Ast 4001, 14 Sept 2009

Homework set 2 -- Notes on astronomical photometry (magnitudes), and some practice exercises -- Don't turn in your answers, just be sure you understand them perfectly. --

A few www sites to read or skim

- Wikipedia "magnitude (astronomy)", "photometry (astronomy)", "UBV photometric system", etc. Google has trouble with this topic, because keywords such as "magnitudes" or "photometry" or "UBVR" may lead to a bunch of specialized research papers that merely have these words in their titles. But if you look around, you may find various course notes from other universities.
- `www.astrophysicsspectator.com/topics/observation/MagnitudesAndColors.html`
- `www.sizes.com/units/magnitude_stellar.htm` (historical notes)
- `aas.org/archives/BAAS/v33n4/aas199/738.htm` (trivial fussing)
- `www.badastronomy.com/bad/misc/badstarlight.html` (popular-level blurb)

Basic facts: If you remember the following points, you can reconstruct all the useful formulae for magnitudes without needing a textbook.

- Magnitude m is a logarithmic representation of physical brightness or radiation flux F. Therefore a magnitude difference $m_2 - m_1$ represents a brightness ratio F_1 / F_2.
- For historical reasons, a smaller magnitude value m corresponds to larger F. Thus $m \approx 1.0$ is a bright star while $m \approx 15$ is faint. (Negative values are OK; apart from the Sun, the brightest star in the sky has visual magnitude -1.4.)
- In order to simplify the calculations, a difference of exactly 5 magnitudes is defined to indicate a brightness factor of exactly 100. Consequently, if you think about it, you'll see the following examples which may be worth remembering:

  Δm = 2.5 magnitudes $\Rightarrow$ brightness factor of exactly $10\times$ (why?);
  Δm = 10 magnitudes $\Rightarrow$ exactly $10^4\times$;
  Δm = 1.0 magnitude $\Rightarrow$ approximately $2.512\times$;
  Δm = 2.0 magnitudes $\Rightarrow$ approximately $2.512^2 \approx 6.31\times$;
  Δm = 1.5 magnitude $\Rightarrow$ close to $4\times$ (calculate the exact number);
  Δm = 0.01 magnitude $\Rightarrow$ approximately $1.01\times$; etc.

  The last example suggests a rough approximation that's very useful for $\Delta m \le 0.3$.
- In order to be physically meaningful, a magnitude refers to radiation in some particular wavelength interval, which depends on the detector, filters, etc. Observers have developed many photometric systems, but the most common is the UBV or UBVR magnitude system; the U ("ultraviolet") interval is centered around $\lambda \approx 0.36$ nm, B ("blue") is near 0.44 nm, and V ("visual") is near 0.55 nm.

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- Astronomical magnitude systems are defined by sets of well-observed reference stars, not lab measurements of radiation fluxes. Absolute flux calibrations are difficult! Suppose, for instance, that a particular star is observed to have $m = 11.34 \pm 0.01$ in some magnitude system. This is 1% accuracy relative to other stars, but the corresponding absolute radiation flux in W per m^2 will usually be much less precise. For many purposes we avoid using absolute calibrations.
- An important exception to the two last points above is bolometric magnitude $m_{\rm bol}$. In principle this represents the total radiation energy flux, including all wavelengths. In practice, no instrument can directly measure $m_{\rm bol}$ for any astronomical object. (Can you think of several reasons why?) Therefore the bolometric magnitude of a star must be estimated from a combination of measurements and theoretical details. For the background to this sub-topic, google "bolometer" or see wikipedia.

Technical details -- some definitions:

- Let $f_\nu \; d\nu \; dA$ = radiative power passing through area dA, in frequency interval $d\nu$; the units of f_ν may be W $\mathrm{m}^{-2}\ \mathrm{Hz}^{-1}$, or erg $\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{Hz}^{-1}$, or whatnot. We often call f_ν "energy flux", but this term can also refer to the related quantities f_λ, f_ε, and F mentioned below.
- If λ = wavelength = c/ν and ε = photon energy = $h\nu$, then f_λ and f_ε are exactly analogous to f_ν. If $d\nu$ and $d\lambda$ are the frequency and wavelength intervals corresponding to photon-energy interval $d\varepsilon$, then $f_\nu \, d\nu = f_\lambda \, d\lambda = f_\varepsilon \, d\varepsilon$.
- Suppose F is the integrated energy flux in some finite, well-defined interval of wavelength or frequency. Then the associated "apparent astronomical magnitude" m is defined by

$$m = -2.5 \log_{10} (F/F_0) \qquad \text{or} \qquad F = 10^{-0.4\,m} \, F_0 ,$$

where F_0 is a more or less arbitrary constant that depends on the way this particular magnitude system was set up.*

* (Pedantic stuff:) Strictly speaking, F is really a weighted integral or weighted average, $F = \int Q(\lambda) \, f_\lambda \, d\lambda$ rather than simply $F = \int f_\lambda \, d\lambda$. The weighting function $Q(\lambda)$ is typically the product of some filter transmission curve, a detector efficiency, and other factors. Moreover, f_λ refers to the flux that a telescope would see *outside the Earth's atmosphere.* (Can you guess how observers make accurate corrections for variable atmospheric absorption?)

Practice problems: It's hard to become genuinely familiar with photometric systems unless you do a few exercises like these.

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1. (a) Work out a simple formula for f_λ in terms of f_ν, for example. (In other words, if f_ν is known, how would you calculate f_λ ?)

   (b) Then write a formula for the quantity λf_λ in terms of νf_ν. What units would you express either of these quantities in?

   (c) Show that λf_λ is the energy flux per *logarithmic* interval of λ (i.e., per $d \ln \lambda$).

2. According to reference books, the star HDE 327331 has apparent visual magnitude $m = 8.53$. However, observations with the Hubble Space Telescope show that it's really a very close double star. According to HST, component A is twice as bright as B at visual wavelengths. (This statement refers to their radiation fluxes F.) Deduce the two stars' individual magnitudes m_A and m_B.

3. A particular distant galaxy normally has apparent magnitude $m = +20.3$. A supernova temporarily causes the galaxy's total brightness to increase to $m = +20.0$. *Estimate the magnitude of the supernova itself,* if we could observe it separately.

4. We often use "Johnson visual" magnitudes, denoted m_V or simply V. For this type of apparent magnitude, as a rough approximation we can identify F in the magnitude equation with f_λ evaluated at wavelength $\lambda = 550$ nm $= 5500$ Å $= 0.55$ µm. *Use the Sun to roughly estimate the calibration constant F_0 for the Johnson-V magnitude system.* In order to do this, adopt the following assumptions, which are fairly realistic:

   - Apparent magnitude of the Sun: $m_V \approx -26.7$.
   - Solar constant $\approx$ 1370 Watts per square meter. That's the total solar energy flux outside Earth's atmosphere, including all wavelengths.
   - The Sun's spectral energy distribution is roughly like a Planckian "blackbody" with temperature $T \approx 5800$ K.

   Your estimated calibration parameter F_0 will be, approximately, f_λ(550 nm) for a star whose apparent magnitude is $m_V = 0$.

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