I. TRITIUM IONS IN A PLASMA-FILLED CYLINDER

A device confines a hot gas of positively charged ions, called plasma, in a very long cylinder with a radius $R = 2.0$ cm. The charge density of the plasma in the cylinder is $\rho = 6.0 \times 10^{-5}$ C/m$^3$. Positively charged tritium ions are to be injected into the plasma perpendicular to the axis of the cylinder in a direction toward the center of the cylinder. Find the speed that a tritium ion should have when it enters the plasma cylinder so that its velocity is zero when it reaches the axis of the cylinder. Tritium is an isotope of Hydrogen with one proton and two neutrons. Note that the charge of a proton and the mass of tritium are $1.6 \times 10^{-19}$ C and $5.0 \times 10^{-27}$ kg.

**Solution.** We use Gauss’s Law to find the electric field inside the cylinder, noting that the electric field is parallel to the top and bottom of our cylindrical Gaussian surface $S$ of radius $r < R$ and arbitrary length $l$, so they contribute no flux.

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

Symmetry implies that $E$ is constant around the cylinder at $r$, so it can be pulled out of the integral, which can then be evaluated and combined with $Q_{enc} = \rho V = \rho \pi r^2 l$ to yield

$$E \cdot 2\pi rl = \frac{\rho \pi r^2 l}{\epsilon_0} \quad \Rightarrow \quad E = \frac{\rho r}{2\epsilon_0}$$

The strategy will now be to equate the kinetic energy lost by the tritium ion to its increase in electrical potential energy. We can write $U = qV$ where

$$V = -\int_0^r \vec{E} \cdot d\vec{r} = -\frac{\rho r^2}{4\epsilon_0}$$

Conservation of energy now tells us that

$$0 = \Delta T + \Delta U = -\frac{mv_0^2}{2} + \frac{q\rho R^2}{4\epsilon_0} \quad \Rightarrow \quad v_0 = \left(\frac{q\rho R^2}{2m\epsilon_0}\right)^{1/2} = 456 \text{ m/s}$$
II. ELECTRIC FLUX THROUGH A CUBE

A given region has an electric field that is the sum of two parts: a field due to a point charge \( q = 5 \times 10^{-8} \) C at the origin plus a uniform electric field of magnitude 3000 N/C directed in the \(-x\) direction. Calculate the flux through each face of a cube with sides of length 20 cm and centered on the origin. The faces are parallel to the \( x, y, z \) axes.

Solution. The uniform field is parallel to the unit normal vector of the surface at \( x = -10 \) cm, antiparallel to the one at \( x = +10 \) cm, and perpendicular to all the rest, so the flux due to the uniform field is just \( \Phi_{\text{unif}} = 3000 \text{ N/C} \times (0.2 \text{ m})^2 = 120 \text{ Nm}^2/\text{C} \) on one face and similarly -120 Nm\(^2/\text{C}\) on the other. We now turn to the flux from the point charge.

**Method 1.** The electric field from the point charge is

\[
E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2} = \frac{1}{4\pi \epsilon_0} \left( \frac{q}{x^2 + y^2 + z^2} \right)
\]

Consider the surface at \( z = 10 \) cm. The flux through this surface will be

\[
\Phi_{\text{chg}} = \int_S (\hat{n} \cdot \vec{E})dA = \frac{q}{4\pi \epsilon_0} \int_{-10}^{10} \int_{-10}^{10} \cos \theta \frac{z}{x^2 + y^2 + z^2} dxdy
\]

Where \( \theta \) is the angle from the \( z \) axis to point of integration. To do this integral we need to write \( \cos \theta = z/r \) to find

\[
\Phi_S = \frac{q}{4\pi \epsilon_0} \int_{-10}^{10} \int_{-10}^{10} \frac{z}{(x^2 + y^2 + z^2)^{3/2}} dxdy = \frac{q}{4\pi \epsilon_0} \times 2.0944
\]

Due to symmetry, all six sides contribute the same flux. Notice that \( 2.0944 \times 6 = 4\pi \). So the total flux from the charge is \( q/(4\pi \epsilon_0) \times 4\pi = q/\epsilon_0 \). This suggests another method we could have used to simplify the problem.

**Method 2.** Use Gauss’s Law, considering the cube as a closed surface that encloses the charge \( q \). Then Gauss’s Law says that the flux of the charge will be

\[
\oint_{\text{cube}} \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}
\]

This immediately gives us the same answer for the flux of the charge through the cube that we found using brute force in Method 1, with one sixth of the flux passing through each face of the cube. The positive and negative contributions from the uniform field cancel when summed and the final answer is

\[
\Phi_{\text{total}} = \Phi_{\text{unif}} + \Phi_{\text{chg}} = (0 + 5647) \text{ Nm}^2/\text{C} = 5647 \text{ Nm}^2/\text{C}
\]