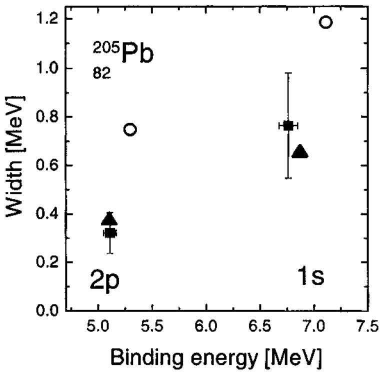

Figure 1. Binding energies and widths of pionic $1s$ and $2p$ states in ^{205}Pb. Experimental data from [1]. Full triangles: results of two-loop in-medium chiral perturbation theory, keeping the explicit energy dependence in the s-wave polarization operator. Open circles: energy independent potential as described in text (see ref. [4] for details). Note that $\mathrm{Re}B_0 = 0$ in both cases.

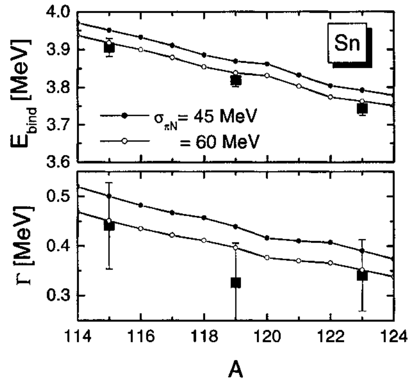

Figure 2. Binding energies and widths of pionic $1s$ states in Sn isotopes. The curves show predictions [10] based on the explicitly energy dependent pionic s-wave polarization operator calculated in two-loop in-medium chiral perturbation theory [4]. The sensitivity to the πN sigma term (input) is also shown. Data from ref. [2].

Using the same (explicitly energy dependent) scheme we have predicted binding energies and widths for pionic $1s$ states bound to a chain of Sn isotopes. These calculations [10] include a careful assessment of uncertainties in neutron distributions. Results are shown in Fig. 2 in comparison with experimental data [2] reported at PANIC'02 after the calculations. This figure also gives an impression of the sensitivity with respect to variations of the (input) πN sigma term.

We now come to an important question of interpretation: do we actually "observe" fingerprints of (partial) chiral symmetry restoration in the high-precision data of deeply bound pionic atoms with heavy nuclei, as anticipated in refs. [7,8]? Is this observation related to the "missing s-wave repulsion" that has been recognized (but not resolved in a consistent way) by scanning the large amount of already existing pionic atom data?

To approach this question, recall that pionic atom calculations are traditionally done with *energy-independent* phenomenological optical potentials instead of explicitly energy dependent pionic polarization functions. Let us examine the connection between these two seemingly different approaches by illustrating the leading-order driving mechanisms.

Consider a zero momentum pion in low density matter. Its energy dependent leading-order polarization operator is $\Pi(\omega) = -\left[T^+(\omega)\,\rho + T^-(\omega)\,\delta\rho\right]$, and the in-medium dispersion equation at $\vec{q} = 0$ is $\omega^2 - m_\pi^2 - \Pi(\omega) = 0$. The chiral low-energy expansion of the off-shell amplitudes $T^{\pm}(\omega)$ at $\vec{q}=0$ implies leading terms of the form:

$$T^{+}(\omega)=\frac{\sigma_N-\beta\,\omega^2}{f_\pi^2}\,,\quad T^{-}(\omega)=\frac{\omega}{2\,f_\pi^2}\,, \tag{3}$$

where $f_\pi = 92.4$ MeV is the pion decay constant in vacuum and $\sigma_N \simeq 0.05$ GeV. The empirical $T^{+}(\omega=m_\pi)=(-0.04\pm0.09)$ fm $\simeq 0$ sets the constraint $\beta\simeq\sigma_N/m_\pi^2$.

Expanding $\Pi(\omega)$ around the threshold, $\omega=m_\pi$, we identify the commonly used effective (energy-independent) s-wave optical potential U_S as:

$$2\,m_\pi\,U_S=\frac{\Pi(\omega=m_\pi,\vec{q}=0)}{1-\partial\Pi/\partial\omega^2}\,, \tag{4}$$

where $\partial\Pi/\partial\omega^2$ is taken at $\omega=m_\pi$. Inserting (3) and assuming $\delta\rho\ll\rho$ one finds:

$$U_S\simeq-\frac{\delta\rho}{4\,f_\pi^2}\left(1-\frac{\sigma_N\,\rho}{m_\pi^2\,f_\pi^2}\right)^{-1}=-\frac{\delta\rho}{4\,f_\pi^{*2}(\rho)}\,, \tag{5}$$

with the replacement $f_\pi\to f_\pi^*(\rho)$ of the pion decay constant representing the in-medium wave function renormalization. The expression (5) is just the one proposed previously in ref. [7] on the basis of the relationship between the in-medium changes of the chiral condensate $<\bar{q}\,q>$ and of the pion decay constant associated with the time component of the axial current. The explicitly energy dependent chiral dynamics encoded in $\Pi(\omega)$ "knows" about these renormalization effects. Their translation into an equivalent, energy-independent potential implies $f_\pi\to f_\pi^*(\rho)$ as given in eq. (5). This statement holds to leading order. Whether (important) higher order corrections permit a similar interpretation needs still to be further explored.

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