In the early Revolutionary War, General Washington gained a decisive tactical advantage in the 11-month Siege of Boston by positioning cannons on Dorchester Heights, overlooking Boston Harbor and out of range from the British fleet.

To use the cannons effectively, it is important to know how the horizontal range, \( R \), depends on the (variable) angle at which the cannon is fired, \( \theta \), the (fixed) vertical distance from the Heights to the Harbor, \( d \), and the (fixed) speed with which the ball is fired, \( v_0 \).

a) Find a relationship relating gravity, \( R \), \( \theta \), \( d \), and \( v_0 \). Ignore the effect of air resistance.

b) Find the maximum range, \( R_m \), as a function of only \( d \) and the angle at maximum range, \( \theta_m \). Simplify your answer using \( \sin(2\theta) = 2\sin\theta\cos\theta \), and \( \cos^2 \theta = \frac{1}{2} (1 + \cos(2\theta)) \).

c) Find the angle for maximum range, \( \theta_m \), in terms of the direct angle below horizontal, \( \Phi \), (easily observed with a protractor) from the Heights to the furthest reachable point in the Harbor.

You might find the following trigonometric relation useful:

\[
\tan \left( \theta_1 + \theta_2 \right) = \frac{\tan \theta_1 + \tan \theta_2}{1 - \tan \theta_1 \tan \theta_2}
\]

The movement to Dorchester Heights took place during the winter, when the weather could severely hamper either side. The colonist's visibility depended on the straight-line distance from the Heights to the Harbor, rather than \( R \) or \( d \).

d) Find \( R_m \) in terms of \( d \) and \( R_0 \equiv v_0^2/g \), and use this to determine the straight line distance as a function of just the known cannon variable, \( R_0 \), and the relatively easily measured height, \( d \).