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FERMIONIC GENERALIZATION OF LINEAL GRAVITY IN CENTRALLY EXTENDED FORMULATION

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We generalize the central extension of the $(1+1)$-dimensional Poincaré algebra by including fermionic charges which obey not supersymmetric algebra, but special graded algebra containing in the right hand side a central element only. We verify selfconsistency of Jacobi identities and derive the Casimir operator. Then we introduce the correspondent gauge fields and construct the classical gauge theory based on this graded algebra, present field transformations and derive the black hole mass in $(1+1)$-dimensions.

KEY WORDS: Poincaré algebra, lineal gravity, central extention, fermionic generator, Casimir operator

The Einstein-type theory in $(1+1)$-dimensions — lineal gravity — provides a great interest in connection e.g. with black hole solutions, while drastic simplification of group theoretical properties in lowest dimension allows a more deep understanding of gravitational effects.

In $(1+1)$-dimensions the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \dfrac{1}{2} g_{\mu\nu} R$ vanishes identically, therefore the standard equations of motion vanish, and one needs to introduce additional fields to endow theory with sensible dynamics. The most studied models of lineal gravity are Liouville theory [1], a simplest non-trivial theory based on the scalar curvature R and additional scalar field, and so-called 'string-inspied' model [2,3], where scalar (dilaton) field arises from string theory. Both the models can be also obtained by dimensional reduction from $(2+1)$-dimensions [4–6]. In [2] it was shown that the lineal gravity in $(1+1)$-dimensional space-time [7, 8] can be treated as a gauge theory with a central extension of the two-dimensional Poincaré algebra taken as a gauge algebra. The quantization of lineal gravity was obtained in [9, 10]. Recently a possible fermionic generalization of the central extension of the Poincaré algebra was proposed [11]. By gauging of this algebra, we give here a special fermionic generalization of the lineal gravity model which differs from the standard supersymmetry.

CENTRAL EXTENSION OF POINCARÉ ALGEBRA

The central extension of the $(1+1)$-dimensional Poincaré algebra is [2, 11]

$$[P_a, P_b] = \epsilon_{ab} Y, \tag{1}$$

$$[P_a, J] = \epsilon_a{}^b P_b, \tag{2}$$

$$[P_a, Y] = 0, \tag{3}$$

$$[J, Y] = 0, \tag{4}$$

where P_a are translation generators, J is a Lorentz generator, Y is a central element

$$Y = \frac{i}{2} \Lambda I, \tag{5}$$

here Λ is some real constant, and $\epsilon^{ab} = -\epsilon^{ba}$, $\epsilon^{01} = 1$ is the totally antisymmetric two-dimensional Levi-Civita tensor, the tangent space indices $a, b, c \ldots = (0, 1)$ are lowered $\epsilon_a{}^b = h_{ac} \epsilon^{cb}$ by means of the flat metric $h_{ab} = \mathrm{diag}\,(-1, 1)$.

The shape of central tensor charges Z_{ab} for the Poincaré group in $1+1$ dimensions

$$Z_{ab} = \epsilon_{ab} Y, \tag{6}$$

was introduced in [2] while investigating the string-inspired lineal gravity, and for any dimensions similar tensor charges were proposed in [11]. Note that for the gauge theory of the de Sitter group $Y = \frac{1}{2} \Lambda J$ [6, 12].

If $Z_{ab} = 0$, the standard way of the supersymmetric extension of the Poincaré algebra (1)-(2) is introducing some fermionic generators $Q_\alpha^{(0)}$ which satisfy anticommuting relations [13]

$$\{Q_\alpha^{(0)}, Q_\beta^{(0)}\} = -2i \left(\gamma^a C\right)_{\alpha\beta} P_a. \tag{7}$$

But in case of nonvanishing tensor charges $Z_{ab} \neq 0$ the Jacobi identity and P-invariance forbid appearance of the standard terms with momenta P_a in the right hand side of (7). However we can introduce fermionic generators with nonvanishing anticommutator in another way [11] (using the central element Y) which satisfy the graded algebra

$$\{Q_\alpha, Q_\beta\} = -q(\gamma_5 C)_{\alpha\beta} Y, \tag{8}$$
$$[Q_\alpha, J] = \frac{1}{2}(\gamma_5 Q)_\alpha, \tag{9}$$
$$[Y, Q_\alpha] = 0, \tag{10}$$
$$[P_a, Q_\alpha] = 0, \tag{11}$$

where C is the charge conjugation matrix, q is some constant (which controls nonanticommutativity of the fermionic charges themselves) and $\gamma_5 = \frac{1}{2}\epsilon^{ab}\gamma_a\gamma_b$, $\sigma^{ab} = \frac{1}{4}\left[\gamma^a, \gamma^b\right]$. As a real (Majorana) representation for the two-dimensional γ-matrices and charge conjugation matrix C we use $\gamma^0 = C = -C^T = -i\sigma_2$, $\gamma^1 = \sigma_1$, $\gamma_5 = \frac{1}{2}\epsilon^{ab}\gamma_a\gamma_b = \sigma_3$, $\{\gamma_a, \gamma_b\} = 2h_{ab}$, $h_{11} = -h_{00} = 1$, $C^{-1}\gamma_a C = -\gamma_a{}^T$, where σ_i are Pauli matrices. The matrices γ_a satisfy

$$\gamma_a\gamma_5 = \epsilon_{ab}\gamma^b, \qquad \gamma_a\gamma_b = h_{ab} - \epsilon_{ab}\gamma_5. \tag{12}$$

The Jacobi identities of the extended Poincaré algebra (1)–(4),(8)-(11) are

$$[P_a, [P_b, P_c]] + [P_c, [P_a, P_b]] + [P_b, [P_c, P_a]] = 0, \tag{13}$$
$$[P_a, [P_b, Q_\alpha]] + [Q_\alpha, [P_a, P_b]] + [P_b, [Q_\alpha, P_a]] = 0, \tag{14}$$
$$[P_a, \{Q_\alpha, Q_\beta\}] - \{Q_\alpha, [P_a, Q_\beta]\} - \{Q_\beta, [P_a, Q_\alpha]\} = 0, \tag{15}$$
$$[J, [P_a, P_b]] + [P_a, [J, P_b]] + [P_b, [J, P_a]] = 0, \tag{16}$$
$$[J, [P_a, J]] + [P_a, [J, J]] + [J, [J, P_a]] = 0, \tag{17}$$
$$[Q_\alpha, \{Q_\beta, Q_\gamma\}] - [Q_\gamma, \{Q_\alpha, Q_\beta\}] - [Q_\beta, \{Q_\gamma, Q_\alpha\}] = 0, \tag{18}$$
$$[J, \{Q_\alpha, Q_\beta\}] - \{Q_\alpha, [J, Q_\beta]\} - \{Q_\beta, [J, Q_\alpha]\} = 0, \tag{19}$$

where (13)-(18) satisfy trivially, while (19) follows from antisymmetry of matrix C.

We note that the full algebra (1)–(4),(8)-(11) is not semi-simple, because it is a semi-direct sum of the subalgebra generated by the Lorentz generator J and the graded ideal consisting of momenta P_a, central charge Y and fermionic generators Q_α. It has the following Casimir operator

$$K = P_a P^a - 2YJ - \frac{1}{2q} Q_\alpha \left(C^{-1}\right)^{\alpha\beta} Q_\beta. \tag{20}$$

GAUGE TRANSFORMATIONS

The gauge 1-form $\mathrm{A} = dx^\mu A_\mu(x)$ can be expanded in terms of the generators, and the gauge connection is

$$A_\mu(x) = e_\mu^a(x) P_a + \omega_\mu(x) J + w_\mu(x) Y + \psi_\mu^\alpha(x) Q_\alpha, \tag{21}$$

where $e_\mu^a(x)$ is the *zweibein* which determines the metric tensor of space-time $g_{\mu\nu}(x) = e_\mu^a(x) e_\nu^b(x) h_{ab}$, $\omega_\mu(x)$ is the spin connection, $w_\mu(x)$ is the gauge field corresponding to the central element Y and $\psi_\mu^\alpha(x)$ is the Grassmann Majorana "gravitino" field associated with Q_α. Greek indices $\mu, \nu \ldots = 0, 1$ refer to the world space.

Infinitesimal gauge transformations corresponding to the full algebra (1)–(4),(8)-(11) are

$$\delta\mathrm{A} = \mathrm{d}\Omega + [\mathrm{A}, \Omega], \tag{22}$$

with the gauge generator

$$\Omega = y^a(x) P_a + \varphi(x) J + z(x) Y + \varepsilon^\alpha(x) Q_\alpha, \tag{23}$$

where $y^a(x)$ are space-time translations, $\varphi(x)$ is the Lorentz boost parameter, $z(x)$ is the central element translation and $\varepsilon^\alpha(x)$ is the fermionic translation. From (1)–(4),(8)-(11) and (22) it follows

$$\delta e_\mu^a(x) = \epsilon^a{}_b \left[-\varphi(x) e_\mu^b(x) + y^b(x)\omega_\mu(x)\right] + \partial_\mu y^a(x), \tag{24}$$
$$\delta\omega_\mu(x) = \partial_\mu\varphi(x), \tag{25}$$
$$\delta w_\mu(x) = e_\mu^a(x) y^b(x)\epsilon_{ab} + q\psi_\mu^\alpha(x)\varepsilon^\beta(x)(\gamma_5 C)_{\alpha\beta} + \partial_\mu z(x), \tag{26}$$
$$\delta\psi_\mu^\alpha(x) = \frac{1}{2}\left[\varphi(x)\psi_\mu^\beta(x) - \omega_\mu(x)\varepsilon^\beta(x)\right](\gamma_5)_\beta{}^\alpha + \partial_\mu\varepsilon^\alpha(x). \tag{27}$$

The finite gauge transformations

$$\mathrm{A} \to \bar{\mathrm{A}} = e^{-\Omega}\mathrm{d}e^{\Omega} + e^{-\Omega}\mathrm{A}e^{\Omega} \tag{28}$$

have the following form:

$$\bar{e}^a_\mu(x) = e^a_\mu(x) - N^a{}_b(x)\left(e^b_\mu(x) - \omega_\mu(x)s^b(x) - \epsilon^b{}_c\partial_\mu s^c(x)\right) + s^a(x)\partial_\mu\varphi(x), \tag{29}$$

$$\bar{\omega}_\mu(x) = \omega_\mu(x) + \partial_\mu\varphi(x), \tag{30}$$

$$\begin{aligned}\bar{w}_\mu(x) = {}& w_\mu(x) - s_a(x)N^a{}_b(x)\left[e^b_\mu(x) - \omega_\mu(x)s^b(x) - \epsilon^b{}_c\partial_\mu s^c(x)\right]\\ &- 2q\left[\psi^\alpha_\mu(x) - \omega_\mu(x)\rho^\alpha(x) + 2\partial_\mu\rho^\gamma(x)(\gamma_5)_\gamma{}^\alpha\right]T_\alpha{}^\beta(x)\rho_\beta\\ &+ \partial_\mu z(x) - \varphi(x)s_a(x)\partial_\mu s^a(x) - 2q\varphi(x)\rho^\alpha(x)\partial_\mu\rho_\alpha(x),\end{aligned} \tag{31}$$

$$\bar{\psi}^\alpha_\mu(x) = \psi^\alpha_\mu(x) - [\psi^\beta_\mu(x) - \omega_\mu(x)\rho^\beta(x) + 2\partial_\mu\rho^\gamma(x)(\gamma_5)_\gamma{}^\beta]T_\beta{}^\alpha(x) + \partial_\mu\varphi(x)\rho^\alpha(x), \tag{32}$$

where the notations

$$N^a{}_b(x) = \delta^a{}_b - \left(M^{-1}(x)\right)^a{}_b, \quad T_\alpha{}^\beta(x) = \delta_\alpha{}^\beta - S_\alpha{}^\beta(x), \tag{33}$$

$$\rho^\alpha(x) = \frac{\varepsilon^\alpha(x)}{\varphi(x)}, \quad s^a(x) = \frac{y^a(x)}{\varphi(x)}, \tag{34}$$

with the finite Lorentz transformations for vectors and spinors respectively

$$M^a{}_b(x) = \delta^a{}_b\cosh\varphi(x) + \epsilon^a{}_b\sinh\varphi(x), \tag{35}$$

$$S_\alpha{}^\beta(x) = \delta_\alpha{}^\beta\cosh\frac{\varphi(x)}{2} + (\gamma_5)_\alpha{}^\beta\sinh\frac{\varphi(x)}{2} \tag{36}$$

are introduced.

The multiplet of the curvature 2-form is

$$\mathrm{F} = \frac{1}{2}dx^\mu \wedge dx^v F_{\mu\nu}(x) = \mathrm{dA} + \mathrm{A}\wedge\mathrm{A} = \frac{1}{2}dx^\mu \wedge dx^v\left[\partial_{[\mu}A_{\nu]}(x) + A_{[\mu}(x)A_{\nu]}(x)\right], \tag{37}$$

where antisymmentrization $[\mu\nu]$ is implied without the factor $1/2$. The field strength F can be expanded in terms of the generators

$$F_{\mu\nu}(x) = f^a_{\mu\nu}(x)P_a + r_{\mu\nu}(x)J + v_{\mu\nu}(x)Y + \xi^\alpha_{\mu\nu}(x)Q_\alpha, \tag{38}$$

with

$$f^a_{\mu\nu}(x) = \partial_{[\mu}e^a_{\nu]}(x) + \epsilon^a{}_b\omega_{[\mu}(x)e^b_{\nu]}(x) \equiv D^f_{[\mu}(\omega)e^a_{\nu]}(x), \tag{39}$$

$$r_{\mu\nu}(x) = \partial_{[\mu}\omega_{\nu]}(x), \tag{40}$$

$$v_{\mu\nu}(x) = \partial_{[\mu}w_{\nu]}(x) + \frac{1}{2}e^a_{[\mu}(x)e^b_{\nu]}(x)\epsilon_{ab} + \frac{q}{2}\psi^\alpha_{[\mu}(x)\psi^\beta_{\nu]}(x)(\gamma_5 C)_{\alpha\beta}, \tag{41}$$

$$\xi^\alpha_{\mu\nu}(x) = \partial_{[\mu}\psi^\alpha_{\nu]}(x) - \frac{1}{2}\omega_{[\mu}\psi^\beta_{\nu]}(x)(\gamma_5)_\beta{}^\alpha \equiv D^\xi_{[\mu}(\omega)\psi^\alpha_{\nu]}(x), \tag{42}$$

where $D^f_\mu(\omega)$ and $D^\xi_\mu(\omega)$ are covariant derivatives corresponding to boson and fermion translations, while $r_{\mu\nu}(x)$ is the strength tensor corresponding to the Lorentz boost, and $v_{\mu\nu}(x)$ is the strength tensor corresponding to the central element Y.

The components of $F_{\mu\nu}(x)$ (39)-(42) are transformed by the adjoint representation of the fermionic generalization (8)-(11) of the centrally extended Poincaré group (1)-(4) as follows

$$\bar{f}^a_{\mu\nu}(x) = f^a_{\mu\nu}(x) - N^a{}_b(x)\left[f^b_{\mu\nu}(x) - r_{\mu\nu}(x)s^b(x)\right], \tag{43}$$

$$\bar{r}_{\mu\nu}(x) = r_{\mu\nu}(x), \tag{44}$$

$$\begin{aligned}\bar{v}_{\mu\nu}(x) = {}& v_{\mu\nu}(x) - s_a(x)N^a{}_b(x)\left[f^b_{\mu\nu}(x) - r_{\mu\nu}(x)s^b(x)\right]\\ &- 2q\left[\xi^\beta_{\mu\nu}(x) - r_{\mu\nu}(x)\rho^\beta(x)\right]T_\beta{}^\alpha(x)\rho_\alpha(x),\end{aligned} \tag{45}$$

$$\bar{\xi}^\alpha_{\mu\nu}(x) = \xi^\alpha_{\mu\nu}(x) - \left[\xi^\beta_{\mu\nu}(x) - r_{\mu\nu}(x)\rho^\beta(x)\right]T_\beta{}^\alpha(x). \tag{46}$$

This transformation law can be equivalently expressed in the form

$$\bar{F}^A = (U^{-1})^A{}_B F^B \tag{47}$$

with
$$U^{A}{}_{B}=\left(\begin{array}{llll} M^{a}{}_{b}\left(x\right) & L^{a}{}_{c}\left(x\right)s^{c}\left(x\right) & 0 & 0\\ 0 & 1 & 0 & 0\\ -s_{c}\left(x\right)L^{c}{}_{b}\left(x\right) & s_{c}\left(x\right)L^{c}{}_{l}\left(x\right)s^{l}\left(x\right)+2q\rho^{\gamma}\left(x\right)R_{\gamma}{}^{\lambda}\rho_{\lambda}\left(x\right) & 1 & 2qR_{\beta}{}^{\gamma}\left(x\right)\rho_{\gamma}\left(x\right)\\ 0 & \rho^{\lambda}\left(x\right)R_{\lambda}{}^{\alpha}\left(x\right) & 0 & \left(S^{-1}\left(x\right)\right)_{\beta}{}^{\alpha} \end{array}\right), \tag{48}$$
where
$$L^{a}{}_{b}\left(x\right)=\delta^{a}{}_{b}-M^{a}{}_{b}\left(x\right),\qquad R_{\alpha}{}^{\beta}\left(x\right)=\delta_{\alpha}{}^{\beta}-\left(S^{-1}\right)_{\alpha}{}^{\beta}\left(x\right). \tag{49}$$
We observe that
$$\mathrm{s}\det U=\frac{\det M}{\det S^{-1}}=1. \tag{50}$$

LAGRANGIAN AND EQUATIONS OF MOTION

An invariant Lagrangian density can be constructed using a multiplet of Lagrange multipliers

$$\mathcal{L}=\frac{1}{2}\epsilon^{\mu\nu}\eta_{A}F_{\mu\nu}^{A}\left(x\right)=\frac{1}{2}\epsilon^{\mu\nu}\left(\eta_{a}f_{\mu\nu}^{a}\left(x\right)+\eta_{2}r_{\mu\nu}\left(x\right)+\eta_{3}v_{\mu\nu}\left(x\right)+\eta_{\alpha}\xi_{\mu\nu}^{\alpha}\left(x\right)\right), \tag{51}$$
$$\eta_{A}=\left(\eta_{a},\eta_{2},\eta_{3},\eta_{\alpha}\right),$$

which obey the coadjoint transformation law $\bar{\eta}_{A}=\eta_{B}U^{B}{}_{A}$, or in manifest form

$$\bar{\eta}_{a}=(\eta_{b}+\eta_{3}s_{b}\left(x\right))M^{b}{}_{a}\left(x\right)-\eta_{3}s_{a}\left(x\right), \tag{52}$$
$$\bar{\eta}_{2}=\eta_{2}+(\eta_{a}+\eta_{3}s_{a}\left(x\right))L^{a}{}_{b}\left(x\right)s^{b}\left(x\right)-\rho^{\alpha}\left(x\right)R_{\alpha}{}^{\beta}\left(x\right)\left(\eta_{\beta}-2q\eta_{3}\rho_{\beta}\left(x\right)\right), \tag{53}$$
$$\bar{\eta}_{3}=\eta_{3}, \tag{54}$$
$$\bar{\eta}_{\alpha}=\left(S^{-1}\right)_{\alpha}{}^{\beta}\left(x\right)\left(\eta_{\beta}-2q\eta_{3}\rho_{\beta}\left(x\right)\right)+2q\eta_{3}\rho_{\alpha}\left(x\right). \tag{55}$$

The corresponding equations of motion are

$$F_{\mu\nu}^{A}\left(x\right)=0, \tag{56}$$
$$\partial_{\mu}\eta^{a}=\left(e_{\mu}^{b}\left(x\right)\eta_{3}+\omega_{\mu}\left(x\right)\eta^{b}\right)\epsilon_{b}{}^{\ a}, \tag{57}$$
$$\partial_{\mu}\eta_{2}=e_{\mu}^{a}\left(x\right)\eta^{b}\epsilon_{ab}-\frac{1}{2}\psi_{\mu}^{\beta}\left(x\right)\left(\gamma_{5}\right)_{\beta}{}^{\ \alpha}\eta_{\alpha}, \tag{58}$$
$$\partial_{\mu}\eta_{3}=0, \tag{59}$$
$$\partial_{\mu}\eta^{\alpha}=\left(\frac{1}{2}\omega_{\mu}\left(x\right)\eta^{\beta}-q\psi_{\mu}^{\beta}\left(x\right)\eta_{3}\right)\left(\gamma_{5}\right)_{\beta}{}^{\ \alpha}. \tag{60}$$

The fermionic generalization of the centrally extended Poincaré algebra (1)–(4),(8)-(11) in the coadjoint representation possesses a nonsingular invariant graded metric

$$h_{AB}=(-1)^{p_{B}(p_{A}+p_{C})}h_{CD}U^{D}{}_{B}U^{C}{}_{A}, \tag{61}$$

which has the form

$$h_{AB}=(-1)^{p_{A}p_{B}}h_{BA}=\left(\begin{array}{cccc} h_{ab} & 0 & 0 & 0\\ 0 & u & -1 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & -2qC_{\alpha\beta} \end{array}\right), \tag{62}$$
$$\mathrm{s}\det h_{AB}=-\frac{\det h_{ab}}{4q^{2}\det C}=\frac{1}{4q^{2}}, \tag{63}$$

where $p_{A}\equiv p(A)$ is a Grassmann parity of the quantity A and u is an arbitrary constant. The inverse metric h^{AB}

$$h^{AB}h_{BC}=\delta_{C}{}^{A} \tag{64}$$

having the property $h^{AB}=(-1)^{p_{A}+p_{B}+p_{A}p_{B}}h^{BA}$ can be used to upper the indices $\eta^{A}=h^{AB}\eta_{B}$.

In the compact notation $X_{A}=\{P_{a},J,Y,Q_{\alpha}\}$ the full algebra (1)–(4),(8)-(11) can be presented as

$$[X_{A},X_{B}\}\equiv X_{A}X_{B}-(-1)^{p_{A}p_{B}}X_{B}X_{A}=f_{AB}^{\ \ C}X_{C}, \tag{65}$$

where ${f_{AB}}^C$ are the structure constants, and nonvanishing ones have the form

$$ {f_{ab}}^Y = \epsilon_{ab}, \quad {f_{aJ}}^b = {\epsilon_a}^b, \quad {f_{\alpha\beta}}^Y = -q\left(\gamma_5 C\right)_{\alpha\beta}, \quad {f_{\alpha J}}^\beta = \frac{1}{2}{\left(\gamma_5\right)_\alpha}^\beta . \tag{66} $$

In terms of $\eta = \eta^A X_A$ the equations of motion (57)–(60) can be written as

$$ \mathrm{d}\eta + [\mathrm{A}, \eta] = 0, \tag{67} $$

where $\mathrm{A} = \mathrm{A}^B X_B$. The invariant quantity

$$ M = -\frac{1}{2\Lambda}\eta_A h^{AB}\eta_B \tag{68} $$

can be interpreted as the black-hole mass [7], if it is constant (Λ is from (5)). Indeed, from the equations of motion (67) we have

$$ \mathrm{d}M = -\frac{1}{\Lambda}\eta_A \mathrm{d}\eta^A = \frac{1}{\Lambda}\eta_A\eta^C \mathrm{A}^B f_{BC}^{\ \ A}. \tag{69} $$

As a consequence of the structure constant properties we obtain $\eta_A\eta^C\mathrm{A}^B f_{BC}^{\ \ A} = 0$, and therefore $M = const$.

Using the inverse metric h^{AB}, the set of independent Casimir operators of the algebra (65) can be presented as

$$ K\left(u\right) = X_A h^{AB} X_B = K - uY^2, \tag{70} $$

which is marked by the additional parameter u from (62).

CONCLUSION

Thus, we have constructed a special fermionic generalization of the lineal gravity which is not the standard supersymmetric two-dimensional gravity [14]. We presented the algebra of generators, the field transformations and found Lagrangian and equation of motion. We obtained the constant black hole mass and derived the Casimir operator which depends on an additional parameter.

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О ФЕРМИОННОМ ОБОБЩЕНИИ ЛИНЕЙНОЙ ГРАВИТАЦИИ В ФОРМУЛИРОВКЕ С ЦЕНТРАЛЬНЫМ РАСШИРЕНИЕМ

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В работе обобщается центральное расширение алгебры Пуанкаре в $(1+1)$ измерениях путем включения фермионных зарядов, которые удовлетворяют не стандартной суперсимметричной алгебре, а специальной градуированной алгебре, которая в правой части содержит только центральные заряды. Проверена самосогласованность тождеств Якоби и получен оператор Казимира. Далее вводятся соответствующие калибровочные поля и построена классическая калибровочная теория, основанная на данной градуированной алгебре, представлены преобразования полей и получена масса черной дыры в $(1+1)$ измерениях.

КЛЮЧЕВЫЕ СЛОВА: алгебра Пуанкаре, линейная гравитация, центральное расширение, фермионный генератор, оператор Казимира