Equation sheet #1

Vectors:
\[ \vec{A} + \vec{B} = (A_x + B_x, A_y + B_y, A_z + B_z) \]
\[ \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \]
\[ \vec{A} \cdot \vec{B} = AB \cos \alpha \]
\( \alpha \) is angle between the vectors:
\[ \vec{A} \times \vec{B} = (A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x) \]
\[ |\vec{A} \times \vec{B}| = AB \sin \alpha \]
Can use right-hand rule for the direction of the cross product result.

Equations of motion (constant acceleration):
\[ v - v_0 = a(t - t_0) \]
\[ x - x_0 = \frac{a}{2} (t - t_0)^2 + v_0(t - t_0) \]
\[ v^2 - v_0^2 = 2a(x - x_0) \]
\[ a = \frac{F_{\text{net}}}{m} \]

Taylor expansion:
\[ f(x) = \sum_{n=0}^{\infty} \left( \frac{d^n f}{d x^n} \right)_{x=x_0} \frac{(x-x_0)^n}{n!} \]

Complex numbers:
\[ c = a + i b = r e^{i \theta} \]
\[ a = r \cos \theta \]
\[ b = r \sin \theta \]
\[ r = \sqrt{a^2 + b^2} \]
\[ \tan \theta = \frac{b}{a} \]
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Equation sheet #2

From Chapter 4:

\[ \nabla \times \vec{F} = 0 \quad \text{(force is then conservative)} \]

\[ \nabla \times \vec{F} = \left( \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \]

\[ \text{Work} = \int \vec{F} \cdot d\vec{r} \]

From Chapter 5:

Relation between inertial and non-inertial reference frame:

\[ \ddot{\vec{a}} = \ddot{\vec{a}}^i + \vec{\dot{\omega}} \times (\dot{\vec{\omega}} \times \vec{r}^i) + 2(\dot{\omega} \times \vec{v}^i) + \frac{d\dot{\omega}}{dt} \times \vec{r}^i \]

Relation between force and acceleration in non-inertial reference frame:

\[ \vec{F}_{\text{physical}} - m\vec{\dot{a}}^i - m(\dot{\vec{\omega}} \times (\dot{\vec{\omega}} \times \vec{r}^i)) - 2m(\dot{\omega} \times \vec{v}^i) - m\left( \frac{d\dot{\omega}}{dt} \right) \times \vec{r}^i = ma^i \]

Earth’s radius = 6.4 x 10^3 km (assume the same everywhere on earth).

Earth’s angular speed = \( \omega = 7.3 \times 10^{-5} \) rads/s.
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Problems

1) Show that the following forces are conservative:

   a) \( \vec{F} = c(\vec{\omega} \times (\vec{\omega} \times \vec{r})) \)
   
   b) \( \vec{F} = d(\vec{\omega} \times \vec{v}) \)

   where \( c \) and \( d \) are constants

   Hints: For the force in a) evaluate the curl of the force. Do only the x-component
   of the curl and we will assume by symmetry the same value for the other
   components. For the force in b) show that the work done by the force along any
   path must be zero. It will be helpful to recall that \( \vec{v}dt = d\vec{r} \).

2) A wheel of radius \( b \) is rounding a curve as shown in the figure. Calculate the
   acceleration relative to the ground of the point at the very bottom of the wheel
   (opposite point \( P \)). The primed coordinate system has an origin fixed to the wheel’s
   center. The \( z' \) axis remains vertical as the wheel travels in a circle about center \( C \)
   (with speed \( V_0 \)).

   ![Diagram](image)

3) You are first-mate on the pirate ship Revenge. After plundering islands along the
   equator you are carrying approximately a metric tonne of gold (10^3 kg). While your
   ship is heading west at 20 knots (10m/s), the captain uses a scale to weigh the gold.
   Several days later your ship is heading east at the same speed. The paranoid captain
   reweighs the gold. Now she has a knife at your neck asking where you hid the gold
   that you stole. Explain why the Captian’s two measurements differed. Calculate the
   difference she saw.