S08 / 1302.500 Final Exam

Instructions

Write your name and student ID clearly on all sheets and fill your name and student ID on the bubble sheet.

Solve all multiple choice questions. No penalty is given for wrong answers.

Solve each Problem on a different sheet of paper.

Turn in all sheets of the exam including the MC questions.

Solutions to the problems should begin from basic physical principles, unless otherwise noted.

Show all steps in the derivation of the answers. Make sure you write neatly and orderly. It is YOUR RESPONSIBILITY to make sure that the grader understands your solution. The graders can not give partial or full credit if they can not follow the solution, even if the final answer is correct.

You can use a calculator (as long as it has no electronic communication capabilities) and an 8.5X11 crib sheet.

Point distribution:

5 points for each multiple choice question
20 points for each text problem

Good Luck

Potentially useful information:

\[
\mu_0 = 4\pi \times 10^{-7} \ TmA^{-1} \quad \frac{1}{4\pi \varepsilon_0} = 9.0 \times 10^9 \ \text{kgm}^3\text{s}^{-2}\text{C}^{-2} \quad c = 3.0 \times 10^8 \ \text{ms}^{-1} \quad G = 6.7 \times 10^{-11} \ \text{m}^3\text{kg}^{-1}\text{s}^{-2} \\
q_e = 1.6 \times 10^{-19} C \quad m_e = 9.1 \times 10^{-31} \text{kg} \quad g = 9.8 \text{ms}^{-2}
\]
Multiple Choice Questions

1. The electric permittivity of paper is 3.7 times larger than the electric permittivity of vacuum. What is the capacitance of a parallel plate capacitor with plates of dimensions 2.0 cm by 3.0 cm, separated by a sheet of paper 1.0 mm in thickness.

(a) 5 pF  
(b) 10 pF  
(c) 15 pF  
(D) 20 pF  
(c) 25 pF

2. A defibrillator uses an RC circuit to store energy and then discharges this energy through a patient to restore normal heart rhythm. A certain model of defibrillator uses a 32.0 μF capacitor and a 47.0 kilo-ohm resistor, resulting in an RC time constant of 1.50 s. If the voltage charging the capacitor is 5000 V, what is the maximum energy stored in the capacitor?

(a) 50 J  
(b) 100 J  
(c) 200 J  
(d) 300 J  
(C) 400 J

3. Which of the following statements about Kirchoff’s Laws are true?

✓ I. The algebraic sum of the currents at a circuit node must be greater than zero.  
✓ II. The algebraic sum of voltages around a loop must be zero.  
✓ III. Kirchoff’s Laws are consequences of the conservation of charge and the conservation of energy.

(a) Statement I only  
(b) Statement II only  
(c) Statement III only  
(d) Statements I and II  
(C) Statements II and III

4. A rectangular coil of dimensions 5.40 cm by 8.50 cm consists of 25 turns of wire and carries a current of 15.0 mA. If the coil is placed in a magnetic field of magnitude 0.350 T, what is the maximum possible torque that will be experienced by the coil?

(a) 6.02 x 10^-4 N-m  
(b) 7.57 x 10^-4 N-m  
(c) 8.23 x 10^-4 N-m  
(d) 9.09 x 10^-4 N-m  
(e) 9.91 x 10^-4 N-m
5. Consider a solenoid that is very long compared to its radius. Which of the following will double the magnitude of the magnetic field in the solenoid without changing the current.

- (a) Double its length, keeping the number of turns per unit length constant.
- (b) Double its length without changing the number of turns.
- (c) Reduce the radius of the solenoid by a factor of 2.
- (d) Reduce the cross-sectional area of the solenoid by a factor of 2.
- (e) Wrap the entire solenoid with a second layer of wire having the same number of turns as the first layer.

6. What is the wavelength of a 40.0 MHz electromagnetic wave? [The speed of light is $3.00 \times 10^8$ m/s.]

(a) 1.25 m  
(b) 2.50 m  
(c) 5.00 m  
(d) 7.50 m  
(e) 9.00 m

7. A circuit consists of a sinusoidal power source connected in series to an air core inductor and a light bulb. A short time after the power source is turned on, the current in the circuit stabilizes and the light bulb glows steadily. What happens if an iron core is inserted into the solenoid?

(a) The brightness of the light bulb increases because the iron core increases the inductance of the solenoid.
- (b) The brightness of the light bulb decreases because the iron core increases the inductance of the solenoid.
(c) The brightness of the light bulb decreases over time because of the time constant associated with an RL circuit.
(d) The brightness of the light bulb does not change because the iron core affects only the solenoid.
(e) The brightness of the light bulb oscillates with time because the power source is close to the natural frequency of the circuit.

8. An 8.0 $\mu$F capacitor is connected to a 60 Hz AC source whose rms voltage is 150 V. What is the rms current in this circuit?

(a) 0.452 A  
(b) 0.569 A  
(c) 0.678 A  
(d) 0.719 A  
(e) 0.845 A
9. The orbital period of the Earth is $3.156 \times 10^7$ s. The distance from the Earth to the Sun is $1.496 \times 10^{11}$ m. The Universal Gravitational Constant $G$ is $6.674 \times 10^{-11}$ N·m$^2$/kg$^2$. What is the mass of the Sun?

(a) $1.99 \times 10^{30}$ kg
(b) $2.72 \times 10^{30}$ kg
(c) $3.79 \times 10^{30}$ kg
(d) $5.00 \times 10^{30}$ kg
(e) $7.14 \times 10^{30}$ kg

10. What amount of kinetic energy is required for a 5,000 kg rocket to completely escape the gravitational attraction of the Earth? [The mass of the Earth is $5.98 \times 10^{24}$ kg.]

(a) $0.875 \times 10^{11}$ J
(b) $1.25 \times 10^{11}$ J
(c) $1.98 \times 10^{11}$ J
(d) $2.47 \times 10^{11}$ J
(e) $3.14 \times 10^{11}$ J

\[
U = 0 = \frac{1}{2} mv^2 - \frac{GMm}{R^2}
\]

\[
\frac{GMm^2}{R^2} = \frac{1}{2} mv^2
\]

\[
v = m \sqrt{\frac{GM}{R}}
\]

\[
R = \frac{GM}{q}
\]

\[
E = 5000 \sqrt{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 9.8}
\]
Problem 1

A resistor with $R = 1 \, \Omega$ is connected to a battery with an e.m.f of 12 V. In addition an ammeter (A) with internal resistance $R_A = 0.1 \, \Omega$ and a voltmeter (V) with internal resistance $R_V = 10 \, \Omega$ are present in the circuit as shown in the figure. The circle labeled with A represents the ammeter, while the circle labeled with V symbolizes the voltmeter. The resistor within each circle represents the internal resistance of the instrument.

a) Determine the current $I_A$ measured by the ammeter and the voltage difference $\Delta V_V$ measured by the voltmeter.

b) Explain briefly why the ratio $\Delta V_V / I_A$ is not exactly equal to the resistance $R$ (one sentence is sufficient).

c) Determine the power dissipated by each resistor

d) Compute the power provided by the battery

\[ R_{tot} = R_{eq} + R_A \]
\[ \frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R_V} \implies R_{eq} = \frac{R_{V} + R}{R_{V}R} \]

\[ I_A = \frac{V_B}{R_{tot}} = 11.9 \, A \]

\[ \Delta V_V = I_A R = I_V R_V + I = I R + I_V \]

\[ I_V = \frac{I R}{R + R_V} \]

\[ \Delta V_V = I \frac{R R_V}{R + R_V} = 11.9A \times \frac{1.2 \times 10 \Omega}{11 \Omega} = 10.4 \, V \]

b) Internal resistance contribution $\Delta V_V / I_A = 0.91 \, A$

c) $I_A R_A = 14 \, W$
\[ I_V R_V = I_A^2 \left( \frac{R}{R + R_V} \right)^2 R_V = I_A^2 \left( \frac{1.2 \Omega}{11 \Omega} \right)^2 10 \Omega = 12 \, W \]
\[ I_V^2 R_V = I_A^2 R_A^2 \]
\[ = (1.08A)^2 (1.2 \Omega)^2 = 117 \, W \]

\[ I_R^2 R = I_V^2 R_V \]
\[ = 117 \frac{12}{14.3} = 8.8 \, W \]

d) $I_A V_B = 148 \, W$

\[ I_V^2 R_V + I_R^2 R + I_A^2 R_A = I^2 \left( \frac{R R_V + R_A}{R_{tot}} \right) \]

\[ = I \cdot I R_{tot} = IV \]
Problem 2

A positive point charge \( +Q \) is located at \( x = -a \).

a) How much work is required to bring a second equal positive point charge \( +Q \) from infinity to \( x = +a \)?

b) With the two equal positive point charges at \( x = -a \) and \( x = +a \), how much work is required to bring a third charge \( -Q \) from infinity to the origin?

c) How much work is required to move the charge \( -Q \) from the origin to the point \( x = 2a \) along the semicircular path shown in the Figure?

\[
\begin{align*}
\text{a) } W &= \text{final electric potential} \times \text{charge} = PE \cdot U \\
U_a &= k \frac{Q^2}{2a} \\
\text{b) } \text{PE additive} &\quad U_b = U_a + k \frac{Q(-Q)}{a} + k \frac{Q(-Q)}{a} \\
U_b - U_a &= -k \frac{Q^2}{2a} \cdot 2 \\
\text{c) } U_c - U_a &= k \frac{Q(-Q)}{a} + k \frac{Q(-Q)}{3a} = -k \frac{Q^2}{2a} \cdot \frac{4}{3} \\
U_c - U_b &= -k \frac{Q^2}{2a} \left( \frac{4}{3} - \frac{1}{3} \right) = +k \frac{Q^2}{a} \cdot \frac{2}{3}
\end{align*}
\]
Problem 3

A square loop of side \( L = 1.4 \text{ cm} \), carrying the current \( I = 20 \text{ mA} \) is half immersed in a uniform magnetic field \( B = 0.25 \text{ T} \) (the magnetic field is everywhere below the dashed line, and it goes inside the page). You may assume that the mass of the current loop is negligible. The loop is kept at rest by a spring, which stretches 0.8 cm. Compute the elastic constant of the spring. What is the total force acting on the loop if you remove the spring and immerse the entire loop in the magnetic field?

\[
F_M = ILE = kx \quad L = 1.4 \text{ cm}
\]

\[
k = \frac{ILE}{x} = \frac{20 \times 10^{-3} \text{ A} \times 1.4 \times 10^{-2} \text{ m} \times 0.25 \text{ T}}{0.8 \times 10^{-2} \text{ m}} = 8.75 \times 10^{-2} \text{ N/m}
\]

\[
F_{\text{tot}} = 0
\]
Problem 4

A conducting bar of mass \( m \) and length \( L \) falls vertically, sliding down two wires without friction and maintaining contact with them. The only resistance has a value \( R \) as shown.

a) Find the terminal velocity with which the bar falls after its initial acceleration.
b) Find the power dissipated in the resistor as a function of time.
c) Find the work done by gravity as a function of time.

\[ F_m = ILB \quad F_G = mg \]

\[ IR = \frac{d\phi}{dt} = \frac{d}{dt} BLv t = BLv \quad \Rightarrow \quad I = \frac{BLv}{R} \]

\[ F_m = F_b \quad \text{v - const}^+ \]

\[ \left( \frac{BLv}{R} \right) LB = mg \quad \Rightarrow \quad v = \frac{mgR}{(BL)^2} \]

b) \[ P_R = I^2R = \left( \frac{BLv}{R} \right)^2 R = \left( \frac{BL}{R} \frac{mg}{BL} \right)^2 R \]

\[ = \frac{(mg)^2R}{(BL)^2} \]

c) \[ W_G = mgvt = mg \cdot \frac{mgR}{(BL)^2} t = \frac{(mg)^2R}{(BL)^2} t = P_R t \]
Problem 5

The circuit shown has a switch that has been closed for some time and is suddenly opened at $t = 0$.

a) Find the steady current in each of the circuit elements before the switch is opened.

b) Find the current as a function of time through the inductor after the switch is opened.

c) Find the current through the resistor $R_2$ after the switch is opened.

d) Find the energy in the inductor as a function of time after the switch is opened and also the power dissipated in the resistor.

\[ I_{1R_1} = V_0 \quad \Rightarrow \quad I_1 = \frac{V_0}{R_1} \quad I_2 = 0 \quad I_L = I_1 \]

\[ I = I_1 e^{-t/(L/R)} \]

\[ I_{LR_2} = I = I_1 e^{-t/(L/R_2)} \]

\[ U = \frac{1}{2} LI_2^2 = \frac{1}{2} L I_1^2 e^{-2t/(L/R_2)} \]

\[ P_R = I_2^2 R_2 = R_2 I_1^2 e^{-2t/(L/R_2)} \]

\[ \text{Note:} \quad P_R = -\frac{dU}{dt} \]