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A special fermionic generalization of lineal gravity

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Abstract: The central extension of the (1+1)-dimensional Poincaré algebra by including fermionic charges which obey not supersymmetric algebra, but a special graded algebra containing in the right hand side a central element only is obtained. The corresponding theory being the fermionic extension of the lineal gravity is proposed. We considered the algebra of generators, the field transformations and found Lagrangian and equation of motion, then we derived the Casimir operator and obtained the constant black hole mass.

Key words: Black hole, Central extension, Lineal gravity, Fermionic generator, Casimir operator
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INTRODUCTION

The models of lineal gravity (the gravity theory in (1+1)-dimensions) are Liouville theory (Jackiw, 1985), a simplest non-trivial theory based on the scalar curvature R and additional scalar field, and so-called 'string-inspired' model (Cangemi and Jackiw, 1992; 1993), where scalar (dilaton) field arises from the string theory. Both models can also be obtained by dimensional reduction from (2+1)-dimensions (Achúcarro, 1993; Cangemi, 1992; Grignani and Nardelli, 1994).

In (Cangemi and Jackiw, 1992) it was shown that the lineal gravity in (1+1)-dimensional space-time (Verlinde, 1992; Callan *et al*., 1992) can be treated as a gauge theory with a central extension of the 2D Poincaré algebra taken as a gauge algebra. The quantization of lineal gravity was obtained in (Cangemi and Jackiw, 1994a; 1994b). Recently a possible fermionic generalization of the central extension of Poincaré algebra was proposed (Soroka and Soroka, 2005). By gauging of this algebra, we give here a special fermionic generalization of the lineal gravity model which differs from the standard supersymmetry.

The central extension of the (1+1)-dimensional Poincaré algebra is given by (Cangemi and Jackiw, 1994a; 1994b; Soroka and Soroka, 2005)

$$[P_a, P_b] = \boldsymbol{\varepsilon}_{ab} Y,\ [P_a, J] = \boldsymbol{\varepsilon}_a{}^b P_b, \tag{1}$$

$$[P_a, Y] = 0,\ [J, Y] = 0, \tag{2}$$

where P_a are translation generators, J is a Lorentz generator, Y is a central element

$$Y = \mathrm{i}\Lambda I/2, \tag{3}$$

here Λ is real constant, and $\boldsymbol{\varepsilon}^{ab} = -\boldsymbol{\varepsilon}^{ba}$, $\boldsymbol{\varepsilon}^{01}=1$ is the totally antisymmetric 2D Levi-Civita tensor, the tangent space indices $a,b,c,\ldots=(0,1)$ are lowered $\boldsymbol{\varepsilon}_a{}^b = \boldsymbol{h}_{ac}\boldsymbol{\varepsilon}^{cb}$ by means of the flat metric $\boldsymbol{h}_{ab}=\mathrm{diag}(-1, 1)$.

The shape of central tensor charges Z_{ab} for the Poincaré group in (1+1)-dimensions $Z_{ab}=\boldsymbol{\varepsilon}_{ab}Y$, was introduced in (Cangemi and Jackiw, 1994a; 1994b) while investigating the string-inspired lineal gravity, and for any dimensions similar tensor charges were proposed in (Soroka and Soroka, 2005).

If $Z_{ab}=0$, the standard way of the supersymmetric extension of the Poincaré algebra Eqs.(1)~(2) is introducing some fermionic generators $Q_\alpha^{(0)}$ which satisfy anticommuting relations

$$\{Q_\alpha^{(0)}, Q_\beta^{(0)}\} = -2\mathrm{i}(\gamma^a \boldsymbol{C})_{\alpha\beta} P_a. \tag{4}$$

In case of nonvanishing tensor charges $Z_{ab}\neq 0$ the Jacobi identity and P-invariance forbid appearance of the standard terms with momenta P_a on the right hand side of Eq.(4). We here introduce fermionic generators with nonvanishing anticommutator in another way (Soroka and Soroka, 2005) (using the central element Y) which satisfies the graded algebra

$$\{Q_\alpha, Q_\beta\} = -q(\gamma_5 \boldsymbol{C})_{\alpha\beta} Y,\ [Q_\alpha, J] = (\gamma_5 Q)_\alpha / 2, \tag{5}$$

$$[Y, Q_\alpha] = 0,\ [P_a, Q_\alpha] = 0, \tag{6}$$

where $\boldsymbol{C}$ is the charge conjugation matrix, q is a constant and $\gamma_5=\boldsymbol{\varepsilon}^{ab}\gamma_a\gamma_b/2$, $\boldsymbol{\sigma}^{ab}=[\gamma^a,\gamma^b]/4$. We use here a real (Majorana) representation for the 2D γ-matrices and charge conjugation matrix $\boldsymbol{C}$ as follows: $\gamma^0=\boldsymbol{C}=-\boldsymbol{C}^{\mathrm{T}}=-\mathrm{i}\boldsymbol{\sigma}_2$, $\gamma^1=\boldsymbol{\sigma}_1$, $\gamma_5=\boldsymbol{\varepsilon}^{ab}\gamma_a\gamma_b/2=\boldsymbol{\sigma}_3$, $\{\gamma_a,\gamma_b\}=2\boldsymbol{h}_{ab}$, $\boldsymbol{h}_{11}=-\boldsymbol{h}_{00}=1$, $\boldsymbol{C}^{-1}\gamma_a\boldsymbol{C}=-\gamma_a^{\mathrm{T}}$, where $\boldsymbol{\sigma}_i$ are Pauli matrices.

The algebra Eqs.(1)~(2), (5)~(6) is not semi-simple, because it is a semi-direct sum of the subalgebra generated by the Lorentz generator J and the graded ideal consisting of momenta P_a, central charge Y and fermionic generators Q_α, and it has the Casimir operator $K=P_aP^a-2YJ-(2q)^{-1}Q_\alpha(\boldsymbol{C}^{-1})^{\alpha\beta}Q_\beta$.

The expansion of the gauge 1-form $\boldsymbol{A}=\mathrm{d}x^\mu\boldsymbol{A}_\mu(x)$ in terms of the generators is

$$\boldsymbol{A}_\mu(x) = \boldsymbol{e}_\mu^a(x)P_a + \boldsymbol{\omega}_\mu(x)J + \boldsymbol{w}_\mu(x)Y + \boldsymbol{\psi}_\mu^\alpha(x)Q_\alpha, \tag{7}$$

where $\boldsymbol{e}_\mu^a(x)$ is the zweibein which determines the metric tensor of space-time $\boldsymbol{g}_{\mu\nu}(x)=\boldsymbol{e}_\mu^a(x)\boldsymbol{e}_\nu^b(x)\boldsymbol{h}_{ab}$, $\omega_\mu(x)$ is the spin connection, $w_\mu(x)$ is the gauge field corresponding to the central element Y and $\boldsymbol{\psi}_\mu^\alpha(x)$ is the Grassmann Majorana "gravitino" field associated with Q_α. Greek indices $\mu,\nu,\ldots=(0,1)$ referring to the world space.

Infinitesimal gauge transformations corresponding to the full algebra Eqs.(1)~(2),(5)~(6) are

$$\delta\boldsymbol{A} = \mathrm{d}\Omega + [\boldsymbol{A}, \Omega], \tag{8}$$

$$\Omega = y^a(x)P_a + \varphi(x)J + z(x)Y + \boldsymbol{\varepsilon}^\alpha(x)Q_\alpha, \tag{9}$$

where $y^a(x)$ are space-time translations, $\varphi(x)$ is the Lorentz boost parameter, $z(x)$ is the central element translation and $\boldsymbol{\varepsilon}^\alpha(x)$ is the fermionic translation. From Eqs.(1)~(2),(5)~(6) and (8) it follows

$$\begin{aligned}
&\delta\boldsymbol{e}_\mu^a(x) = \boldsymbol{\varepsilon}_b^a[-\varphi(x)\boldsymbol{e}_\mu^b(x) + y^b(x)\omega_\mu(x)] + \partial_\mu y^a(x),\\
&\delta\omega_\mu(x) = \partial_\mu\varphi(x),\\
&\delta w_\mu(x) = \boldsymbol{e}_\mu^a(x)y^b(x)\boldsymbol{\varepsilon}_{ab} + q\boldsymbol{\psi}_\mu^\alpha(x)\boldsymbol{\varepsilon}^\beta(x)(\gamma_5\boldsymbol{C})_{\alpha\beta} + \partial_\mu z(x),\\
&\delta\boldsymbol{\psi}_\mu^\alpha(x) = [\varphi(x)\boldsymbol{\psi}_\mu^\beta(x) - \omega_\mu(x)\boldsymbol{\varepsilon}^\beta(x)](\gamma_5)_\beta^\alpha/2 + \partial_\mu\boldsymbol{\varepsilon}^\alpha(x).
\end{aligned} \tag{10}$$

The finite gauge transformations $\boldsymbol{A}\rightarrow\bar{\boldsymbol{A}}= e^{-\Omega}\mathrm{d}e^{\Omega} + e^{-\Omega}\boldsymbol{A}e^{\Omega}$ have the following component form:

$$\begin{aligned}
\bar{\boldsymbol{e}}_\mu^a(x) &= \boldsymbol{e}_\mu^a(x) - N_b^a(x)(\boldsymbol{e}_\mu^b(x) - \omega_\mu(x)s^b(x) - \boldsymbol{e}_c^b\partial_\mu s^c(x))\\
&\quad + s^a(x)\partial_\mu\varphi(x),\\
\bar{\omega}_\mu(x) &= \omega_\mu(x) + \partial_\mu\varphi(x),\\
\bar{\omega}_\mu(x) &= \omega_\mu(x) - s_a(x)N_b^a(x)[\boldsymbol{e}_\mu^b(x) - \omega_\mu(x)s^b(x)\\
&\quad - \boldsymbol{e}_c^b\partial_\mu s^c(x)] - 2q[\boldsymbol{\psi}_\mu^\alpha(x) - \omega_\mu(x)\rho^\alpha(x)\\
&\quad + 2\partial_\mu\rho^\gamma(x)(\gamma_5)_\gamma^\alpha]T_\alpha^\beta(x)\rho_\beta + \partial_\mu z(x)\\
&\quad - \varphi(x)s_a(x)\partial_\mu s^a(x) - 2q\varphi(x)\rho^\alpha(x)\partial_\mu\rho_\alpha(x),\\
\bar{\boldsymbol{\psi}}_\mu^\alpha(x) &= \boldsymbol{\psi}_\mu^\alpha(x) - [\boldsymbol{\psi}_\mu^\beta(x) - \omega_\mu(x)\rho^\beta(x) + 2\partial_\mu\rho^\gamma(x)(\gamma_5)_\gamma^\beta]\\
&\quad \times T_\beta^\alpha(x) + \partial_\mu\varphi(x)\rho^\alpha(x),
\end{aligned} \tag{11}$$

where the notations

$$\begin{aligned}
&N_b^a(x) = \boldsymbol{\delta}_b^a - (M^{-1}(x))_b^a,\ T_\alpha^\beta(x) = \boldsymbol{\delta}_\alpha^\beta - S_\alpha^\beta(x),\\
&\rho^\alpha(x) = \frac{\varepsilon^\alpha(x)}{\varphi(x)},\ s^a(x) = \frac{y^a(x)}{\varphi(x)},
\end{aligned} \tag{12}$$

with the finite Lorentz transformations for vectors and spinors respectively

$$\begin{aligned}
&M_b^a(x) = \boldsymbol{\delta}_b^a\cosh\varphi(x) + \boldsymbol{\varepsilon}_b^a\sinh\varphi(x),\\
&S_\alpha^\beta(x) = \boldsymbol{\delta}_\alpha^\beta\cosh\frac{\varphi(x)}{2} + (\gamma_5)_\alpha^\beta\sinh\frac{\varphi(x)}{2}
\end{aligned} \tag{13}$$

are introduced. The multiplet of the curvature 2-form is

$$\boldsymbol{F}=\frac{1}{2}\mathrm{d}x^{\mu}\wedge\mathrm{d}x^{\nu}F_{\mu\nu}(x)=\mathrm{d}\boldsymbol{A}+\boldsymbol{A}\wedge\boldsymbol{A}$$
$$=\frac{1}{2}\mathrm{d}x^{\mu}\wedge\mathrm{d}x^{\nu}[\partial_{[\mu}A_{\nu]}(x)+A_{[\mu}(x)A_{\nu]}(x)], \quad (14)$$

where antisymmentrization $[\mu\nu]$ is implied without the factor 1/2. The field strength $\boldsymbol{F}$ can be expanded in terms of the generators

$$\boldsymbol{F}_{\mu\nu}(x)=\boldsymbol{f}^{a}_{\mu\nu}(x)P_a+\boldsymbol{r}_{\mu\nu}(x)J+\boldsymbol{v}_{\mu\nu}(x)Y+\xi^{\alpha}_{\mu\nu}(x)Q_{\alpha}, \quad (15)$$

with

$$\boldsymbol{f}^{a}_{\mu\nu}(x)=\partial_{[\mu}\boldsymbol{e}^{a}_{\nu]}(x)+\boldsymbol{\varepsilon}^{a}_{b}\omega_{[\mu}(x)\boldsymbol{e}^{b}_{\nu]}(x)\equiv\boldsymbol{D}^{f}_{[\mu}(\omega)\boldsymbol{e}^{a}_{\nu]}(x), \quad (16)$$

$$\boldsymbol{r}_{\mu\nu}(x)=\partial_{[\mu}\omega_{\nu]}(x), \quad (17)$$

$$\boldsymbol{v}_{\mu\nu}(x)=\partial_{[\mu}w_{\nu]}(x)+\frac{1}{2}\boldsymbol{e}^{a}_{[\mu}(x)\boldsymbol{e}^{b}_{\nu]}(x)\boldsymbol{\varepsilon}_{ab}+\frac{q}{2}\boldsymbol{\psi}^{\alpha}_{[\mu}(x)\boldsymbol{\psi}^{\beta}_{\nu]}(x)(\gamma_5\boldsymbol{C})_{\alpha\beta}, \quad (18)$$

$$\xi^{\alpha}_{\mu\nu}(x)=\partial_{[\mu}\boldsymbol{\psi}^{\alpha}_{\nu]}(x)-\frac{1}{2}\omega_{[\mu}\boldsymbol{\psi}^{\beta}_{\nu]}(x)(\gamma_5)^{\alpha}_{\beta}\equiv\boldsymbol{D}^{\xi}_{[\mu}(\omega)\boldsymbol{\psi}^{\alpha}_{\nu]}(x), \quad (19)$$

where $\boldsymbol{D}^{f}_{\mu}(\omega)$ and $\boldsymbol{D}^{\xi}_{\mu}(\omega)$ are covariant derivatives corresponding to boson and fermion translations, while $\boldsymbol{r}_{\mu\nu}(x)$ is the strength tensor corresponding to the Lorentz boost, and $\boldsymbol{v}_{\mu\nu}(x)$ is the strength tensor corresponding to the central element Y.

The components of $\boldsymbol{F}_{\mu\nu}(x)$ Eqs.(16)~(19) are transformed by the adjoint representation of the fermionic generalization Eqs.(5)~(6) of the centrally extended Poincaré group Eqs.(1)~(2) as follows

$$\begin{aligned}\bar{\boldsymbol{f}}^{a}_{\mu\nu}(x)&=\boldsymbol{f}^{a}_{\mu\nu}(x)-N^{a}_{b}(x)[\boldsymbol{f}^{b}_{\mu\nu}(x)-\boldsymbol{r}_{\mu\nu}(x)s^{b}(x)],\\ \bar{\boldsymbol{r}}_{\mu\nu}(x)&=\boldsymbol{r}_{\mu\nu}(x),\\ \bar{\boldsymbol{v}}_{\mu\nu}(x)&=\boldsymbol{v}_{\mu\nu}(x)-s_a(x)N^{a}_{b}(x)[\boldsymbol{f}^{b}_{\mu\nu}(x)-\boldsymbol{r}_{\mu\nu}(x)s^{b}(x)]\\ &\quad-2q[\boldsymbol{\xi}^{\beta}_{\mu\nu}(x)-\boldsymbol{r}_{\mu\nu}(x)\rho^{\beta}(x)]T^{\alpha}_{\beta}(x)\rho_{\alpha}(x),\\ \bar{\xi}^{\alpha}_{\mu\nu}(x)&=\xi^{\alpha}_{\mu\nu}(x)-[\boldsymbol{\xi}^{\beta}_{\mu\nu}(x)-\boldsymbol{r}_{\mu\nu}(x)\rho^{\beta}(x)]T^{\alpha}_{\beta}(x).\end{aligned} \quad (20)$$

This transformation law can be expressed in the concise form

$$\bar{\boldsymbol{F}}^{A}=(\boldsymbol{U}^{-1})^{A}_{B}\boldsymbol{F}^{B}. \quad (21)$$

The invariant Lagrangian density can be constructed using a multiplet of Lagrange multipliers

$$\begin{aligned}\mathcal{L}&=\frac{1}{2}\boldsymbol{\varepsilon}^{\mu\nu}\boldsymbol{\eta}_{A}\boldsymbol{F}^{A}_{\mu\nu}(x)=\frac{1}{2}\boldsymbol{\varepsilon}^{\mu\nu}[\eta_a\boldsymbol{f}^{a}_{\mu\nu}(x)+\eta_2\boldsymbol{r}_{\mu\nu}(x)\\ &\quad+\eta_3\boldsymbol{v}_{\mu\nu}(x)+\eta_{\alpha}\xi^{\alpha}_{\mu\nu}(x)],\\ \boldsymbol{\eta}_A&=(\eta_a,\eta_2,\eta_3,\eta_{\alpha}),\end{aligned} \quad (22)$$

which obey the coadjoint transformation law $\bar{\boldsymbol{\eta}}_A=\boldsymbol{\eta}_B\boldsymbol{U}^{B}_{A}$. The corresponding equations of motion are

$$\boldsymbol{F}^{A}_{\mu\nu}(x)=0, \quad (23)$$

$$\partial_{\mu}\eta_a=(\boldsymbol{e}^{b}_{\mu}(x)\eta_3+\omega_{\mu}(x)\eta^{b})\boldsymbol{\varepsilon}^{a}_{b}, \quad (24)$$

$$\partial_{\mu}\eta_2=\boldsymbol{e}^{a}_{\mu}(x)\eta^{b}\boldsymbol{\varepsilon}_{ab}-\frac{1}{2}\boldsymbol{\psi}^{\beta}_{\mu}(x)(\gamma_5)^{\alpha}_{\beta}\eta_{\alpha}, \quad (25)$$

$$\partial_{\mu}\eta_3=0, \quad (26)$$

$$\partial_{\mu}\eta_{\alpha}=\left(\frac{1}{2}\omega_{\mu}(x)\eta^{\beta}-q\boldsymbol{\psi}^{\beta}_{\mu}(x)\eta_3\right)(\gamma_5)^{\alpha}_{\beta}. \quad (27)$$

The fermionic generalization of the centrally extended Poincaré algebra Eqs.(1)~(2), (5)~(6) in the coadjoint representation possesses a nonsingular invariant graded metric $\boldsymbol{h}_{AB}=(-1)^{p_B(p_A+p_C)}\boldsymbol{h}_{CD}\boldsymbol{U}^{D}_{B}\boldsymbol{U}^{C}_{A}$, which has the form

$$\boldsymbol{h}_{AB}=(-1)^{p_A+p_B}\boldsymbol{h}_{BA}=\begin{pmatrix}h_{ab}&0&0&0\\0&u&-1&0\\0&-1&0&0\\0&0&0&-2qC_{\alpha\beta}\end{pmatrix}, \quad (28)$$

where $p_A\equiv p(A)$ is a Grassmann parity of the quantity A and u is an arbitrary constant. The inverse metric $\boldsymbol{h}^{AB}$ can be obtained from $\boldsymbol{h}^{AB}\boldsymbol{h}_{BC}=\boldsymbol{\delta}^{A}_{C}$ and has the property $\boldsymbol{h}^{AB}=(-1)^{p_A+p_B+p_Ap_B}\boldsymbol{h}^{BA}$ (which can be used to upper the indices $\boldsymbol{\eta}^{A}=\boldsymbol{h}^{AB}\boldsymbol{\eta}_B$).

In the compact notation $X_A=\{P_a,J,Y,Q_{\alpha}\}$ the full algebra Eqs.(1)~(2), (5)~(6) can be presented as

$$[X_A,X_B]\equiv X_AX_B-(-1)^{p_Ap_B}X_BX_A=f^{\ C}_{AB}X_C, \quad (29)$$

where f_{AB}^{C} are the structure constants, and nonvanishing ones have the form

$$f_{ab}^{Y}=\boldsymbol{\varepsilon}_{ab},\ f_{aJ}^{b}=\boldsymbol{\varepsilon}_{a}^{b},\ f_{\alpha\beta}^{Y}=-q(\gamma_5\boldsymbol{C})_{\alpha\beta},\ f_{\alpha J}^{\beta}=(\gamma_5)_{\alpha}^{\beta}/2.$$

In terms of $\eta=\eta^A X_A$ the equations of motion Eqs.(24)~(27) can be written as $\mathrm{d}\eta+[\boldsymbol{A},\eta]=0$, where $\boldsymbol{A}=\boldsymbol{A}^B X_B$. The invariant quantity

$$M=-\frac{1}{2\Lambda}\eta_A\boldsymbol{h}^{AB}\eta_B \tag{30}$$

can be interpreted as the black-hole mass (Verlinde, 1992), if it is constant (Λ is from Eq.(3)). Indeed, from the equations of motion we have

$$\mathrm{d}M=-\frac{1}{\Lambda}\eta_A\mathrm{d}\eta^A=\frac{1}{\Lambda}\eta_A\eta^C\boldsymbol{A}^B f_{BC}^{A}. \tag{31}$$

As a consequence of the structure constant properties we obtain $\eta_A\eta^C\boldsymbol{A}^B f_{BC}^{A}=0$, and therefore M=const.

Using the inverse metric $\boldsymbol{h}^{AB}$, we can define the set of independent Casimir operators

$$K_h(u)=(-1)^{p_A+p_B}X_A\boldsymbol{h}^{AB}X_B=K-uY^2, \tag{32}$$

which is marked by the additional parameter u from Eq.(28).

CONCLUSION

We propose here a special fermionic extension of the lineal gravity which is not the standard supersymmetric 2D gravity (Cangemi and Leblanc, 1994), (because of absence of momenta on the right hand side of the fermionic anticommutator), which can be treated as a new kind of "degenerated" fermionic-bosonic system. We considered the algebra of generators, the field transformations and found Lagrangian and equation of motion, then we derived the Casimir operator and obtained the constant black hole mass.

References

Achúcarro, A., 1993. Lineal gravity from planar gravity. *Phys. Rev. Lett.*, **70**(8):1037-1040. [doi:10.1103/PhysRevLett.70.1037]

Callan, C.G., Giddings, S.B., Harvey, J.A., Strominger, A., 1992. Evanescent black holes. *Phys. Rev.*, **D45**: 1005-1009.

Cangemi, D., 1992. One formulation for both lineal gravities through a dimensional reduction. *Phys. Lett.*, **B297**: 261-267.

Cangemi, D., Jackiw, R., 1992. Gauge invariant formulations of lineal gravity. *Phys. Rev. Lett.*, **69**(2):233-236. [doi:10.1103/PhysRevLett.69.233]

Cangemi, D., Jackiw, R., 1993. Poincaré gauge theory for gravitational forces in (1+1)-dimensions. *Ann. Phys.*, **225**(2):229-263. [doi:10.1006/aphy.1993.1058]

Cangemi, D., Jackiw, R., 1994a. Quantal analysis of string inspired lineal gravity with matter fields. *Phys. Lett.*, **B337**:271-278.

Cangemi, D., Jackiw, R., 1994b. Quantum states of string inspired lineal gravity. *Phys. Rev.*, **D50**:3913-3922.

Cangemi, D., Leblanc, M., 1994. Two dimensional gauge theoretic supergravities. *Nucl. Phys. B*, **420**(1-2):363-386.

Grignani, G., Nardelli, G., 1994. Poincaré gauge theories for lineal gravity. *Nucl. Phys. B*, **412**(1-2):320-344. [doi:10.1016/0550-3213(94)90505-3]

Jackiw, R., 1985. Lower dimensional gravity. *Nucl. Phys. B*, **252**:343-356. [doi:10.1016/0550-3213(85)90448-1]

Soroka, D.V., Soroka, V.A., 2005. Tensor extension of the Poincaré algebra. *Phys. Lett.*, **B607**:302-305.

Verlinde, H., 1992. Black Holes and Strings in Two Dimensions. *In*: Sato, M., Nakamura, T.(Eds.), Sixth Marcel Grossmann Meeting on General Relativity. World Scientific, Singapore, p.214-223.