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Covariant Meson–Baryon Scattering with Chiral and Large N_c Constraints[1]

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We give a review of recent progress on the application of the relativistic chiral SU(3) Lagrangian to meson–baryon scattering. It is shown that a combined chiral and $1/N_c$ expansion of the Bethe–Salpeter interaction kernel leads to a good description of the kaon–nucleon, antikaon–nucleon and pion–nucleon scattering data typically up to laboratory momenta of $p_{\rm lab} \simeq 500\,MeV$. We solve the covariant coupled channel Bethe–Salpeter equation with the interaction kernel truncated to chiral order Q^3 where we include only those terms which are leading in the large N_c limit of QCD.

1. INTRODUCTION

An outstanding but still thrilling problem of modern particle physics is to unravel systematically the properties of quantum chromodynamics (QCD) in its low-energy phase where the effective degrees of freedom are hadrons rather than quarks and gluons. There are two promising paths along which one achieved and expects significant further progress. A direct evaluation of many observable quantities is feasible by large scale numerical simulations where QCD is put on a finite lattice. Though many static properties, like hadron ground-state properties, have been successfully reproduced by lattice calculations, the description of the wealth of hadronic scattering data is still outside the scope of that approach.[(1)] That is a challenge since many of the most exciting phenomena of QCD, like the zoo of meson and

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baryon resonances, are reflected directly in the scattering data. Here a complementary approach, effective field theory, is more promising. Rather than solving low-energy QCD directly in terms of quark and gluonic degrees of freedom, inefficient at low energies, one aims at constructing an effective, field theory in terms of hadrons directly. The idea is to constrain that theory by as many properties of QCD as possible. That leads to a significant parameter reduction and a predictive power of the effective field theory approach. In this spirit many effective field theory models, not all of which are fully systematic, have been constructed and applied to the data set. Given the many empirical data points to be described, it is not always necessary to build in all constraints of QCD. Part of QCD's properties may enter the model indirectly once it successfully describes the data,. The most difficult part in constructing an acceptable effective field theory is the identification of its applicability domain and its accuracy level. An effective field theory, which meets the above criterium, is the so called chiral perturbation theory (χPT) applicable in the flavor $SU(2)$ sector of low-energy QCD.

The merit of standard χPT is that first it is based on an effective Lagrangian density constructed in accordance with all chiral constraints of QCD and second that it permits a systematic evaluation applying formal power counting rules.(2) There is mounting empirical evidence that the QCD ground state breaks the chiral $SU(2)$ symmetry spontaneously in the limiting case where the up and down current quark masses of the QCD Lagrangian vanish. For instance the observation that hadrons do not occur in parity doublet states directly reflects that phenomenon. Also the smallness of the pion masses with $m_\pi \simeq 140$ MeV much smaller than the nucleon mass $m_N \simeq 940$ MeV, naturally fits into this picture once the pions are identified to be the Goldstone bosons of that spontaneously broken chiral symmetry. In χPT the finite but small values of the up and down quark masses $m_{u,d} \simeq 10$ MeV are taken into account as a small perturbation defining the finite masses of the Goldstone bosons. The smallness of the current quark masses on the typical chiral scale of 1 GeV explains the success of standard χPT. For early applications of χPT to pion–nucleon scattering see Refs. 3–5.

It is of course tempting to generalize the successful chiral $SU(2)$ scheme to the $SU(3)$ flavor group. To construct the appropriate chiral $SU(3)$ Lagrangian is mathematically straightforward and has been done long ago (see e.g., Ref. 6). The mass $m_s \simeq 10 m_{u,d}$ of the strange quark, though much larger than the up and down quark masses, is still small as compared to the typical chiral scale of 1 GeV.(23) The required approximate Goldstone boson octet is readily found with the pions, kaons and the eta–meson. Nevertheless, important predictions of standard χPT as applied to the $SU(3)$ flavor group are in stunning conflict with empirical observations. Most spectacular is the failure of the Weinberg–Tomozawa theorem[7] which predicts an attractive K^--proton scattering length, rather than the observed large and repulsive scattering length.

Progress was made upon accepting the crucial observation that the power counting rules must not be applied to a certain subset of Feynman diagrams.[8,9] Whereas for irreducible diagrams the chiral power counting rules are well justified, this is no longer necessarily the case for the reducible diagrams.[2] The latter diagrams are enhanced as compared to irreducible diagrams and therefore may require a systematic resummation scheme in particular in the strangeness sectors. The first intriguing works taking up this idea in the chiral context are given in Refs. 10–13.

In this contribution we report on our recent application of the relativistic chiral $SU(3)$ Lagrangian[12,14] to meson–baryon scattering. Preliminary results of that work were presented by one of the authors (M.F.M.L.) at this meeting. As to our knowledge this is the first application of the chiral $SU(3)$ Lagrangian density to the kaon–nucleon and antikaon–nucleon system including systematically constraints from the pion–nucleon sector. In our scheme, which considers constraints from chiral and large N_c sum rules, the baryon decuplet field with $J^P = \frac{3}{2}^+$ is an important ingredient, because it is part of the baryon ground state multiplet which arises in the large N_c limit of QCD.[15,16] We solve the Bethe–Salpeter equation for the scattering amplitude with the interaction kernel truncated at chiral order Q^3 where we include only those terms which are leading in the large N_c limit of QCD.[15–19] The renormalization scheme is an essential input of our chiral $SU(3)$ dynamics, because it leads to consistency with chiral counting rules and an approximate crossing symmetry of the subthreshold scattering amplitudes. The existing low-energy cross section data on kaon–nucleon and antikaon–nucleon scattering including angular distributions are reproduced to good accuracy. At the same time we achieve a good description of the low-energy *s*- and *p*-wave pion–nucleon phase shifts as well as the empirical axial-vector coupling constants of the baryon octet states.

2. RELATIVISTIC CHIRAL *SU*(3) INTERACTION TERMS IN LARGE N_c QCD

Details on the systematic construction principle for the chiral Lagrangian can be found, e.g., in Refs. 6, 20, and 21. Here we collect the interaction terms of the relativistic chiral $SU(3)$ Lagrangian density relevant for the meson–baryon scattering process. The basic building blocks of the chiral Lagrangian are

$$U_\mu = \tfrac{1}{2}\, e^{-i\frac{\Phi}{2f}}\, (\partial_\mu e^{i\frac{\Phi}{f}} + i[A_\mu, e^{i\frac{\Phi}{f}}]_+)\, e^{-i\frac{\Phi}{2f}}, \qquad B, \qquad \Delta_\mu \tag{1}$$

where we include the pseudo-scalar meson octet field $\Phi(J^P = 0^-)$, the baryon octet field $B(J^P = \frac{1}{2}^+)$ and the baryon decuplet field $\Delta_\mu(J^P = \frac{3}{2}^+)$. In (1) we consider an external axial-vector source function A_μ which is required for the systematic evaluation of matrix elements of the axial-vector current. A corresponding term for the vector current is not shown in (1) because it will not be needed in this work. The axial-vector source function $A^\mu = \sum A^\mu_a \lambda^{(a)}$, the meson octet field $\Phi = \sum \Phi_a \lambda^{(a)}$ and the baryon octet fields $B = \sum B_a \lambda^{(a)}/\sqrt{2}$ are decomposed with the Gell–Mann matrices λ_a normalized with tr $\lambda_a \lambda_b = 2\delta_{ab}$. The baryon decuplet field Δ^{abc} is completely symmetric

$$\begin{aligned} &\Delta^{111} = \Delta^{++}, \quad \Delta^{113} = \Sigma^+/\sqrt{3}, \quad \Delta^{133} = \Xi^0/\sqrt{3}, \quad \Delta^{333} = \Omega^- \\ &\Delta^{112} = \Delta^+/\sqrt{3}, \quad \Delta^{123} = \Sigma^0/\sqrt{6}, \quad \Delta^{233} = \Xi^-/\sqrt{3} \\ &\Delta^{122} = \Delta^0/\sqrt{3}, \quad \Delta^{223} = \Sigma^-/\sqrt{3}, \quad \Delta^{222} = \Delta^- \end{aligned} \tag{2}$$

The parameter f in (1) can be related to the weak decay constant of the charged pions and kaons modulo chiral $SU(3)$ correction terms. Taking the average of the empirical decay parameters $f_\pi = 92.42 \pm 0.33$ MeV and $f_K \simeq 113.0 \pm 1.3$ MeV[22] one obtains the naive estimate $f \simeq 104$ MeV. This value is still within reach of the more detailed analysis[23] which leads to $f_\pi/f = 1.07 \pm 0.12$. As was emphasized in Ref. 24 the precise value of f is subject to large uncertainties.

Explicit chiral symmetry-breaking effects are included in terms of scalar and pseudo-scalar source fields $\chi_\pm$ proportional to the quark-mass matrix of QCD,

$$\chi_\pm = \tfrac{1}{2}\,(e^{+i\frac{\Phi}{2f}}\chi_0 e^{+i\frac{\Phi}{2f}} \pm e^{-i\frac{\Phi}{2f}}\chi_0 e^{-i\frac{\Phi}{2f}}) \tag{3}$$

where $\chi_0 \sim \mathrm{diag}(m_u, m_d, m_s)$. All fields in (1) and (3) have identical properties under chiral $SU(3)$ transformations. The chiral Lagrangian consists of all possible interaction terms, formed with the fields U_μ, B, Δ_μ and $\chi_\pm$ and their respective covariant derivatives. Derivatives of the fields must be included in compliance with the chiral $SU(3)$ symmetry. This leads to the notion of a covariant derivative $\mathcal{D}_\mu$ which is identical for all fields in (1) and (3),

$$[\mathscr{D}_\mu, B]_- = \partial_\mu B + \frac{1}{2}\,[e^{-i\frac{\Phi}{2f}}(\partial_\mu e^{+i\frac{\Phi}{2f}}) + e^{+i\frac{\Phi}{2f}}(\partial_\mu e^{-i\frac{\Phi}{2f}}), B]_-$$

$$+\frac{i}{2}\,[e^{-i\frac{\Phi}{2f}}A_\mu e^{+i\frac{\Phi}{2f}} - e^{+i\frac{\Phi}{2f}}A_\mu e^{-i\frac{\Phi}{2f}}, B]_- \qquad (4)$$

where we illustrated the action of $\mathscr{D}_\mu$ at hand of the baryon octet field.

The chiral Lagrangian becomes a powerful tool once it is combined with appropriate power counting rules leading to a systematic approximation strategy. One aims at describing hadronic interactions at low energy by constructing an expansion in small momenta and the small pseudo-scalar meson masses. The infinite set of Feymnan diagrams are sorted according to their chiral powers. The minimal chiral power Q^ν of a given relativistic Feynman diagram,

$$\nu = 2 - \tfrac{1}{2}\,E_B + 2L + \sum_i V_i\,(d_i + \tfrac{1}{2}\,n_i - 2) \qquad (5)$$

is given in terms of the number of loops, L, the number, V_i, of vertices of type i with d_i "small" derivatives and n_i baryon fields involved, and the number of external baryon lines E_B.[2] Here one calls a derivative small if it acts on the pseudo-scalar meson field or if it probes the virtuality of a baryon field. Explicit chiral symmetry-breaking effects are perturbative and included in the counting scheme with $\chi_0 \sim Q^2$. For a discussion of the relativistic chiral Lagrangian and its required systematic regrouping of interaction terms we refer to Refs. 14 and 25. The relativistic chiral Lagrangian requires a non-standard renormalization scheme. The MS or $\overline{MS}$ minimal subtraction schemes of dimensional regularization do not comply with the chiral counting rule.[3] However, an appropriately modified subtraction scheme for relativistic Feynman diagrams leads to manifest chiral counting rules.[14, 25–27] Alternatively one may work with the chiral Lagrangian in its heavy-fermion representation[28] where an appropriate frame-dependent redefinition of the baryon fields lead to a more immediate manifestation of the chiral power counting rule (5).

In the πN sector the $SU(2)$ chiral Lagrangian was successfully applied[3, 4] demonstrating good convergence properties of the perturbative chiral expansion. In the $SU(3)$ sector the situation is more involved due in part to the relatively large kaon mass $m_K \simeq m_N/2$. Here the perturbative evaluation of the chiral Lagrangian cannot be justified and one must change the expansion strategy. Rather than expanding directly the scattering amplitude one may expand the interaction kernel according to chiral power counting rules.[2, 29] The scattering amplitude then follows from the solution of a scattering equation like the Lipmann–Schwinger or the Bethe–Salpeter equation. This is by analogy with the treatment of the e^+e^- bound-state problem of QED where a perturbative evaluation of the interaction kernel can be justified. The rationale behind this change of scheme lies in the observation that reducible diagrams are typically enhanced close to their unitarity threshold. The enhancement factor of $(2\pi)^n$, measured relative to a reducible diagram with the same number of independent loop integrations, is given by the number, n, of reducible meson–baryon pairs in the diagram, i.e., the number of unitary iterations implicit in the diagram. In the πN sector this enhancement factor does not prohibit a perturbative treatment, because the typical expansion parameter $m_\pi^2/(8\pi f^2) \sim 0.1$ remains sufficiently small. In the $\bar{K}N$ sector, on the other hand, the factor $(2\pi)^n$ invalidates a perturbative treatment, because the typical expansion parameter would be $m_K^2/(8\pi f^2) \sim 1$. This is in contrast to irreducible diagrams. They yield the typical expansion parameters $m_\pi/(4\pi f)$ and $m_K/(4\pi f)$ which justifies the perturbative evaluation of the scattering kernels.

In our work we consider all terms of chiral order Q^3 in the scattering kernel which are leading in the large N_c limit. According to this philosophy loop corrections to the Bethe–Salpeter kernel need not be evaluated, because they carry minimal chiral order Q^3 but are at the same time $1/N_c$ suppressed. In the following we will discuss all chiral interaction terms relevant to chiral order Q^2. Further terms of order Q^3 considered in our approach can be found in Ref. 14. Therein the interested reader may also find the many technical developments needed to solve the Bethe–Salpeter scattering equation.

The effective chiral Lagrangian

$$\mathscr{L} = \sum_n \mathscr{L}^{(n)} + \sum_n \mathscr{L}_\chi^{(n)} \tag{6}$$

falls into different classes $\mathscr{L}^{(n)}$ and $\mathscr{L}_\chi^{(n)}$. With an upper index n in $\mathscr{L}^{(n)}$ we indicate the number of fields in the interaction vertex. The lower index χ signals terms with explicit chiral symmetry breaking. We assume charge conjugation symmetry and parity invariance in this work. At leading chiral order the following interaction terms are required:

$$\begin{aligned}\mathscr{L}^{(2)} &= \mathrm{tr}\, \bar{B}(i\partial\!\!\!/ - \mathring{m}_{[8]})\, B + \frac{1}{4}\,\mathrm{tr}(\partial^\mu \Phi)(\partial_\mu \Phi)\\ &\quad + \mathrm{tr}\, \bar{\Delta}_\mu \cdot ((i\partial\!\!\!/ - \mathring{m}_{[10]})\, g^{\mu\nu} - i(\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) + i\gamma^\mu \partial\!\!\!/ \gamma^\nu + \mathring{m}_{[10]} \gamma^\mu \gamma^\nu)\, \Delta_\nu\end{aligned}$$

$$\mathcal{L}^{(3)} = \frac{F_{[8]}}{2f}\,\mathrm{tr}\,\bar{B}\gamma_5\gamma^\mu[(\partial_\mu\Phi), B]_- + \frac{D_{[8]}}{2f}\,\mathrm{tr}\,\bar{B}\gamma_5\gamma^\mu[(\partial_\mu\Phi), B]_+$$
$$-\frac{C_{[10]}}{2f}\,\mathrm{tr}\left\{(\bar{\Delta}_\mu\cdot(\partial_\nu\Phi))\left(g^{\mu\nu}-\frac{1}{2}Z_{[10]}\gamma^\mu\gamma^\nu\right)B+\text{h.c.}\right\}$$
$$\mathcal{L}^{(4)} = \frac{i}{8f^2}\,\mathrm{tr}\,\bar{B}\gamma^\mu[[\Phi, (\partial_\mu\Phi)]_-, B]_- \tag{7}$$

where we use the notations $[A, B]_\pm = AB \pm BA$ for $SU(3)$ matrices A and B. The last term in (7) reproduces the low-energy theorems of meson–nucleon scattering derived first by Weinberg and Tomazawa applying current-algebra techniques.(7) Note that the complete chiral interaction terms which lead to the terms in (7) are easily recovered by replacing $i\partial_\mu\Phi/f \to U_\mu$. Also, a derivative acting on a baryon field must be replaced by the covariant derivative with $\partial_\mu B \to [\mathcal{D}_\mu, B]_-$ and $\partial_\mu \Delta_\nu \to [\mathcal{D}_\mu, \Delta_\nu]_-$ in (7). With $\mathring{m}_{[8]}$ and $\mathring{m}_{[10]}$ in (7) we denote the baryon masses in the chiral $SU(3)$ limit. Furthermore the products of an anti-decuplet field $\bar{\Delta}$ with a decuplet field Δ and an octet field Φ transform as $SU(3)$ octets

$$(\bar{\Delta}\cdot\Delta)^a_b = \bar{\Delta}_{bcd}\,\Delta^{acd}, \qquad (\bar{\Delta}\cdot\Phi)^a_b = \epsilon^{kla}\,\bar{\Delta}_{knb}\Phi^n_l$$
$$(\Phi\cdot\Delta)^a_b = \epsilon_{klb}\Phi^l_n\,\Delta^{kna} \tag{8}$$

where ϵ_{abc} is the completely anti-symmetric pseudo-tensor.

The parameters $F_{[8]} \simeq 0.45$ and $D_{[8]} \simeq 0.80$ are constrained by the weak decay widths of the baryon octet states(30) and $C_{[10]} \simeq 1.6$ can be estimated from the hadronic decay width of the baryon decuplet states. The parameter $Z_{[10]}$ in (7) may be best determined in an $SU(3)$ analysis of meson–baryon scattering. While in the pion–nucleon sector it can be absorbed into the quasi-local 4-point interaction terms to chiral accuracy Q^2 (31) this is no longer possible in the $SU(3)$ scheme. Our detailed analysis reveals that the parameter $Z_{[10]}$ is relevant already at order Q^2 if a simultaneous chiral analysis of the pion–nucleon and kaon–nucleon scattering processes is performed.

2.1. Large N_c Counting

We briefly recall a powerful expansion strategy which follows from QCD if the numbers of colors (N_c) is considered as large number. We present a formulation best suited for an application to the chiral Lagrangian leading to a significant parameter reduction. The large N_c scaling of a chiral interaction term is easily worked out using the operator analysis proposed in Ref. 32. Interaction terms involving baryon fields are represented by matrix elements of many-body operators in the large N_c ground-state baryon multiplet $|\mathscr{B}\rangle$. A n-body operator is the product of n factors formed exclusively in terms of the bilinear quark-field operators J_i, $G_i^{(a)}$ and $T^{(a)}$ being characterized fully by their commutation relations

$$
\begin{gathered}
[G_i^{(a)}, G_j^{(b)}]_- = \tfrac{1}{4}\, i\delta_{ij}\, f^{ab}{}_c T^{(c)} + \tfrac{1}{2}\, i\varepsilon_{ij}{}^k (\tfrac{1}{3}\,\delta^{ab} J_k + d^{ab}{}_c G_k^{(c)}) \\
[J_i, J_j]_- = i\varepsilon_{ij}{}^k J_k, \qquad [T^{(a)}, T^{(b)}]_- = i f^{ab}{}_c T^{(c)} \\
[T^{(a)}, G_i^{(b)}]_- = i f^{ab}{}_c G_i^{(c)}, \qquad [J_i, G_j^{(a)}]_- = i\varepsilon_{ij}{}^k G_k^{(a)}, \qquad [J_i, T^{(a)}]_- = 0
\end{gathered} \tag{9}
$$

The algebra (9), which reflects the so-called contracted spin-flavor symmetry of QCD, leads to a transparent derivation of the many sum rules implied by the various infinite subclasses of QCD quark-gluon diagrams as collected at a given order in the $1/N_c$ expansion. A convenient realization of the algebra (9) is obtained in terms of non-relativistic, flavor-triplet and color N_c-multiplet field operators q and $q^\dagger$,

$$
\begin{gathered}
J_i = q^\dagger \left(\frac{\sigma_i^{(q)}}{2} \otimes 1 \right) q, \qquad T^{(a)} = q^\dagger \left(1 \otimes \frac{\lambda^{(a)}}{2} \right) q \\
G_i^{(a)} = q^\dagger \left(\frac{\sigma_i^{(q)}}{2} \otimes \frac{\lambda^{(a)}}{2} \right) q
\end{gathered} \tag{10}
$$

If the fermionic field operators q and $q^\dagger$ are assigned anti-commutation rules the algebra (9) follows. The Pauli spin matrices $\sigma_i^{(q)}$ act on the two-component spinors of the fermion fields q, $q^\dagger$ and the Gell–Mann matrices λ_a on its flavor components. Here one needs to emphasize that the non-relativistic quark-field operators q and $q^\dagger$ must not be identified with the quark-field operators of the QCD Lagrangian.[17–19] Rather, they constitute an effective tool to represent the operator algebra (9) which allows an efficient derivation of the large N_c sum rules of QCD. A systematic truncation scheme results in the operator analysis, because a n-body operator is assigned the suppression factor N_c^{1-n}. The analysis is complicated by the fact that matrix elements of $T^{(a)}$ and $G_i^{(a)}$ may be of order N_c in the baryon ground state $|\mathscr{B}\rangle$. That implies for instance that matrix elements of the $(2n+1)$-body operator $(T_a T^{(a)})^n T^{(c)}$ are not suppressed relative to the matrix elements of the one-body operator $T^{(c)}$. The systematic large N_c operator analysis relies on the observation that matrix elements of the spin operator J_i, on the other hand, are always of order N_c^0. Then a set of identities shows how to systematically represent the infinite set of many-body operators, which one may write down at a given order in the $1/N_c$ expansion, in terms of a finite number of operators. This leads to a convenient scheme with only a finite number of operators at given order.[32] We recall typical examples of the required operator identities,

$$
\begin{aligned}
&[T_a, T^{(a)}]_+ - [J_i, J^{(i)}]_+ = \tfrac{1}{6} N_c (N_c + 6), \qquad [T_a, G_i^{(a)}]_+ = \tfrac{2}{3} (3 + N_c)\, J_i \\
&27 [T_a, T^{(a)}]_+ - 12 [G_i^{(a)}, G_a^{(i)}]_+ = 32 [J_i, J^{(i)}]_+ \\
&d_{abc} [T^{(a)}, T^{(b)}]_+ - 2 [J_i, G_c^{(i)}]_+ = -\tfrac{1}{3} (N_c + 3)\, T_c \\
&d^a{}_{bc} [G_a^{(i)}, G_i^{(b)}]_+ + \tfrac{9}{4} d_{abc} [T^{(a)}, T^{(b)}]_+ = \tfrac{10}{3} [J_i, G_c^{(i)}]_+ \\
&d_{ab}{}^c [T^{(a)}, G_i^{(b)}]_+ = \tfrac{1}{3} [J_i, T^{(c)}]_+ - \tfrac{1}{3} \varepsilon_{ijk} f_{ab}{}^c [G_a^{(j)}, G_b^{(k)}]_+
\end{aligned}
\tag{11}
$$

For instance the first identity in (11) shows how to avoid that infinite tower $(T_a T^{(a)})^n\, T^{(c)}$ discussed above. Note that the "parameter" N_c enters in (11) as a mean to classify the possible realizations of the algebra (9).

As a first and simple example we recall the large N_c structure of the 3-point vertices. One readily establishes two operators with parameters g and h at leading order in the $1/N_c$ expansion[32]

$$
\langle \mathscr{B}' |\, \mathscr{L}^{(3)} \,| \mathscr{B} \rangle = \frac{1}{f} \langle \mathscr{B}' |\, g G_i^{(c)} + h J_i T^{(c)} \,| \mathscr{B} \rangle \operatorname{tr} \lambda_c\, \nabla^{(i)} \Phi + \mathscr{O}\left(\frac{1}{N_c}\right) \tag{12}
$$

Further possible terms in (12) are either redundant or suppressed in the $1/N_c$ expansion. For example, the two-body operator $i f_{abc} G_i^{(a)} T^{(b)} \sim N_c^0$ is reduced by

$$
i f_{ab}{}^c [G_i^{(a)}, T^{(b)}]_- = i f_{ab}{}^c\, i f^{ab}{}_d\, G_i^{(d)} = -3 G_i^{(c)}
$$

In order to make use of the large N_c result it is necessary to evaluate the matrix elements in (12) at $N_c = 3$ where one has a **56**-plet with $|\mathscr{B}\rangle = |B(a), \Delta(ijk)\rangle$. Most economically this is achieved with the completeness identity $1 = |B\rangle\langle B| + |\Delta\rangle\langle\Delta|$ in conjunction with

$$
\begin{aligned}
T_c\, |B_a(\chi)\rangle &= i f_{abc}\, |B^{(b)}(\chi)\rangle \\
J^{(i)}\, |B_a(\chi)\rangle &= \frac{1}{2} \sigma^{(i)}_{\chi'\chi}\, |B_a(\chi')\rangle \\
G_c^{(i)}\, |B_a(\chi)\rangle &= \left(\frac{1}{2} d_{abc} + \frac{1}{3} i f_{abc} \right) \sigma^{(i)}_{\chi'\chi}\, |B^{(b)}(\chi')\rangle \\
&\quad + \frac{1}{\sqrt{2}\,2} (\varepsilon_l^{jk} \lambda^{(c)}_{mj} \lambda^{(a)}_{nk})\, S^{(i)}_{\chi'\chi}\, |\Delta^{(lmn)}(\chi')\rangle
\end{aligned}
\tag{13}
$$

where $S_i S_j^\dagger = \delta_{ij} - \sigma_i \sigma_j/3$ and $\lambda_a \lambda_b = \frac{2}{3}\,\delta_{ab} + (i f_{abc} + d_{abc})\,\lambda^{(c)}$. In (13) the baryon octet states $|B_b(\chi)\rangle$ are labeled according to their $SU(3)$ octet index $a = 1,\ldots, 8$ with the two spin states represented by $\chi = 1, 2$. Similarly the decuplet states $|\Delta_{lmn}(\chi')\rangle$ are listed with $l, m, n = 1, 2, 3$ as defined in (2). Note that the expressions (13) may be verified using the quark-model wave functions for the baryon octet and decuplet states. It is important to realize, however, that the result (13) is much more general than the quark-model, because it reflects the structure of the ground-state baryons in the large N_c limit of QCD only. Matching the interaction vertices of the relativistic chiral Lagrangian onto the static matrix elements arising in the large N_c operator analysis requires a non-relativistic reduction. It is standard to decompose the 4-component Dirac fields B and Δ_μ into baryon octet and decuplet spinor fields $B(\chi)$ and $\Delta(\chi)$:

$$(B, \Delta_\mu) \to \begin{pmatrix} \left(\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{\nabla^2}{M^2}}\right)^{1/2} (B(\chi), S_\mu\, \Delta(\chi)) \\ \frac{(\sigma\cdot\nabla)}{\sqrt{2}\, M}\left(1+\sqrt{1+\frac{\nabla^2}{M^2}}\right)^{-1/2} (B(\chi), S_\mu\, \Delta(\chi)) \end{pmatrix} \tag{14}$$

where M denotes the baryon octet and decuplet mass in the large N_c limit. At leading order one finds $S_\mu = (0, S_i)$ with the transition matrices S_i introduced in (13). It is then straightforward to expand in powers of ∇/M and achieve the desired matching. This leads for example to the identification $D_{[8]} = g$, $F_{[8]} = 2g/3 + h$ and $C_{[10]} = 2g$. The empirical values of $F_{[8]}$, $D_{[8]}$ and $C_{[10]}$ are quite consistent with those relations.[33] Note that operators at subleading order in (12) then parameterize the deviation from $C \simeq 2D$.

2.2. Quasi-Local Interaction Terms

We turn to the two-body interaction terms at chiral order Q^2. From phase space consideration it is evident that at this order there are only terms which contribute to the meson–baryon s-wave scattering lengths, the s-wave effective range parameters and the p-wave scattering volumes. Higher partial waves are not involved at this order. The various contributions are distributed with:

$$\mathscr{L}_2^{(4)} = \mathscr{L}^{(S)} + \mathscr{L}^{(V)} + \mathscr{L}^{(T)} \tag{15}$$

where the lower index k in $\mathscr{L}_k^{(n)}$ denotes the minimal chiral order of the interaction vertex. In the relativistic framework one observes mixing of the partial waves in the sense that for instance $\mathscr{L}^{(S)}$, $\mathscr{L}^{(V)}$ contribute to the s-wave channels and $\mathscr{L}^{(S)}$, $\mathscr{L}^{(T)}$ to the p-wave channels. We write:

$$
\begin{aligned}
\mathscr{L}^{(S)} &= \frac{g_0^{(S)}}{8f^2}\,\mathrm{tr}\,\bar{B}B\;\mathrm{tr}(\partial_\mu\Phi)(\partial^\mu\Phi) + \frac{g_1^{(S)}}{8f^2}\,\mathrm{tr}\,\bar{B}(\partial_\mu\Phi)\;\mathrm{tr}(\partial^\mu\Phi)\,B \\
&\quad + \frac{g_F^{(S)}}{16f^2}\,\mathrm{tr}\,\bar{B}[[(\partial_\mu\Phi),(\partial^\mu\Phi)]_+,B]_- + \frac{g_D^{(S)}}{16f^2}\,\mathrm{tr}\,\bar{B}[[(\partial_\mu\Phi),(\partial^\mu\Phi)]_+,B]_+ \\
\mathscr{L}^{(V)} &= \frac{g_0^{(V)}}{16f^2}\,(\mathrm{tr}\,\bar{B}i\gamma^\mu(\partial^\nu B)\;\mathrm{tr}(\partial_\nu\Phi)(\partial_\mu\Phi) + \mathrm{h.c.}) \\
&\quad + \frac{g_1^{(V)}}{32f^2}\,\mathrm{tr}\,\bar{B}i\gamma^\mu((\partial_\mu\Phi)\;\mathrm{tr}(\partial_\nu\Phi)(\partial^\nu B) + (\partial_\nu\Phi)\;\mathrm{tr}(\partial_\mu\Phi)(\partial^\nu B) + \mathrm{h.c.}) \\
&\quad + \frac{g_F^{(V)}}{32f^2}\,(\mathrm{tr}\,\bar{B}i\gamma^\mu[[(\partial_\mu\Phi),(\partial_\nu\Phi)]_+,(\partial^\nu B)]_- + \mathrm{h.c.}) \\
&\quad + \frac{g_D^{(V)}}{32f^2}\,(\mathrm{tr}\,\bar{B}i\gamma^\mu[[(\partial_\mu\Phi),(\partial_\nu\Phi)]_+,(\partial^\nu B)]_+ + \mathrm{h.c.}) \\
\mathscr{L}^{(T)} &= \frac{g_1^{(T)}}{8f^2}\,\mathrm{tr}\,\bar{B}(\partial_\mu\Phi)\;i\sigma^{\mu\nu}\;\mathrm{tr}(\partial_\nu\Phi)\,B \\
&\quad + \frac{g_D^{(T)}}{16f^2}\,\mathrm{tr}\,\bar{B}i\sigma^{\mu\nu}[[(\partial_\mu\Phi),(\partial_\nu\Phi)]_-,B]_+ \\
&\quad + \frac{g_F^{(T)}}{16f^2}\,\mathrm{tr}\,\bar{B}i\sigma^{\mu\nu}[[(\partial_\mu\Phi),(\partial_\nu\Phi)]_-,B]_-
\end{aligned}
\tag{16}
$$

It is clear that if the heavy-baryon expansion is applied to (16) the quasi-local 4-point interactions can be mapped onto corresponding terms of the heavy-baryon formalism presented for example in Ref. 34.

We apply the large N_c counting rules in order to estimate the relative importance of the quasi-local Q^2-terms in (16). Terms which involve a single-flavor trace are enhanced as compared to the double-flavor trace terms. This is obvious, because a flavor trace in an interaction term is necessarily accompanied by a corresponding color trace if visualized in terms of quark and gluon lines. A color trace signals a quark loop and therefore provides the announced $1/N_c$ suppression factor.[15,16] The counting rules are nevertheless subtle, because a certain combination of double trace expressions can be rewritten in terms of a single-flavor trace term[35]

$$\begin{aligned} &\mathrm{tr}(\bar{B}B)\,\mathrm{tr}(\Phi\Phi)+2\,\mathrm{tr}(\bar{B}\Phi)\,\mathrm{tr}(\Phi B) \\ &\quad = \mathrm{tr}[\bar{B},\Phi]_-\,[B,\Phi]_- +\tfrac{3}{2}\,\mathrm{tr}\,\bar{B}[[\Phi,\Phi]_+,B]_+ \end{aligned} \tag{17}$$

Thus one expects for example that both parameters $g_0^{(S)}$ and $g_1^{(S)}$ may be large separately but the combination $2g_0^{(S)}-g_1^{(S)}$ should be small. A more detailed operator analysis leads to

$$\begin{aligned} \langle \mathscr{B}'|\,\mathscr{L}_2^{(4)}\,|\mathscr{B}\rangle &= \frac{1}{16f^2}\,\langle \mathscr{B}'|\,O_{ab}(g_1,g_2)\,|\mathscr{B}\rangle\,\mathrm{tr}[(\partial_\mu\Phi),\lambda^{(a)}]_-\,[(\partial^\mu\Phi),\lambda^{(b)}]_- \\ &\quad +\frac{1}{16f^2}\,\langle \mathscr{B}'|\,O_{ab}(g_3,g_4)\,|\mathscr{B}\rangle\,\mathrm{tr}[(\partial_0\Phi),\lambda^{(a)}]_-\,[(\partial_0\Phi),\lambda^{(b)}]_- \\ &\quad +\frac{1}{16f^2}\,\langle \mathscr{B}'|\,O_{ab}^{(ij)}(g_5,g_6)\,|\mathscr{B}\rangle\,\mathrm{tr}[(\nabla_i\Phi),\lambda^{(a)}]_- \\ &\quad \times[(\nabla_j\Phi),\lambda^{(b)}]_- \\ O_{ab}(g,h) &= g\,d_{abc}T^{(c)}+h[T_a,T_b]_+ +\mathcal{O}\left(\frac{1}{N_c}\right) \\ O_{ab}^{(ij)}(g,h) &= i\varepsilon^{ijk}i\,f_{abc}(gG_k^{(c)}+hJ_kT^{(c)})+\mathcal{O}\left(\frac{1}{N_c}\right) \end{aligned} \tag{18}$$

We checked that other forms for the coupling of the operators O_{ab} to the meson fields do not lead to new structures. It is straightforward to match the coupling constants $g_{1,\dots,6}$ onto the ones in (16). Identifying the leading terms in the non-relativistic expansion we obtain:

$$\begin{aligned} &g_0^{(S)}=\frac{1}{2}\,g_1^{(S)}=\frac{2}{3}\,g_D^{(S)}=-2g_2, \qquad g_F^{(S)}=-3g_1 \\ &g_0^{(V)}=\frac{1}{2}\,g_1^{(V)}=\frac{2}{3}\,g_D^{(V)}=-2\,\frac{g_4}{M}, \qquad g_F^{(V)}=-3\,\frac{g_3}{M} \\ &g_1^{(T)}=0, \qquad g_F^{(T)}=-g_5-\frac{3}{2}\,g_6, \qquad g_D^{(T)}=-\frac{3}{2}\,g_5 \end{aligned} \tag{19}$$

where M is the large N_c value of the baryon octet mass. We conclude that at chiral order Q^2 there are only six leading large N_c coupling constants.

Part of the predictive power of the chiral Lagrangian results, because chiral $SU(3)$ symmetry selects certain subsets of all $SU(3)$ symmetric tensors at a given chiral order.

2.3. Explicit Chiral Symmetry Breaking

There remain the interaction terms proportional to $\chi_\pm$ which break the chiral $SU(3)$ symmetry explicitly. We collect here all relevant terms of chiral order Q^2.[3, 10] It is convenient to visualize the symmetry-breaking fields $\chi_\pm$ of (3) in their expanded forms:

$$\begin{aligned} \chi_+ &= \chi_0 - \frac{1}{8f^2}\,[\Phi, [\Phi, \chi_0]_+]_+ + \mathcal{O}(\Phi^4) \\ \chi_- &= \frac{i}{2f}\,[\Phi, \chi_0]_+ + \mathcal{O}(\Phi^3) \end{aligned} \tag{20}$$

We begin with the 2-point interaction vertices which all result exclusively from chiral interaction terms linear in χ_+. They read

$$\begin{aligned} \mathcal{L}_\chi^{(2)} &= -\frac{1}{4}\,\mathrm{tr}\,\Phi[\chi_0, \Phi]_+ + 2\,\mathrm{tr}\,\bar{B}(b_D[\chi_0, B]_+ + b_F[\chi_0, B]_- + b_0 B\,\mathrm{tr}\,\chi_0) \\ &\quad + 2d_D\,\mathrm{tr}(\bar{\Delta}_\mu \cdot \Delta^\mu)\,\chi_0 + 2d_0\,\mathrm{tr}(\bar{\Delta}_\mu \cdot \Delta^\mu)\,\mathrm{tr}\,\chi_0 \\ &\quad + \mathrm{tr}\,\bar{B}(i\partial\!\!\!/ - \mathring{m}_{[8]})(\zeta_0 B\,\mathrm{tr}\,\chi_0 + \zeta_D[B, \chi_0]_+ + \zeta_F[B, \chi_0]_-) \\ \chi_0 &= \frac{1}{3}\,(m_\pi^2 + 2m_K^2)\,1 + \frac{2}{\sqrt{3}}\,(m_\pi^2 - m_K^2)\,\lambda_8 \end{aligned} \tag{21}$$

where we normalized χ_0 to give the pseudo-scalar mesons their isospin averaged masses. The first term in (21) leads to the finite masses of the pseudo-scalar mesons. Note that to chiral order Q^2 one has $m_\eta^2 = 4(m_K^2 - m_\pi^2)/3$. The parameters b_D, b_F, and d_D are determined to leading order by the baryon octet and decuplet mass splitting

$$\begin{aligned} m_{[8]}^{(\Sigma)} - m_{[8]}^{(\Lambda)} &= \tfrac{16}{3}\,b_D(m_K^2 - m_\pi^2), \qquad m_{[8]}^{(\Xi)} - m_{[10]}^{(N)} = -8b_F(m_K^2 - m_\pi^2) \\ m_{[10]}^{(\Sigma)} - m_{[10]}^{(\Delta)} &= m_{[8]}^{(\Xi)} - m_{[10]}^{(\Sigma)} = m_{[10]}^{(\Omega)} - m_{[10]}^{(\Xi)} = -\tfrac{4}{3}\,d_D(m_K^2 - m_\pi^2) \end{aligned}$$

The empirical baryon masses lead to the estimates $b_D \simeq 0.06\ \mathrm{GeV}^{-1}$, $b_F \simeq -0.21\ \mathrm{GeV}^{-1}$, and $d_D \simeq -0.49\ \mathrm{GeV}^{-1}$. For completeness we recall the leading large N_c operators for the baryon mass splitting (see e.g., Ref. 33)

$$\langle \mathscr{B}' |\, \mathscr{L}_\chi^{(2)} \,| \mathscr{B} \rangle = \langle \mathscr{B}' |\, b_1 T^{(8)} + b_2 [J^{(i)}, G_i^{(8)}]_+ \,| \mathscr{B} \rangle + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

$$b_D = -\frac{\sqrt{3}}{16}\frac{3b_2}{m_K^2 - m_\pi^2}, \qquad b_F = -\frac{\sqrt{3}}{16}\frac{2b_1 + b_2}{m_K^2 - m_\pi^2}, \qquad d_D = -\frac{3\sqrt{3}}{8}\frac{b_1 + 2b_2}{m_K^2 - m_\pi^2} \tag{22}$$

where we matched the symmetry-breaking parts with λ_8. One observes that the empirical values for $b_D + b_F$ and d_D are remarkably consistent with the large N_c sum rule $b_D + b_F \simeq \frac{1}{3} d_D$. The parameters b_0 and d_0 are more difficult to access. They determine the deviation of the octet and decuplet baryon masses from their chiral $SU(3)$ limit values $\mathring{m}_{[8]}$ and $\mathring{m}_{[10]}$

$$\begin{aligned} m_{[8]}^{(N)} &= \mathring{m}_{[8]} - 2m_\pi^2 (b_0 + 2b_F) - 4m_K^2 (b_0 + b_D - b_F) \\ m_{[10]}^{(\Delta)} &= \mathring{m}_{[10]} - 2m_\pi^2 (d_0 + d_D) - 4m_K^2 d_0 \end{aligned} \tag{23}$$

where terms of chiral order Q^3 are neglected. The size of the parameter b_0 is commonly encoded into the pion–nucleon sigma term

$$\sigma_{\pi N} = -2m_\pi^2 (b_D + b_F + 2b_0) + \mathcal{O}(Q^3)$$

Note that the former standard value $\sigma_{\pi N} = (45 \pm 8)$ MeV of Ref. 36 is currently under debate.[37]

The predictive power of the chiral Lagrangian lies in part in the strong correlation of vertices of different degrees as implied by the non-linear fields U_μ and $\chi_\pm$. A powerful example is given by the two-point vertices introduced in (21). Since they result from chiral interaction terms linear in the χ_+-field (see (20)) they induce particular meson–octet baryon–octet interaction vertices

$$\begin{aligned} \mathscr{L}_\chi^{(4)} = &-\frac{1}{4f^2} \operatorname{tr} \bar{B} (b_D [[\Phi, [\Phi, \chi_0]_+]_+, B]_+ + b_F [[\Phi, [\Phi, \chi_0]_+]_+, B]_-) \\ &-\frac{b_0}{4f^2} \operatorname{tr} \bar{B} B \operatorname{tr} [\Phi, [\Phi, \chi_0]_+]_+ \end{aligned} \tag{24}$$

At chiral order Q^2 there are no further four-point interaction terms with explicit chiral symmetry breaking.

Closing this section we would like to emphasize that one should not be discouraged by the many fundamental parameters introduced in this section. The empirical data set includes many hundreds of data points and will be reproduced rather accurately. Our scheme makes many predictions for poorly known or not known observable quantities like for example the p-wave scattering volumes of the kaon–nucleon scattering processes or the many $SU(3)$ reactions like $\pi\Lambda \to \pi\Sigma$. Also one should realize that in a more conventional meson-exchange approach, besides lacking a systematic approximation scheme, many parameters are implicit in the so-called form factors. In a certain sense the parameters used in the form factors reflect the more systematically constructed and controlled quasi-local counter terms of the chiral Lagrangian.

3. MESON–BARYON SCATTERING

We present a selection of results obtained in our detailed chiral $SU(3)$ analysis of the low-energy meson–baryon scattering data. Before delving into details we briefly summarize the main features and crucial arguments of our approach. We consider the number of colors (N_c) in QCD as a large parameter relying on a systematic expansion of the interaction kernel in powers of $1/N_c$. The coupled-channel Bethe–Salpeter kernel is evaluated in a combined chiral and $1/N_c$ expansion to chiral order Q^3. The scattering amplitudes for the meson–baryon scattering processes are obtained from the solution of the coupled-channel Bethe–Salpeter scattering equation. Approximate crossing symmetry of the amplitudes is guaranteed by a renormalization program which leads to the matching of subthreshold amplitudes. After giving a discussion of baryon-resonance generation in the coupled-channel approach, we present the parameters as they are adjusted to the data set. In the subsequent sections we confront our results with the empirical data. Our theory is referred to as the "χ-BS(3)" approach for chiral Bethe–Salpeter dynamics of the flavor $SU(3)$ symmetry.

3.1. Dynamical Generation of Baryon Resonances

A qualitative understanding of the typical strength in the various channels can be obtained already at leading chiral order Q. In particular the $\Lambda(1405)$ resonance is formed as a result of the coupled-channel dynamics defined by the leading order Weinberg–Tomozawa interaction vertices ($\mathscr{L}^{(4)}$ in (7)). If taken as input for the multi-channel Bethe–Salpeter equation a rich structure of the scattering amplitude arises. Figure 1 shows the s-wave solution of the multi-channel Bethe–Salpeter as a function of the kaon mass. For physical kaon masses the isospin zero scattering amplitude exhibits a resonance structure at energies where one would expect the $\Lambda(1405)$ resonance. We point out that the resonance structure disappears as the kaon mass is decreased. Already at a hypothetical kaon mass of

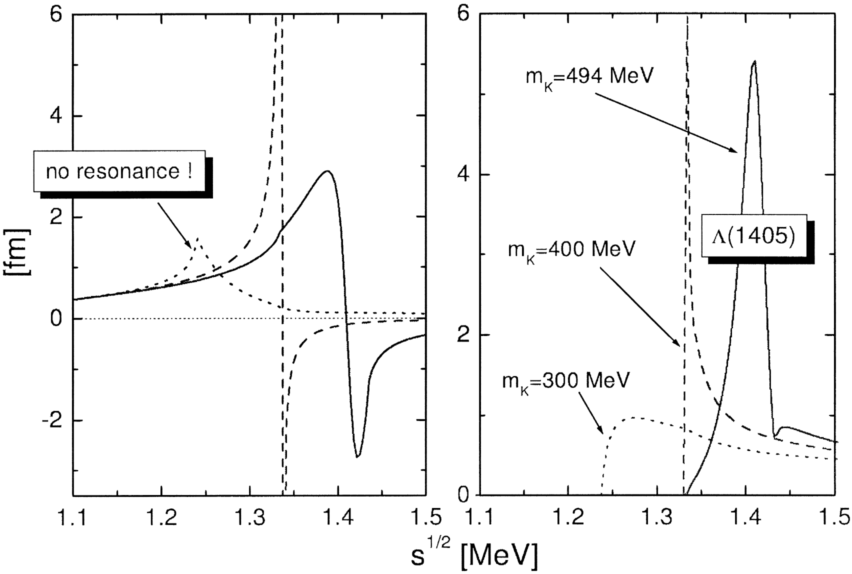

Fig. 1. Real (l.h.s.) and imaginary (r.h.s.) part of the isospin zero s-wave K^--nucleon scattering amplitude as it follows from the $SU(3)$ Weinberg–Tomozawa interaction term in a coupled-channel calculation. We use here $f = 93$ MeV.

300 MeV the $\Lambda(1405)$ resonance is not formed anymore. Figure 1 nicely demonstrates that the chiral $SU(3)$ Lagrangian is necessarily non-perturbative in the strangeness sector.[10–12, 38–40] Unitary (reducible) loop diagrams are typically enhanced by a factor of 2π close to threshold relatively to irreducible diagrams and must therefore be summed to infinite orders.

An immediate question arises: what is the generic nature of the nucleon and hyperon resonances? We successfully generated the s-wave resonance $\Lambda(1405)$ in terms of chiral $SU(3)$ coupled-channel dynamics. Are more resonances of dynamic origin? We expect all baryon resonances, with the important exception of those resonances which belong to the large N_c baryon ground states, to be generated by coupled-channel dynamics. This conjecture is based on the observation that unitary (reducible) loop diagrams are typically enhanced by a factor of 2π close to threshold relatively to irreducible diagrams. That factor invalidates the perturbative evaluation of the scattering amplitudes and leads necessarily to a non-perturbative scheme with reducible diagrams summed to all orders.

A more conservative approach would be to explicitly incorporate additional resonance fields in the chiral Lagrangian. That causes, however, severe problems as there is no longer a, straightforward systematic approximation strategy. Given that the baryon octet and decuplet states are degenerate in the large N_c limit, it is natural to impose $m_{[10]} - m_{[8]} \sim Q$. In contrast with that there is no fundamental reason to insist on $m_{N^*} - m_N \sim Q$. But, only with $m_{N^*} - m_{[8]} \sim Q$ is it feasible to establish consistent power counting rules needed for the systematic evaluation of the chiral Lagrangian. Note that the presence of the nucleon resonance states in the large N_c limit of QCD is far from obvious. Since reducible diagrams are typically enhanced by a factor of 2π relatively to irreducible diagrams, there are a priori two possibilities: the baryon resonances are a consequence of important coupled-channel dynamics or they are already present in the interaction kernel. Our opinion differs here from the one expressed in Refs. 41 and 42 where for instance the d-wave baryon resonance states are considered as part of an excited large N_c **70**-plet. An expansion in $2\pi/N_c$, in our world with $N_c = 3$, does not appear useful. Also the fact that baryon resonances exhibit large hadronic decay widths may be taken as an indication that the coupled-channel dynamics is the driving mechanism for the creation of baryon resonances. Related arguments have been put forward in Refs. 43 and 44. Indeed, for instance the d-wave $N(1520)$ resonance, was successfully described in terms of coupled-channel dynamics, including the vector–meson nucleon channels as an important ingredient,[45] but without assuming a preformed resonance structure in the interaction kernel. The successful description of the $\Lambda(1405)$ resonance in our scheme (see Fig. 1) supports the above arguments. For a recent discussion of the competing picture in which the $\Lambda(1405)$ resonance is considered as a quark-model state we refer to Ref. 46.

The description of resonances has a subtle consequence for the treatment of the u-channel baryon resonance exchange contribution. If a resonance is formed primarily by the coupled-channel dynamics one should not include an explicit bare u-channel resonance contribution in the interaction kernel. The necessarily strong energy dependence of the resonance self-energy would invalidate the use of the bare u-channel resonance contribution, because for physical energies $\sqrt{s} > m_N + m_\pi$ the resonance self-energy is probed far off the resonance pole. Our discussion has non-trivial implications for the chiral $SU(3)$ dynamics. Naively one may object that the effects of the u-channel baryon resonance exchange contributions can be absorbed to good accuracy into chiral two-body interaction terms in any case. However, while this is true in a chiral $SU(2)$ scheme, this is no longer possible in a chiral $SU(3)$ approach. This follows because chiral symmetry selects a, specific subset of all possible $SU(3)$-symmetric two-body interaction terms to given chiral order. For that reason we discriminate two possibilities. In scenario I we conjecture that the baryon resonance states are primarily generated by the coupled-channel dynamics. This is analogous to the treatment of the $\Lambda(1405)$ resonance, which is generated dynamically in the chiral $SU(3)$ scheme (see Fig. 1). Here a s-channel pole term is generated by the coupled-channel dynamics but the associated u-channel term is effectively left out as a much suppressed contribution. In scenario II we explicitly include the s- and the u-channel resonance exchange contributions, thereby assuming that the resonance was preformed already in the large N_c limit of QCD and only slightly affected by the coupled-channel dynamics. In a scheme based on scenario I, which however does not consider important channels required to generate a specific resonance, we suggest to include only the s-channel resonance contribution as a reminiscence of the neglected channels. An example is the d-wave $J^P = \frac{3}{2}^-$ baryon nonet resonance which we believe to be generated primarily by the vector–meson baryon channels.[43–45]

Our detailed analyses of the data set clearly favor scenario I. The inclusion of a u-channel baryon–nonet d-wave resonance exchange contributions appears to destroy the subtle balance of chiral s-wave range terms and makes it impossible to obtain a reasonable fit to the data set. Thus all results reviewed in this work are based on scenario I. In our present scheme we include contributions of s- and u-channel baryon octet and decuplet states explicitly but only the s-channel contributions of an explicit d-wave $J^P = \frac{3}{2}^-$ baryon nonet resonance states. Thereby we consider the s-channel baryon nonet contributions to the interaction kernel as a reminiscence of additional inelastic, channels not included in the present scheme like for example the $K\Delta_\mu$ or $K_\mu N$ channel. It is gratifying to find precursor effects for the s-wave $\Sigma(1750)$ and the p-wave $\Lambda(1600)$, $\Lambda(1890)$ and p-wave $\Sigma(1660)$ resonances once agreement with the low-energy data set with $p_{\rm lab} < 500$ MeV was achieved. A more accurate description of the latter resonances requires the extension of the χ-BS(3) approach including more inelastic channels. These finding strongly support the conjecture that all baryon resonances but the decuplet ground states are a consequence of coupled-channels dynamics.

3.2. Parameters

The set of parameters is well determined by the available scattering data and weak decay widths of the baryon octet states. We aimed at a uniform quality of the data description. In Tables I and II we present the parameter set to chiral order Q^2 of our best fit to the data collection. Note that part of the parameters are predetermined to a large extent and therefore fine-tuned only in a small interval.

A qualitative understanding of the typical strength in the various channels can be obtained already at leading chiral order Q. In particular, as

Table I. Leading Chiral Parameters which Contribute to Meson–Baryon Scattering at Order Q

f [MeV]	C_R	F_R	D_R
90.04	1.734	0.418	0.748

shown in Fig. 1, the $\Lambda(1405)$ resonance is formed as a result of the coupled-channel dynamics defined by the Weinberg–Tomozawa interaction vertices ($\mathscr{L}^{(4)}$ in (7)). There are four parameters relevant at that order f, C_R, F_R, and D_R. Their respective values as given in Table I are the result of our global fit to the data set including all parameters of the χ-BS(3) approach. At leading chiral order the parameter f determines the weak pion- and kaon-decay processes and at the same time the strength of the Weinberg–Tomozawa interaction vertices. At subleading order the Weinberg–Tomozawa terms and the weak-decay constants of the pseudo-scalar meson octet receive independent correction terms. The result $f \simeq 90$ MeV is sufficiently close to the empirical pion and kaon weak-decay constants $f_\pi \simeq 92.4$ MeV and $f_K \simeq 113.0$ MeV to expect that the chiral correction terms lead indeed to values rather close to the empirical decay constants. Our value for f is consistent with the estimate of Ref. 23 which leads to $f_\pi/f = 1.07 \pm 0.12$. The baryon octet and decuplet s- and u-channel exchange contributions to the interaction kernels are determined by the F_R, D_R and C_R parameters at leading order. Note that F_R and D_R predict the baryon octet weak-decay processes and C_R the strong decay widths of the baryon decuplet states also at this order.

A quantitative description of the data set requires the inclusion of higher order terms. Initially we tried to establish a consistent picture of the existing low-energy meson–baryon scattering data based on a truncation of the interaction kernels to chiral order Q^2. This attempt failed due to the insufficient quality of the kaon–nucleon scattering data, at low energies. In particular some of the inelastic K^--proton differential cross sections are strongly influenced by the d-wave $\Lambda(1520)$ resonance at energies where the

Table II. Chiral Q^2-Parameters Resulting from a Fit to Low-Energy Meson–Baryon Scattering Data. Further Parameters at This Order Are Determined by the Large N_c Sum Rules

$g_F^{(V)}$[GeV^{-2}]	0.293	$g_F^{(S)}$[GeV^{-1}]	−0.198	$g_F^{(T)}$[GeV^{-1}]	1.106	$Z_{[10]}$	0.719
$g_D^{(V)}$[GeV^{-2}]	1.240	$g_D^{(S)}$[GeV^{-1}]	−0.853	$g_D^{(T)}$[GeV^{-1}]	1.607	—	—

data points start to show smaller error bars. We conclude that, on the one hand, one must include an effective baryon–nonet resonance field and, on the other hand, perform minimally a chiral Q^3 analysis to extend the applicability domain to somewhat higher energies. Since the effect of the d-wave resonances is only necessary in the strangeness minus one sector, they are only considered in that channel.

At subleading order Q^2 the chiral $SU(3)$ Lagrangian predicts the relevance of 12 basically unknown parameters, $g^{(S)}$, $g^{(V)}$, $g^{(T)}$ and $Z_{[10]}$, which all need to be adjusted to the empirical scattering data. It is important to realize that chiral symmetry is largely predictive in the $SU(3)$ sector in the sense that it reduces the number of parameters beyond the static $SU(3)$ symmetry. For example one should compare the six tensors which result from decomposing $8 \otimes 8 = 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus \overline{10} \oplus 27$ into its irreducible components with the subset of SU(3) structures selected by chiral symmetry in a given partial wave. Thus static $SU(3)$ symmetry alone would predict 18 independent terms for the s-wave and two p-wave channels rather than the 11 chiral Q^2 background parameters, $g^{(S)}$, $g^{(V)}$ and $g^{(T)}$. In our work the number of parameters was further reduced significantly by insisting on the large N_c sum rules

$$g_1^{(S)} = 2g_0^{(S)} = 4g_D^{(S)}/3, \qquad g_1^{(V)} = 2g_0^{(V)} = 4g_D^{(V)}/3, \qquad g_1^{(T)} = 0$$

for the symmetry conserving quasi-local two body interaction terms (see (19)). In Table II we collect the values of all free parameters as they result from our best global fit. We point out that the large N_c sum rules derived in Sec. 2 implicitly assume that additional inelastic channels like $K\Delta_\mu$ or $K_\mu N$ are not too important. The effect of such channels can be absorbed to some extent into the quasi-local counter terms, however possibly at the price that their large N_c sum rules are violated. It is therefore a highly non-trivial result that we obtain a successful fit imposing (2). Note that the only previous analysis,[10] which truncated the interaction kernel to chiral order Q^2 but did not include p-waves, found values for the s-wave range parameters largely inconsistent with the large N_c sum rules. This may be due in part to the use of channel dependent cutoff parameters and the fact that that analysis missed octet and decuplet exchange contributions, which are important for the s-wave interaction kernel already to chiral order Q^2.

The parameters b_0, b_D and b_F to this order characterize the explicit chiral symmetry-breaking effects of QCD via the finite current quark masses. The parameters b_D and b_F are well estimated from the baryon octet mass splitting (see (22)) whereas b_0 must be extracted directly from the meson–baryon scattering data. It drives the size of the pion–nucleon sigma term for which conflicting values are still being discussed in the literature.[37] Our values

$$b_0 = -0.346\ \mathrm{GeV}^{-1}, \qquad b_D = 0.061\ \mathrm{GeV}^{-1}, \qquad b_F = -0.195\ \mathrm{GeV}^{-1} \tag{25}$$

are rather close to values expected from the baryon octet mass splitting (22). The pion–nucleon sigma terra $\sigma_{\pi N}$ if evaluated at leading chiral order Q^2 would be $\sigma_{\pi N} \simeq 32$ MeV. That value should not be compared directly with $\sigma_{\pi N}$ as extracted usually from pion–nucleon scattering data at the Cheng–Dashen point. The required subthreshold extrapolation involves further poorly convergent expansions.[37] Here we do not attempt to add anything new to this ongoing debate. We turn to the analogous symmetry-breaking parameters d_0 and d_D for the baryon decuplet states. Like for the baryon octet states we use the isospin averaged empirical values for the baryon masses in any u-channel exchange contribution. In the s-channel decuplet expressions we use the slightly different values $m_{10}^{(\Delta)} = 1223.2$ MeV and $m_{10}^{(\Sigma)} = 1374.4$ MeV to compensate for a small mass renormalization induced by the unitarization. Those values are rather consistent with $d_D \simeq -0.49\ \mathrm{GeV}^{-1}$ (see (22)). Moreover note that all values used are quite compatible with the large N_c sum rule

$$b_D + b_F = d_D/3$$

The parameter d_0 is not determined by our analysis. Its determination required the study of the meson baryon–decuplet scattering processes.

At chiral order Q^3 the number of parameters increases significantly unless additional constraints from QCD are imposed. Recall for example that Ref. 47 presents a large collection of already 102 chiral Q^3 interaction terms. A systematic expansion of the interaction kernel in powers of $1/N_c$ leads to a much reduced parameter set. For example we established 23 additional parameters describing the refined 3-point meson–baryon vertices, including in particular explicit $SU(3)$ symmetry-breaking effects. A detailed analysis of the 3-point vertices in the $1/N_c$ expansion of QCD reveals that in fact only eleven additional parameters, rather than the 23 parameters, are relevant at leading order in that expansion. Since the leading parameters F_R, D_R together with the symmetry-breaking parameters describe at the same time the weak decay widths of the baryon octet and decuplet ground states, the number of free parameters does not increase significantly at the Q^3 level if the large N_c limit is applied. Similarly the $1/N_c$ expansion leads to only four additional parameters describing the refined two-body interaction vertices. The values of all Q^3 parameters can be found in Ref. 14.

3.3. Pion–Nucleon Scattering

We begin with a detailed account of the strangeness zero sector. For a review of pion–nucleon scattering within the conventional meson exchange picture we refer to Ref. 49. The various chiral approaches will be discussed more explicitly below. Naively one may want to include the pion–nucleon threshold parameters in a global $SU(3)$ fit. In conventional chiral perturbation theory the latter are evaluated perturbatively to subleading orders in the chiral expansion.[4, 50] Here the small pion mass justifies the perturbative treatment. Unfortunately there is no unique set of threshold parameters available. This is due to the difficulty in extrapolating the empirical data set down to threshold, the subtle electromagnetic effects and also some inconsistencies in the data set itself.[37, 51] A collection of mutually contradicting threshold parameters is collected in Table III. In order to obtain an estimate of systematic errors in the various analyses we confront the threshold values with the chiral sum rules:

$$
\begin{aligned}
4\pi\left(1+\frac{m_\pi}{m_N}\right) a^{(\pi N)}_{[S_-]} &= \frac{m_\pi}{2f^2} + \mathcal{O}(Q^3) \\
4\pi\left(1+\frac{m_\pi}{m_N}\right) b^{(\pi N)}_{[S_-]} &= \frac{1}{4f^2 m_\pi} - \frac{2g_A^2+1}{4f^2 m_N} + \frac{C^2}{18f^2}\frac{m_\pi}{m_N(\mu_\Delta+m_\pi)} + \mathcal{O}(Q) \\
4\pi\left(1+\frac{m_\pi}{m_N}\right) (a^{(\pi N)}_{[P_{13}]} - a^{(\pi N)}_{[P_{31}]}) &= \frac{1}{4f^2 m_N} + \mathcal{O}(Q) \\
4\pi\left(1+\frac{m_\pi}{m_N}\right) a^{(\pi N)}_{\rm SF} &= -\frac{3g_A^2}{2f^2 m_\pi}\left(1+\frac{m_\pi}{m_N}\right) \\
&\quad -\frac{2}{3}\frac{C^2}{f^2}\frac{m_\pi m_N}{m_\Delta(\mu_\Delta^2-m_\pi^2)} + \mathcal{O}(Q)
\end{aligned}
\tag{26}
$$

where $\mu_\Delta = m_\Delta - m_N$. We confirm the result of Ref. 4 that the spin-flip scattering volume $a^{(\pi N)}_{\rm SF} = a^{(\pi N)}_{[P_{11}]} + 2a^{(\pi N)}_{[P_{31}]} - a^{(\pi N)}_{[P_{13}]} - 2a^{(\pi N)}_{[P_{33}]}$ and the combination $a^{(\pi N)}_{[P_{13}]} - a^{(\pi N)}_{[P_{31}]}$ in (26) are independent of the quasi-local 4-point interaction strengths at leading order. Confronting the analyses in Table III with the chiral sum rules quickly reveals that only the EM98 analysis[53] appears consistent with the sum rules within 20%. The analysis in Refs. 51 and 54 badly contradict the chiral sum rules (26), valid at leading chiral orders, and therefore would require unnaturally large correction terms possibly discrediting the convergence of the chiral expansion in the pion–nucleon sector. The KA86 analysis is consistent with the two p-wave sum rules but appears inconsistent with the s-wave range parameter $b_{[S_-]}$. The recent π^-

Table III. Pion–Nucleon Threshold Parameters

	χ-BS(3)	KA86 [52]	EM98 [53]	SP98 [54]	GMORW [51]
$a^{(\pi N)}_{[S_-]}$ [fm]	0.124	0.130	0.109 ± 0.001	0.125 ± 0.001	0.116 ± 0.004
$a^{(\pi N)}_{[S_+]}$ [fm]	-0.014	-0.012	0.006 ± 0.001	0.000 ± 0.001	0.005 ± 0.006
$b^{(\pi N)}_{[S_-]}$ [m_π^{-3}]	-0.007	0.008	0.016	0.001 ± 0.001	-0.009 ± 0.012
$b^{(\pi N)}_{[S_+]}$ [m_π^{-3}]	-0.028	-0.044	-0.045	-0.048 ± 0.001	-0.050 ± 0.016
$a^{(\pi N)}_{[P_{11}]}$ [m_π^{-3}]	-0.083	-0.078	-0.078 ± 0.003	-0.073 ± 0.004	-0.098 ± 0.005
$a^{(\pi N)}_{[P_{31}]}$ [m_π^{-3}]	-0.045	-0.044	-0.043 ± 0.002	-0.043 ± 0.002	-0.046 ± 0.004
$a^{(\pi N)}_{[P_{13}]}$ [m_π^{-3}]	-0.038	-0.030	-0.033 ± 0.003	-0.013 ± 0.004	-0.000 ± 0.004
$a^{(\pi N)}_{[P_{33}]}$ [m_π^{-3}]	0.198	0.214	0.214 ± 0.002	0.211 ± 0.002	0.203 ± 0.002

hydrogen atom experiment[55] gives rather precise values for the π^--proton scattering lengths

$$\begin{aligned} a_{\pi^- p \to \pi^- p} &= a^{(\pi N)}_{S_-} + a^{(\pi N)}_{S_+} = \quad 0.124 \pm 0.001 \text{ fm} \\ a_{\pi^- p \to \pi^0 n} &= -\sqrt{2}\, a^{(\pi N)}_{S_-} \quad = -0.180 \pm 0.008 \text{ fm} \end{aligned} \tag{27}$$

These values are in conflict with the s-wave scattering lengths of the EM98 analysis. For a comprehensive discussion of further constraints from the pion–deuteron scattering lengths as derived from recent pionic atom data we refer to Ref. 56. All together the emerging picture is complicated and inconclusive at present. Related arguments are presented by Fettes and Meißner in their work[50] which considers low-energy pion–nucleon phase shifts to chiral order Q^4. A resolution for this puzzle may be offered by the most recent work of Fettes and Meißner[57] where they consider electromagnetic correction terns within the χPT scheme to order Q^3.

We turn to a further important aspect to be discussed. Even though the EM98 analysis is rather consistent with the chiral sum rules (26) does it imply background terms of natural size? This can be addressed by considering a further combination of p-wave scattering volumes

$$\begin{aligned} 4\pi\left(1+\frac{m_\pi}{m_N}\right)\left(a^{(\pi N)}_{[P_{11}]} - 4a^{(\pi N)}_{[P_{31}]}\right) &= \frac{3}{2f^2 m_N} + B + \mathcal{O}(Q) \\ B = -\frac{5}{12 f^2}\left(2\tilde{g}^{(S)}_0 + \tilde{g}^{(S)}_D + \tilde{g}^{(S)}_F\right) &- \frac{1}{3f^2}\left(\tilde{g}^{(T)}_D + \tilde{g}^{(T)}_F\right) \end{aligned} \tag{28}$$

where we absorbed the Z-dependence into the tilde couplings. The naturalness assumption would estimate $B \sim 1/(f^2 m_\rho)$, a typical size which is compatible with the background term $B \simeq 0.92 m_\pi^{-3}$ of the EM98 solution.

In order to avoid the ambiguities of the threshold parameters we decided to include the single energy pion–nucleon phase shifts of Ref. 54 in our global fit. The phase shifts are evaluated in the χ-BS(3) approach including all channels suggested by the $SU(3)$ flavor symmetry. The single energy phase shifts are fitted up to $\sqrt{s} \simeq 1200$ MeV. In Fig. 2 we confront the result of our fit with the empirical phase shifts. All s- and p-wave phase shifts are well reproduced up to $\sqrt{s} \simeq 1300$ MeV with the exception of the S_{11} phase for which our result agrees with the partial-wave analysis only up to about $\sqrt{s} \simeq 1200$ MeV. We emphasize that one should not expect quantitative agreement for $\sqrt{s} > m_N + 2m_\pi \simeq 1215$ MeV where the inelastic pion production process, not included in this work, starts. The missing higher order range terms in the S_{11} please are expected to be induced by additional inelastic channels or the nucleon resonances $N(1520)$ and $N(1650)$. We confirm the findings of Refs. 10 and 59 that the coupled $SU(3)$ channels predict considerable strength in the S_{11} channel around $\sqrt{s} \simeq 1500$ MeV where the phase shift shows a resonance like structure. Note, however that it is expected that the nucleon resonances $N(1520)$ and $N(1650)$ couple strongly to each other[(60)] and therefore one should not expect a quantitative

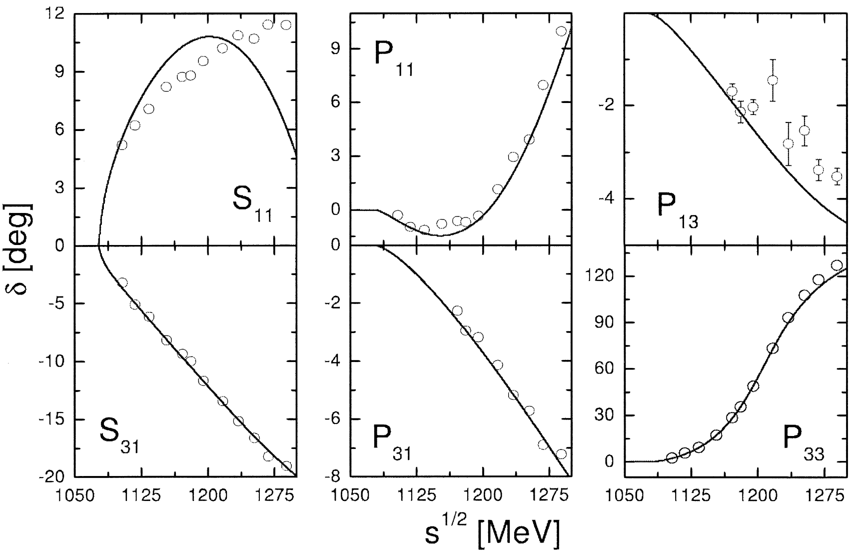

Fig. 2. s- and p-wave pion–nucleon phase shifts. The single energy phase shifts are taken from Ref. 58.

description of the S_{11} phase too far away from threshold. Similarly we observe considerable strength in the P_{11} channel leading to a resonance like structure around $\sqrt{s} \simeq 1500$. We interpret this phenomenon as a precursor effect of the p-wave $N(1440)$ resonance. We stress that our approach differs significantly from the recent work[59] in which the coupled $SU(3)$ channels are applied to pion induced η and kaon production which require much larger energies $\sqrt{s} \simeq m_\eta + m_N \simeq 1486$ MeV or $\sqrt{s} \simeq m_K + m_\Sigma \simeq$ 1695 MeV. We believe that such high energies can be accessed reliable only upon inclusion of additional inelastic channels. The discussion of the pion–nucleon sector is closed by returning to the threshold parameters. In Table III our extracted threshold parameters are presented in the second row. We conclude that all threshold parameters are within the range suggested by the various analyses.

3.4. K^+-Nucleon Scattering

We turn to the strangeness plus one channel. Since it is impossible to give here a comprehensive discussion of the many works dealing with kaon–nucleon scattering we refer to the review article by Dover and Walker[61] which is still up to date in many respects. To summarize the data situation: there exist precise low-energy differential cross sections for K^+p scattering but no scattering data for the K^+-deuteron scattering process at low energies. Thus all low-energy results in the isospin zero channel necessarily follow from model dependent extrapolations. In our global fit we include the available differential cross section.

At this place it is instructive to consider the threshold amplitudes in some detail. In the χ-BS(3) approach the s-wave scattering lengths are renormalized strongly by unitarization. At leading order the s-wave scattering lengths are

$$
\begin{aligned}
4\pi\left(1+\frac{m_K}{m_N}\right) a^{(KN)}_{S_{21}} &= -m_K\left(f^2+\frac{m_K^2}{8\pi}\left(1-\frac{1}{\pi}\ln\frac{m_K^2}{m_N^2}\right)\right)^{-1} \\
4\pi\left(1+\frac{m_K}{m_N}\right) a^{(KN)}_{S_{01}} &= 0
\end{aligned}
\tag{29}
$$

This leads to $a^{(KN)}_{S_{21}} \simeq -0.22$ fm and $a^{(KN)}_{S_{01}} = 0$ fm close to our final values at subleading orders as given in Table IV. In that table we collected typical results for the p-wave scattering volumes also. The large differences in the isospin zero channel reflect the fact that that channel is not constrained by scattering data directly.[61] We find that our p-wave scattering volumes,

Table IV. K^+-Nucleon Threshold Parameters. The Values of the χ-BS(3) Analysis Are Given in the First Row. The Last Two Rows Recall the Threshold Parameters as Given in Refs. 62 and 63

	$a_{S_{01}}^{(KN)}$ [fm]	$a_{S_{21}}^{(KN)}$ [fm]	$a_{P_{01}}^{(KN)}$ $[m_\pi^{-3}]$	$a_{P_{21}}^{(KN)}$ $[m_\pi^{-3}]$	$a_{P_{03}}^{(KN)}$ $[m_\pi^{-3}]$	$a_{P_{23}}^{(KN)}$ $[m_\pi^{-3}]$
χ-BS(3)	0.06	−0.30	0.033	−0.017	−0.003	0.012
[62]	0.0	−0.33	0.028	−0.056	−0.046	0.025
[63]	−0.04	−0.32	0.030	−0.011	−0.007	0.007

also shown in Table IV, differ in part significantly from the values given by previous analyses. Such discrepancies may be explained by important cancellation mechanisms among the *u*-channel baryon octet and decuplet contributions. An accurate description of the scattering volumes requires a precise input for the meson–baryon 3-point vertices. Since the χ-BS(3) approach describes the 3-point vertices in accordance with all chiral constraints and large N_c sum rule of QCD we believe our values for the scattering volumes to be quite reliable.

In Fig. 3 we confront our *s*- and *p*-wave K^+-nucleon phase shifts with the most recent analyses by Hyslop *et al.*[62] and Hashimoto.[64] We find

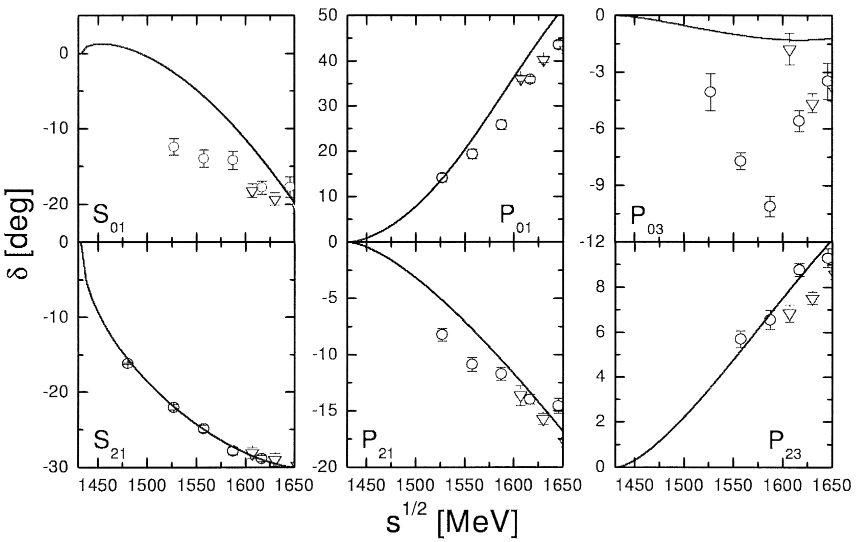

Fig. 3. *s*- and *p*-wave K^+-nucleon phase shifts. The solid lines represent the results of the χ-BS(3) approach. The open circles are from the Hyslop analysis[62] and the open triangles from the Hashimoto analysis.[64]

that our partial wave shifts are reasonably close to the single energy phase shifts of Refs. 62 and 64 except the P_{03} phase for which we obtain much smaller strength. Note however, that at higher energies we smoothly reach the single energy phase shifts of Hashimoto.[64] A possible ambiguity in that phase shift is already suggested by the conflicting scattering volumes found in that channel in earlier works (see Table IV). The isospin one channel, on the other hand, seems well established even though the data, set does not include polarization measurements close to threshold which are needed to unambiguously determine the p-wave scattering volumes.

3.5. K^--Nucleon Scattering

It is left to report on our results in the strangeness minus one sector. The antikaon–nucleon scattering process shows a large variety of intriguing phenomena. Inelastic channels are already open at threshold leading to a rich coupled-channel dynamics. Also the $\bar{K}N$ state couples to many of the observed hyperon resonances for which competing dynamical scenarios are conceivable. We fit directly to the available data set rather than to any partial wave analysis. Comparing for instance the energy dependent analyses in Refs. 65 and 66 one finds large uncertainties in the s- and p-waves in particular at low energies. This reflects on the one hand a model dependence of the analysis and on the other hand an insufficient data set. A partial wave analysis of elastic and inelastic antikaon–nucleon scattering data without further constraints from theory is inconclusive at present.[61, 67] For a detailed overview of the more ancient theoretical analyses we suggest the review article by Dover and Walker.[61]

In Fig. 4 we present the result of our fit for the elastic and inelastic K^-p cross sections. The data set is nicely reproduced including the rather precise data points for laboratory momenta $250\text{ MeV} < p_{\text{lab}} < 500\text{ MeV}$. In Fig. 4 the s-wave contribution to the total cross section is shown with a dashed line. Sizeable p-wave contributions are found at low energies only in the $\Lambda\pi^0$ production cross section. The $\Lambda\pi^0$ channel carries isospin one and therefore provides a valuable constraint on the poorly known K^--neutron interaction. Note that according to Ref. 71 the inelastic channel $K^-p \to \Lambda\pi\pi$, not included in this work, is no longer negligible at a quantitative level for $p_{\text{lab}} > 300$ MeV.

Important information on the p-wave dynamics is provided by angular distributions for the inelastic K^-p reactions. The available data are represented in terms of coefficients A_n characterizing the differential cross section

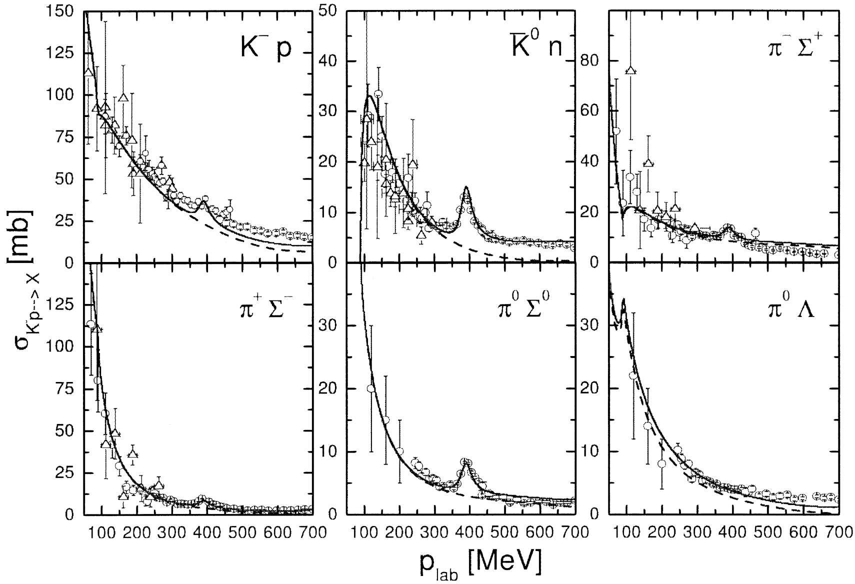

Fig. 4. K^--proton elastic and inelastic cross sections. The data are taken from Refs. 68–70. The solid lines show the results of our χ-BS(3) theory including all effects of s-, p- and d-waves. The dashed lines represent the s-wave contributions only. We fitted the data points given by open circles.[68, 69] Further data points represented by open triangles[70] were not considered in the global fit.

$d\sigma(\cos\theta, \sqrt{s})$ as a function of the center of mass scattering angle θ and the total energy $\sqrt{s}$

$$\frac{d\sigma(\sqrt{s}, \cos\theta)}{d\cos\theta} = \sum_{n=0}^{\infty} A_n(\sqrt{s})\, P_n(\cos\theta) \tag{30}$$

where $P_n(\cos\theta)$ are the Legendre polynomials. In Fig. 5 we compare the empirical ratios A_1/A_0 and A_2/A_0 with the results of the χ-BS(3) approach. Note that for $p_{\rm lab} < 300$ MeV the empirical ratios with $n \geqslant 3$ are compatible with zero within their given errors. A large A_1/A_0 ratio is found only in the $K^- p \to \pi^0 \Lambda$ channel demonstrating again the importance of p-wave effects in the isospin one channel. The dotted lines of Fig. 5 confirm the importance the $\Lambda(1520)$ resonance for the angular distributions in the isospin zero channel. The fact that this resonance appears more important in the differential cross sections than in the total cross sections follows simply because the tail of the resonance is enhanced if probed via an interference

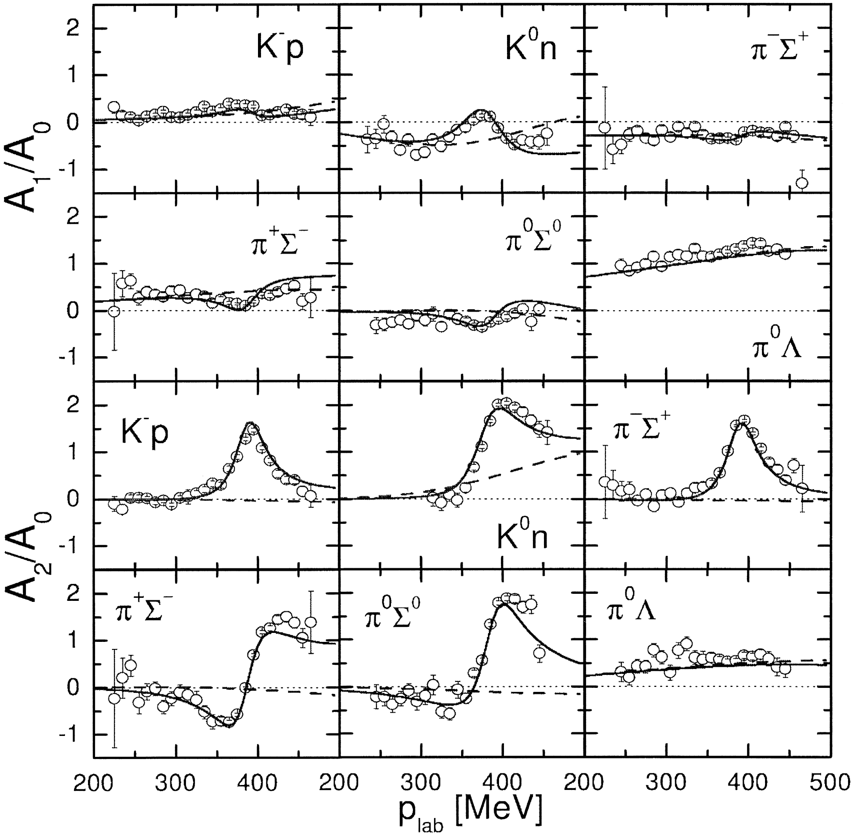

Fig. 5. Coefficients A_1 and A_2 for the $K^-p \to \pi^0\Lambda$, $K^-p \to \pi^\mp\Sigma^\pm$ and $K^-p \to \pi^0\Sigma^0$ differential cross sections. The data are taken from Ref. 68.

term. In the differential cross section the $\Lambda(1520)$ propagator enters linearly whereas the total cross section probes the squared propagator only.

4. SUMMARY

We summarize the main issues reported on in this review. The recently developed χ-BS(3) approach was presented in some detail demonstrating its successful application to meson–baryon scattering. In that approach the Bethe–Salpeter interaction kernel was evaluated to chiral order Q^3 where the number of parameters was reduced significantly by performing a systematic $1/N_c$ expansion of the interaction kernel. The scattering amplitudes, used to obtain the differential cross sections, are the solution of the covariant Bethe–Salpeter scattering equation. The first reliable estimates of previously poorly known s- and p-wave parameters of the chiral $SU(3)$ Lagrangian were obtained. It is a highly non-trivial and novel result that the strength of all quasi-local 2-body interaction terms are consistent with the expectation from the large N_c sum rules of QCD. Moreover, all parameters established prove the chiral $SU(3)$ flavor symmetry to be an extremely useful and accurate tool. Explicit symmetry breaking effects are quantitatively important but sufficiently small to permit an evaluation within our χ-BS(3) approach. This confirms a beautiful analysis by Hamilton and Oades[72] who strongly supported the $SU(3)$ flavor symmetry by a discrepancy analysis of kaon–nucleon scattering data. The χ-BS(3) approach establishes a unified description of pion–nucleon and kaon–nucleon scattering describing a large amount of empirical scattering data including the axial-vector coupling constants for the baryon octet ground states. An important test of the χ-BS(3) analysis could be provided by new data on kaon–nucleon scattering from the DAΦNE facility.[73]

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