TOWARDS GAUGE PRINCIPLE FOR SEMIGROUPS

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The standard gauging procedure is extended to the (super)semigroups consisting of a disjoint subgroup being the ordinary group of internal symmetry and an ideal containing noninvertible transformations arising in supersymmetric theories. The analogue of the conjugation is brought in and used when deriving the nondiagonal gauge-like transformations which are proper for the nondiagonal covariance proposed. The latter is the origin of a new unavoidable field lying in the same representation as the usual gauge field and giving rise to the effect like torsion. The operator bilinear in these fields, which generalizes the covariant derivative in the adjoined representation, is given. The possible form of the Lagrangian and equation of motion is outlined.

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1. Introduction

Many modern physical models are based on the gauge principle which introduces field interactions by localizing symmetries of a theory. This procedure, called the gauging of a corresponding group, allows one to build (internal) gauge theories of strong, weak and electromagnetic interactions [1], where gauge transformations act on internal degrees of freedom, and (external) gauge theories of gravity [2], where gauge transformations are regarded as coordinate changes. Also the string theories can be identified as a gauge theory of the conformal group in two dimensions [3]. When supersymmetry comes into play [4], groups are replaced by supergroups [5], and all variables of the theory are given the same prefix without paying attention to difficulties [6] and new key features arising. One of the latter is the fact that uncommon noninvertible and degenerate transformations come into existence. They form the ideal of the supersemigroup which is the result of extending the supergroup formed by the invertible transformations and restricted properly (see [5] for details). In evident cases the implied "super" will be omitted as a rule. The semigroup of superconformal transformations playing an essential role in the superstring investigations was introduced in [7], where explicit examples of noninvertible transformations were given.

In this paper we extend the gauging procedure to semigroups (for the background of their algebraic and topological properties see [8]). Our rather schematic account is simplified intentionally, and only the main points are indicated. By employing [9] the geometric interpretation of the constructions presented can be provided. For short we do not include fermionic variables and other immaterial complications, which do not change the principal statement: it is natural and consistent from the algebraic viewpoint to make "super" and "semi" generalizations of a physical theory, simultaneously.

2. Semiconjugation

To begin with we define objects which will be employed below. Let S be a semigroup [8] with a disjoint subgroup G, which means that $S = G \cup I$ and $G \cap I = \phi$, where I is the ideal of S, *i.e.* $I = \{i \in S : \; is \in I, \; s \in S\}$. In physical language S can be treated as the semigroup of transformations acting on fields defined in a superspace [4]. Invertible transformations lie in the subgroup G which is some generally accepted Lie group, and noninvertible ones are in the ideal I. The latter can contain the subsemigroup of the nilpotent transformations [10] defined by $N_p = \{n \in I; \; n^p = z\}$, where p is a degree of nilpotency and z is the two-sided zero: $zs = sz = z, \; s \in S$. As it is well-known [5, 11] the basic mapping defined for groups is the conjugation $A_g[h] = ghg^{-1}$, where $g, h \in G$. Now we introduce the nondiagonal mapping

$$B_{sg}[h] = shg^{-1}, \quad s \in S, \quad g \in G \tag{1}$$

which can be named a "semiconjugation". It is determined by two distinct elements s and g and has the following properties: $B_{gg}[h] = A_g[h]$ and $B_{ig}[h] \in I, \; i \in I$, so that the last noninvertible mapping is a new one, as a matter of fact. Certainly, the relation (1) can be commonly interpreted as the action of the direct product $S \times G$ by the left and right shifts [12], but we will treat it as a unified mapping defined for any semigroup and study its properties useful for applications, as seen below. So, there is the dual semiconjugation $\overline{B}_{sg}[h] = g^{-1}hs$ which cannot be expressed through the semiconjugation (1), since in general case the element s should not be invertible (*cf.* $\bar{A}_g[h] = g^{-1}hg = A_{g^{-1}}[h]$). Further the set of the elements from (1) for h fixed can be called a semiconjugation class of h (*cf.* the double conjugation class for groups [12]). Another useful object is a "nondiagonal unit" $e_{sg} = sg^{-1}$ (the dual one is $\bar{e}_{sg} = g^{-1}s$) having the properties $e_{gg} = \bar{e}_{gg} = e$, where e is the ordinary unit of S: $es = se = s, \; s \in S$, and $e_{ig} \in I$, $\bar{e}_{ig} \in I$. It is seen that $\bar{e}_{sg}$ (e_{sg}) is the right (left) inverse element for g with respect to s, since $s = g\bar{e}_{sg} = e_{sg}g$, and they are connected by means of the conjugation as follows: $\bar{e}_{sg} = \bar{A}_g[e_{sg}]$ and $e_{sg} = A_g[\bar{e}_{sg}]$. An intriguing property of the semiconjugation is to turn the ordinary unit into the nondiagonal one:

$$B_{sg}[e] = e_{sg}\,, \tag{2}$$

$$\overline{B}_{sg}[e] = \bar{e}_{sg}\,, \tag{3}$$

while, for the conjugation, the unit is the fixed point: $A_g[e] = e$. Then the multiplication table for the nondiagonal units takes the form

$$e_{s'g}\bar{e}_{sg} = s's\,, \tag{4}$$

$$\bar{e}_{s'g}e_{sg} = \bar{A}_g[s's]\,. \tag{5}$$

Furthermore, the elements s and g lie in the same left (right) "semicoset" in case they form the fixed nondiagonal unit, *i.e.* $e_{sg} = h(\bar{e}_{sg} = h)$, and an invariant subsemigroup H can be defined as the set of the elements satisfying $B_{sg}[h] \in H$ for all $h \in H \subset S$, $s \in S$ and $g \in G$. Hereafter, we concentrate our attention on the inner semiconjugation (1) for which $h \in S$.

It is very uncommon that the semiconjugation (1) is not a homomorphism, since it does not preserve the multiplication

$$B_{sg}[h'h] = B_{sg}[h']\,A_g[h]\,. \tag{6}$$

But the exciting relation

$$B_{sg}[h']\,B_{sg}[h] = B_{sg}[h' \otimes h] \tag{7}$$

holds, where

$$h' \otimes h = h'\bar{e}_{sg}h \tag{8}$$

is a "sg-product" used below. Consequently, the relations (1) and (7) can be treated as the definition of a mapping more general than a homomorphism. Its kernel is $\ker B_{sg} = \{h \in S : B_{sg}[h] = e_{sg}\}$ and image is $\operatorname{Im} B_{sg} = \{B_{sg}[h] : h \in S\}$. When considering the set of the elements $Y_{sg} = \operatorname{Im} B_{sg}$ with s and g fixed we observe that the multiplication in Y_{sg} given by (7) is well-defined (since $h' \otimes h \in S$, then $B_{sg}[h' \otimes h] \in Y_{sg}$) and its associative holds true evidently, and so Y_{sg} is the semigroup which can be called a nondiagonal "sg-semigroup". For an invertible s the ideal of Y_{sg} is $I_{sg} = \{B_{sg}[i] : i \in I\}$, while if $s = i \in I$, then $Y_{sg} = I_{sg}$, that is Y_{ig} does not contain any subgroup. For some $y \in Y_{sg}$ one can define the "sg-inverse" element $y' \in Y_{sg}$ by $y'y = e_{sg}$, which requires their prototypes to be mutually inverse in the sense of the sg-product $h' \otimes h = e$. For the dual semiconjugation it is also possible to define the dual "sg-semigroup" by $\overline{\mathrm{Y}}_{sg} = \mathrm{Im}\,\overline{\mathrm{B}}_{sg}$ and study it in the same fashion. It is worthwhile to observe the connection between them in case of the nilpotent transformations s=n, $n^2 = z$, *viz.* $\bar{y}y = z$, $y \in \mathrm{Y}_{sg}$, $\bar{y} \in \overline{\mathrm{Y}}_{sg}$ (there is no dependence on the prototypes), which means that all elements from $\overline{\mathrm{Y}}_{ng}$ are left divisors of zero for those from Y_{ng}.

3. Semicovariance and gauge fields

Now let us proceed to the action of the semigroup S on a matter field $\Phi(x)$ defined in some superspace X, $x_\mu \in X$, $\mu = 1, \dots, d$. Its dimension d and the signature are immaterial in our context, also fermionic coordinates are not marked out. In other words the elements of S are the transformations of the algebra of the functions $\Phi(x)$ (see [13, 14] for more details). We imply that the fields under consideration are realized in some way, but a definite representation for S will not be used here (*e.g.* for the matrix representation R the semiconjugation can give rise to the "semiadjoint" representation, that is $\mathrm{Bd}_{sg}[M] = \mathrm{s}M\mathrm{g}^{-1}$, $M \in \mathrm{R}$, nevertheless we will use the notion (1) also for the action on fields). Thus we have

$$\Phi'(x) = \mathrm{s}_1\Phi(x)\,, \quad \mathrm{s}_1 \in \mathrm{S}\,. \tag{9}$$

By the conventional definition [9, 11] the covariant derivative D_μ is a first-order differential operator acting on the field $\Phi(x)$ so that the result of this action should transform as the field itself (9). We slightly change this definition (the reason will be clear below) by requiring the "nondiagonal covariance"

$$D'_\mu\Phi'(x) = \mathrm{s}_2 D_\mu\Phi(x)\,, \quad \mathrm{s}_2 \in \mathrm{S}\,, \tag{10}$$

where s_2 is not necessarily equal to s_1. For such D_μ the following ansatz

$$D_\mu = a(x)\partial_\mu + qb_\mu(x) \tag{11}$$

is natural, where $a(x)$ and $b_\mu(x)$ are some functions lying in the same representation of S, and q is the charge of $b_\mu(x)$ which is physically treated as the gauge field ($\partial_\mu \equiv \partial/\partial x_\mu$). The operator D_μ (11) can be viewed as the nondiagonal generalization of the ordinary covariant derivative [9].
From (9)–(11) we derive the system of equations

$$a'(x)\mathrm{s}_1 = \mathrm{s}_2 a(x)\,, \tag{12}$$

$$qb'_\mu(x)\mathrm{s}_1 + a'(x)\cdot\partial_\mu\mathrm{s}_1 = q\mathrm{s}_2 b_\mu(x)\,. \tag{13}$$

In case $s_1 = s_2 = g \in G$ this system can be solved as usual [1, 11]

$$a'(x) = A_g[a(x)], \tag{14}$$

$$b'_\mu(x) = A_g\left[b_\mu(x) - \frac{1}{q}a(x)\cdot C_\mu\right], \tag{15}$$

where $C_\mu = g^{-1}\partial_\mu g$. As a rule, $a(x) = 1$ (or the unit element of a chosen representation), which is clear here from the fact that $A_g[1] = 1$, and so the constant $a(x)$ solves (14) and can be dropped out by normalizing (of course, there are the rigorous explanations [9, 14]).

Then (15) becomes the standard gauge transformation and $D^{st}_\mu = 1\cdot\partial_\mu + q\cdot b_\mu(x)$. When trying to solve the system (12)–(13) out of group G we clash with the fact that the elements from the ideal I are noninvertible transformations. Although in the case $s_1 = s_2 = i \in I$ we put $a(x) = 1$ again, the Eq. (13) cannot be solved with respect to $b'_\mu(x)$ in general, hence additional assumptions and an explicit shape of the transformations are needed. Consequently, the only way to solve the system (12)–(13) out of G is to employ the nondiagonal choice $s_1 = g \in G$ and $s_2 = s \in S$ (even though the new case $s_2 = i \in I$ is interesting, the pure group choice $s_{1,2} = g_{1,2} \in G$, $g_1 \neq g_2$, is also worthy of note). Then we obtain the nondiagonal gauge transformations

$$a'(x) = B_{sg}[a(x)], \tag{16}$$

$$b'_\mu(x) = B_{sg}\left[b_\mu(x) - \frac{1}{q}a(x)C_\mu\right], \tag{17}$$

where B_{sg} is the semiconjugation (1), which clears why it was brought in. When comparing (14)–(15) with (16)–(17), at their face value, one may conclude that only the formal substitution of B_{sg} for A_g has been made. Nevertheless, this results in the intriguing fact that the introducing of $a(x)$ other then the commonly used unit element of the representation of S cannot be kept off, since (2) is the case. Recall that $a(x)$ is charged due to (16) and it is not the conventional *vierbein* of the curved superspace [4] (actually, the factor before the derivative in (11) can be written as $a_{\mu\nu}(x)$ in general, but we confine ourselves to the diagonal case $a_{\mu\nu}(x) = a(x)\delta_{\mu\nu}$ for simplicity). Therefore, $a(x)$ could be interpreted as an additional and *unavoidable* field accompanying the usual gauge field $b_\mu(x)$ and lying in the same representation of S, in case the "nondiagonal covariance" is under consideration.

Further, to find an analogue of the field strength $F_{\mu\nu}$ one has to commute the "nondiagonal derivatives" (11) in the standard way [1]

$$[D_\mu, D_\nu] = qF_{\mu\nu} + T_{\mu\nu}, \tag{18}$$

where

$$F_{\mu\nu} = a(x)\,(\partial_\mu b_\nu(x) - \partial_\nu b_\mu(x)) + q[b_\mu(x),\, b_\nu(x)]\,, \tag{19}$$

$$T_{\mu\nu} = T_\mu \partial_\nu - T_\nu \partial_\mu \tag{20}$$

and

$$T_\mu = \overline{D}_\mu a(x)\,. \tag{21}$$

Here

$$\overline{D}_\mu = a(x)\partial_\mu + q[b_\mu(x),\quad] \tag{22}$$

(evidently, $a(x)$ and $b_\mu(x)$ do not commute), and T_μ indicates an effect like torsion [15]. However, $a(x)$ is not invertible sometimes, and so the division by it cannot be applied in expressing ∂_μ through D_μ in (20), we imply this, while for the invertible $a(x)$ the standard torsion $T_\mu^{st} = a(x)^{-1}\overline{D}_\mu a(x)$ and field strength $F_{\mu\nu}^{st} = F_{\mu\nu} - T_\mu^{st} b_\nu(x) + T_\nu^{st} b_\mu(x)$ can be derived [9, 15]. The relation between our $F_{\mu\nu}$ and T_μ is

$$\begin{aligned} \overline{D}_\mu T_\nu - \overline{D}_\nu T_\mu &= [\overline{D}_\mu,\, \overline{D}_\nu] a(x) \\ &= q[F_{\mu\nu},\, a(x)] + T_{\mu\nu} a(x)\,, \end{aligned} \tag{23}$$

and the analogue of the Bianchi identity takes the form

$$\overline{D}_\mu F_{\mu\rho} + \text{permutations} = T_{\mu\nu} b_\rho(x) + \text{permutations}\,, \tag{24}$$

where the r.h.s. of Eq. (24) cannot be expressed usually through $F_{\mu\nu}$ for the same reason. The special condition for $a(x)$,

$$T_\mu = \overline{D}_\mu a(x) = 0\,, \tag{25}$$

can be called a "torsionless gauge". Using it and (17) we obtain a "pure gauge" in the following way

$$\partial_\mu a(x) = [a(x),\, C_\mu]\,, \tag{26}$$

$$b_\mu(x) = \frac{1}{q} a(x) C_\mu \tag{27}$$

which results in $[D_\mu,\, D_\nu] = 0$.

The consideration of $F_{\mu\nu}$ and $T_{\mu\nu}$ variations under (16)–(17) requires the analogue of the nondiagonal gauge covariance for quantities which are bilinear in the fields. Thinking of (7) and (16) we name W a gauge "semi-covariant" quantity iff it transforms under the nondiagonal gauge transformations as

$$W' = \mathrm{B_{sg}}[W^{\otimes}]\,, \tag{28}$$

where the superscript $\otimes$ means that all multiplications between the fields inside W are replaced by (8) and $\partial_\mu \to \partial_\mu(\bar{e}_{sg})$. Observe that for W being linear in the fields one has $W^{\otimes} = W$ (see *e.g.* (16) and the semicovariance of the operator (11): $D'_\mu = \mathrm{B}_{sg}[D_\mu]$).

Proceeding to $F_{\mu\nu}$ and $T_{\mu\nu}$ we see that they are not semicovariant separately, since

$$F'_{\mu\nu} = \mathrm{B}_{sg}\left[F^{\otimes}_{\mu\nu} - \frac{1}{q}Q_{\mu\nu}\right] \qquad (29)$$

and

$$T'_{\mu\nu} = \mathrm{B}_{sg}[T^{\otimes}_{\mu\nu} + Q_{\mu\nu}], \qquad (30)$$

where

$$Q_{\mu\nu} = \overline{D}^{\otimes}_{\mu} a(x) \cdot C_\nu - \overline{D}^{\otimes}_{\nu} a(x) \cdot C_\mu . \qquad (31)$$

and $T_{\mu\nu}$ acts on a matter field transformed as in (9). Nevertheless, the combination (18) is semicovariant as usual. It is significant that the "derivative" (22) of the semicovariant quantity (28) is not semicovariant in general

$$\overline{D}'_\mu W' = \mathrm{B}_{sg}\left[D^{\otimes}_\mu W^{\otimes} + [W^{\otimes}, a(x)]^{\otimes} \cdot C_\mu\right], \qquad (32)$$

(*cf.* (22)), where $[\mathrm{A}, \mathrm{B}]^{\otimes} = \mathrm{A} \otimes \mathrm{B} - \mathrm{B} \otimes \mathrm{A}$ is a "sg-comutator" (see (8)). The condition of vanishing of the last term in (32), that is $[W^{\otimes}, a(x)]^{\otimes} = 0$, is a significant constraint imposed on W. As, for instance, in case $W = a(x)$ this relation holds, hence $T_\mu = \overline{D}_\mu a(x)$ is semicovariant as such, also $W = a^m(x)$ satisfies it. In searching for the semicovariant differentiation of a semicovariant unconstrained quantity we face the difficulty that there is no such operator required which would be linear in the fields. However, we can construct its generalization $\overline{\overline{D}}_\mu$ which is semicovariant for any W and *bilinear* in the fields as follows

$$\overline{\overline{D}}_\mu W = a(x) \cdot \partial_\mu\big(W \cdot a(x)\big) + q\big(F_\mu(x) \cdot W \cdot a(x) - a(x) \cdot W \cdot F_\mu(x)\big), \qquad (33)$$

where the fact that the field $a(x)$ has no space-time indices is crucial. Note that in the "pure gauge" (26)–(27) one can derive the following connection of $\overline{\overline{D}}_\mu$ with the standard covariant derivative

$$\overline{\overline{D}}_\mu W = a(x) \cdot D^{st}_\mu W \cdot a(x), \qquad (34)$$

which clarifies its notation. Consider some properties of $\overline{\overline{D}}_\mu$. Its action on a product of semicovariant objects is

$$\overline{\overline{D}}_\mu(W \cdot V) = \overline{\overline{D}}_\mu W \cdot V + a(x) \cdot W \cdot \overline{D}_\mu V + D^{W}_\mu [V, a(x)], \qquad (35)$$

where

$$D_\mu^W = W \cdot \overline{D}_\mu + \overline{D}_\mu W + [a(x), W], \tag{36}$$

which should be compared with

$$\overline{D}_\mu(W \cdot V) = \overline{D}_\mu W \cdot V + W \cdot \overline{D}_\mu V + [a(x), W]. \tag{37}$$

In particular,

$$D_\mu^a V = a(x) \cdot \overline{D}_\mu V + T_\mu \cdot V \tag{38}$$

and

$$\overline{\overline{D}}_\mu(W \cdot a(x)) = \overline{\overline{D}}_\mu W \cdot a(x) + a(x) \cdot W \cdot T_\mu . \tag{39}$$

Now the analogue of the "torsionless gauge" (25) is

$$\overline{\overline{D}}_\mu a(x) = 0 \tag{40}$$

which gives together with (27) the generalized "pure gauge"

$$\partial_\mu a^2(x) = \left[a^2(x), C_\mu\right] . \tag{41}$$

Furthermore, the remarkable relation for $\overline{\overline{D}}$ is

$$\overline{\overline{D}}_\mu a(x) = \overline{D}_\mu a^2(x) . \tag{42}$$

It follows that the generalized "torsionless gauge" condition (40) and the generalized "pure gauge" one (41) have the topologically disjoint solution $a^2(x) = 0$, which means that the field $a(x)$ is nilpotent of second degree. Such fields arise due to the existence of a set of nilpotent transformations $\mathrm{N}_p \subset \mathrm{I} \subset \mathrm{S}$ (see Section 2). The general properties of the semigroups of nilpotent transformations are given in Refs [10, 16]. As to their representations one can pay attention to the nilpotent algebras representations [17] which are recently used in studying the non-compact supersymmetric σ-models [18]. Obviously, the nilpotent $a(x)$ is noninvertible, and so the standard torsion and field strength cannot be determined (see the text following Eq. (22)), but our definitions (19) and (21) remain valid.

Further, since T_μ is semicovariant (see (32) and below), one can apply the operator (33) to it and obtain

$$\overline{\overline{D}}_\mu T_\nu - \overline{\overline{D}}_\nu T_\mu = q[F_{\mu\nu}, a^2(x)] + [T_{\mu\nu} a(x), a(x)] - q[U_{\mu\nu}, a(x)], \tag{43}$$

where $U_{\mu\nu} = T_\mu b_\nu(x) - T_\nu b_\mu(x)$. The generalized analogue of the Bianchi identity is (*cf.* (24))

$$\overline{\overline{D}}_\mu F_{\nu\rho} + \text{permutations} = q[F_{\mu\nu} b_\rho(x), a(x)] \\ + a(x) \cdot F_{\mu\nu} \partial_\rho a(x) + T_{\mu\nu} b_\rho(x) \cdot a(x) + \text{permutations} . \tag{44}$$

It follows from (43) that the combination

$$L_{\mu\nu} = F_{\mu\nu}a(x) + \frac{1}{q}T_{\mu\nu}a(x) - U_{\mu\nu} \tag{45}$$

is semicovariant by construction. To obtain a Lorentz invariant object one should square it

$$L = \tfrac{1}{4}L^2_{\mu\nu}\,, \tag{46}$$

which can be interpreted as the semicovariant analogue of the gauge fields Lagrangian, after the use of some suitable representation and the nondiagonal version of a scalar product in it. Alternatively, one can treat $\overline{\overline{D}}_\mu L_{\mu\nu} = 0$ as the semicovariant "equation of motion" for the theory.

4. Conclusions

To summarize, the nondiagonal analogue of the conjugation, that is the semiconjugation, is introduced and studied as a single whole. Certainly, that could be interpreted as the nonsymmetrical action I × G (or G × G), where G and I is the subgroup and the ideal of the semigroup S of gauge-type transformations, respectively. Analogous invertible nonsymmetrical actions were considered when studying, *e.g.*, the Higgs field transformations in the topological supergravity [19], the principal chiral field transformations in nonabelian superstrings, where the nonsymmetry results in some new current-like objects [20], and the bicanonical transformations for non-Hermitian quantum systems [21] (it is also worthwhile to note Refs [22, 23]). In our case the nonsymmetrical action is called for the nondiagonal covariance (or semicovariance) to hold, which is a possible way for inclusion of noninvertible gauge-like transformations into the theory. As a result, a new field $a(x)$ should be introduced together with the standard gauge field $b_\mu(x)$. It belongs to the same representation as $b_\mu(x)$ and has some properties of the *vierbein* [15], therefore methods of the theories with a nonvanishing torsion [15, 24] or the dynamical one [25] could be used here.

Further possible applications of the above approach can be found in various supersymmetric extensions of gauge theories.

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