1 Week 8 - A Toy Hurricane

A principal feature of tropical storms is the presence of a central low pressure zone surrounded by a zone of higher pressure. Assume the pressure is given by:

\[ P(s) = P_0 \left(1 - e^{-s/\lambda}\right) + P_{\text{min}}e^{-s/\lambda}, \]  

where \( s \) is the distance from the center of the storm and \( \lambda \) is a length characterizing the size of the storm.

a) Plot this pressure. What should we take for the value of \( P_0 \)?

Generally speaking, air masses like to move from high to low pressure. For our toy model, we’ll relate this to an effective potential by assuming the following:

\[ V_P = \frac{3}{2n} P, \]  

where \( n \) is a constant.

b) What are the units of \( n \)? Calculate the force resulting from this potential, \( F_P \), and make a qualitative plot.

Recall that Newton’s 2nd Law transforms to a non-inertial frame as:

\[ \vec{F}_{\text{R,eff}} = \vec{F} - m\vec{A}_0 - 2m\vec{\omega} \times \vec{v}_R - m\vec{\omega} \times \vec{r}_R - m\vec{\omega} \times (\vec{\omega} \times \vec{r}_R), \]  

where the subscript \( R \) denotes quantities defined in the non-inertial frame.

c) For a particle moving north to south parallel to the surface of the Earth, which way does the effective force point? Which way do tropical storms rotate in each hemisphere? What is the magnitude of this force at a given latitude? (Hint: Now is a good time to think about the various coordinate systems involved, if you haven’t already. Ignore the centripetal term).

d) The winds will move roughly in a circle around the eye. Evaluate Eq. (3) in a frame co-rotating with the storm. What is the wind velocity at a given distance from the center? Make a rough plot. Where are the winds strongest? What is the wind speed near the center?

e) Hurricane Katrina featured a lowest pressure of about 0.89 atmospheres. Calculate the maximum wind speed and compare to the actual value of 175 mph (78 m/s). You may take \( mn = 1 \text{ kg/m}^3 \) as a reasonable density for warm, humid air, and \( \lambda = 25 \text{ mi} = 40 \text{ km} \). The latitude of New Orleans is about 30 deg N.