If you spin a pail of liquid about its vertical axis, viscous forces will eventually cause the liquid to reach an equilibrium state where all the liquid rotates as a rigid body (all the bits of liquid rotate with the same angular velocity). There is a force normal to the surface, $F'$, where the surface of the liquid is in contact with air, due to the pressure gradient.

Derive a formula for the shape \( z(r) \) of the liquid's surface when it is in this equilibrium state.

The equation of motion for a noninertial reference frame is

$$\ddot{F} - m \ddot{\mathbf{A}}_0 - 2m \dot{\mathbf{\omega}} \times \mathbf{v}' - m \dot{\mathbf{\omega}} \times \mathbf{r}' - m \mathbf{\omega} \times (\dot{\mathbf{\omega}} \times \mathbf{r}') = m \ddot{\mathbf{a}}'$$

where the primed terms are in the noninertial frame, and \( \mathbf{A}_0 \) is the translational acceleration of the noninertial frame.