**Group problem 1.**

Your friend has just been in a traffic accident and is trying to negotiate with the insurance company to pay for fixing the car. Your friend asked your help to prove that it was the other driver's fault. At the scene of the crash you determined what happened. Your friend was traveling toward North and entered an intersection with no stop sign. At the center of the intersection, his car was struck by another car that was traveling toward East. The speed limit on both roads entering the intersection is 80 km/hr. From the visible skid straight marks, the cars skidded locked together for 17 m at an angle of 30° north of east before stopping. At the library you determine that the weight of your friend's car was 1180 kg and that of the other car was 1000 kg, where you included the driver's weight in each case. The coefficient of kinetic friction for a rubber tire skidding on dry pavement is 0.80. It is not enough to prove that the other driver was speeding, you must also show that your friend was under the speed limit.

**Solution:**

Start with a sketch of the accident before the impact, just after the impact and at the end of the slid. Choose a coordinate system with the origin at the impact. We call $v_1$ and $m_1$ the velocity and mass of the car’s friend, $v_2$ and $m_2$ the velocity and mass the other car.
The two cars coalesce, it means that the collision is perfectly inelastic. The kinetic energy is not conserved. The conservation of momentum has to be applied from situation 1 to situation 2, before the collision and just after the collision (just before the 2 cars start skidding).

Before applying the conservation of momentum, we can calculate the velocity of the 2 cars just after collision, we call this velocity \( v_i \). We use the work-kinetic energy theorem from situation 2 to situation 3: \( W = \Delta K = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 \), where \( v_f \) is the speed at the end of skidding: \( v_f = 0 \).

So: \( W = - \frac{1}{2} (m_1 + m_2)v_i^2 \).

During the skidding, only one force does work: friction.

The work done by friction is the dot product: \( W_f = f \cdot s \), where \( f \) is the vector force of friction and \( s \) the vector displacement.

The direction of \( f \) and \( s \) are opposite: \( W_f = f s \cos \pi = -f s \), here \( f \) and \( s \) are the magnitude of the force and the displacement (here \( s \) is the length of skidding).

We know that: \( f = \mu_k N \), where \( \mu_k \) is the coefficient of kinetic friction and \( N \) the normal force. Newton’s second law gives immediately that \( N = mg \), the magnitude of the normal force equals the magnitude of the weight. Finally: \( W_f = - \mu_k mg s = \mu_k (m_1 + m_2)g s \)

The work kinetic energy theorem gives:

\[
W_f = - \frac{1}{2} (m_1 + m_2)v_i^2 \rightarrow - \mu_k (m_1 + m_2)g s = - \frac{1}{2} (m_1 + m_2)v_i^2
\]

Finally: \( v_i = (2 \mu_k g s)^{1/2} = (2 \times 0.8 \times 9.8 \times 17)^{1/2} = 16.3 \text{ m/s} = 58.8 \text{ km/hr} \)

Now we apply the conservation of momentum in the two directions X and Y, in the coordinate system:

Just before collision in X: \( p_{ix} = m_2v_2 \)
Just before collision in Y: \( p_{iy} = m_1v_1 \)

Just after collision in X: \( p_{ix} = (m_1 + m_2)v_i \cos \theta \)
Just after collision in Y: \( p_{ix} = (m_1 + m_2)v_i \sin \theta \)

In X the conservation gives: \( m_2v_2 = (m_1 + m_2)v_i \cos \theta \rightarrow v_2 = v_i \cos \theta (m_1 + m_2) / m_2 \)
In Y the conservation gives: \( m_1v_1 = (m_1 + m_2)v_i \sin \theta \rightarrow v_1 = v_i \sin \theta (m_1 + m_2) / m_1 \)

Where \( v_i = (2 \mu_k g s)^{1/2} = (2 \times 0.8 \times 9.8 \times 17)^{1/2} = 16.3 \text{ m/s} = 58.8 \text{ km/hr} \)

We plug the numbers in:
\( v_2 = 111 \text{ km/hr} \)
\( v_1 = 54.3 \text{ km/hr} \)

Conclusion: indeed, your friend was driving at 54.3 km/hr below the limit while the other driver was speeding at 111 km/hr, above the limit.