Instructions

• Write the names and student ids of all members of the group on the first answer sheet. You should show your work carefully – most of the points depend on your problem solving process.

• Solutions to the problems should begin from the following basic physical principles:
  – If $\vec{x}(t)$ is the position of the object as a function of time then velocity is $\vec{v}(t) = \frac{d\vec{x}}{dt}$ and acceleration is $\vec{a}(t) = \frac{d^2\vec{x}}{dt^2}$.
  – When the acceleration is a constant $\vec{a}$ then $\vec{x}(t) = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$.
  – Newton’s Laws: $\vec{F} = m \vec{a}$ and $\vec{F}_{12} = -\vec{F}_{21}$. Momentum: $\vec{p} = m \vec{v}$.
  – Common forces include static friction ($F \leq \mu_s F_N$), kinetic friction ($F = \mu_k F_N$), gravitational force ($F = mg$), drag ($F = \frac{1}{2} \rho AC_D v^2$) and the spring force ($F = -kx$).
  – Kinetic energy is $\frac{1}{2} m v^2$, work is $W = \int \vec{F} \cdot d\vec{x}$, gravitational potential energy is $U_g = mgh$, and spring potential energy is $U_s = \frac{1}{2} kx^2$.
  – Rotational physics: $K = \frac{1}{2} I \omega^2$, $\tau = I \alpha = Fr \sin \theta_F$, $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$, $L = I \omega$.
  – Moments of inertia: $I = \sum_i m_i R_i^2$. $MR^2$ – hollow cylinder, $\frac{1}{2} MR^2$ – solid cylinder, $\frac{3}{2} MR^2$ – solid sphere, $\frac{2}{3} MR^2$ – hollow sphere, $\frac{1}{12} ML^2$ – thin rod.

• Show all steps in the derivation of the answers. Make sure you write neatly and orderly. It is YOUR RESPONSIBILITY to make sure that the grader understands your solution. S/he will not give full points if they can not follow the solution, even if the final answer is correct.

• The acceleration due to gravity on Earth is 9.8 m/s$^2$.

• The solutions to the quadratic equation $0 = ax^2 + bx + c$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

• You can use a calculator.

• You can discard this page – it does not need to be handed in.

Problem on back side of sheet
Problem (25 points)

This year you have a summer job working for the National Park Service. Since they know that you have taken physics, they start you off in the laboratory which tests possible new equipment. Your first job is to test a small cannon. During the winter, small cannons are used to prevent avalanches in populated areas by shooting down heavy snow concentrations overhanging the sides of mountains. In order to determine the range of the cannon, it is necessary to know the velocity with which the projectile leaves the cannon (muzzle velocity). The cannon you are testing has a mass of 200 kg and shoots a 18-kg projectile. During the lab tests the cannon is held horizontally in a rigid support so that it cannot move. Under those conditions, you measure the magnitude of the muzzle velocity to be 400 m/s. When the cannon is actually used in the field, however, it is mounted so that it is free to move (recoil) horizontally when it is fired. Your boss asks you to calculate the projectile’s speed leaving the cannon under field conditions, when it is allowed to recoil. She tells you to take the case where the cannon is fired at 45° using cannon shells which are identical in chemical energy to those used in the laboratory test.
With identical cannon shells, the same energy will be expended – generating kinetic energy for the cannon and the shell both. The cannon and shell will conserve the x-momentum, which provides a second constraint.

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K = \frac{1}{2} m_s v_i^2
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\[
K = \frac{1}{2} m_c v_c^2 + \frac{1}{2} m_s v_s^2
\]

\[
p_i = 0 = p_f
\]

\[
= m_c (-v_c) + m_s v_s \cos 45^\circ
\]

\[
v_c = \frac{m_s}{m_c} v_s \cos 45^\circ
\]

\[
K = \frac{1}{2} m_c \left( \frac{m_s}{m_c} v_s \cos 45^\circ \right)^2 + \frac{1}{2} m_s v_s^2
\]

\[
\frac{1}{2} m_s v_i^2 = \frac{1}{2} m_s v_s^2 \left( \frac{m_s}{2m_c} + 1 \right)
\]

\[
v_s = v_i \frac{\frac{m_s}{2m_c} + 1}{\sqrt{\frac{m_s}{2m_c} + 1}}
\]

\[
= 391 \text{ m/s}
\]