The gravitational attraction from the moon generates a slightly different force on each side of the earth, creating tides on the earth's oceans. In what follows, let \( r \) be the radius of the earth, \( R \) be the distance from the center of the earth to the center of the moon, and neglect effects due to the rotation of the earth about its axis and about the earth-moon center of mass.

I) Find the terms up to order \( r/R^3 \) for the total force on a bit of matter at a latitude \( \lambda \) and longitude \( \phi \) when the moon is over Greenwich. Use the center of the earth as the origin of your (noninertial) coordinate system.

II) Since gravitational forces are conservative, this force can be written as the negative gradient of a potential. Find this potential.

III) Consider a simple model of the earth where the earth is entirely covered by oceans. Find the height of the tide as a function of \( \lambda \) and \( \phi \). Hint: How will the potential vary across the surface of the oceans from point to point?

IV) Given that the moon has about 1/81.5 times the mass of the earth, the earth-moon distance is about \( 3.8 \times 10^8 \) km, and that the radius of the earth is 6,371 km, find the heights for the high and low tide at our latitude \( (\lambda = 45^\circ) \).

While we normally think of the moon as producing the tides, the sun also generate different forces on each side of the earth.

V) How large are the tides produced by the sun relative to the tides produced by the moon? The earth-sun distance is \( 1.5 \times 10^8 \) km, and the sun is about \((1/3) \times 10^6\) larger than the mass of the earth.

When the sun and the moon line up (new moon and full moon), the tide is particularly large and is called a spring tide (the term “spring” comes from the verb for “to jump” or “to leap up” rather than from the season). When the sun and moon tides partially cancel (first quarter, last quarter), the less extreme tides that result are called neap tides. The tides also vary with the earth-moon distance; when the moon is new or full and at perigee (nearest approach), the tides have their largest magnitudes and are called perigean tides.