Non-linear analysis of chaotic oscillations observed in DC glow discharge Plasma

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## Contents

1 Introduction ............................................. 2

2 Waves in Plasmas ........................................ 3

3 Description of Experimental Set up ...................... 3
   3.1 Plasma producing part ............................... 3
   3.2 Data acquisition system .................................. 3

4 Some Nonlinear time series analysis techniques[++] .......... 4
   4.1 Phase Space .......................................... 5
   4.2 Fourier Spectrum ..................................... 5
   4.3 Correlation dimension ................................ 5
   4.4 Largest Lyapunov exponent ........................... 6
   4.5 Rescaled range analysis ............................. 7
   4.6 0-1 test for chaos .................................. 7

5 Results and analysis ..................................... 8
   5.1 Paschen curve ........................................ 8
   5.2 Langmuir probe I-V characteristic ..................... 9
   5.3 Floating potential observations −− > ........................ 10
       5.3.1 For varying discharge voltages ....................... 10
       5.3.2 For varying magnetic field ........................ 12

6 Conclusion and further works .......................... 15

7 Acknowledgement ....................................... 16

8 References .............................................. 16
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Abstract
In this paper first of all DC glow discharge plasma is characterized by finding out plasma parameters. This involves study of Paschen curve and Langmuir probe characteristic. After that floating potential fluctuations are obtained by using Langmuir probe at different values of pressure of gas, external magnetic field and discharge voltage. The analysis of these nonlinear oscillations involves nonlinear time series analysis. Analysis is carried out by several techniques like determining correlation dimensional analysis, Fourier spectrum analysis, rescaled range analysis (for Hurst exponent), determining Largest Lyapunov exponent (by Rosenstein method), 0-1 test for chaos etc.

1 Introduction
Plasma is a quasi neutral assembly of charged particles (ions and electrons) and neutrals which has the property of collective behavior. Quasi neutral means number of electrons per unit volume are same as number of ions per unit volume\((n_i \approx n_e)\). Collective behavior leads to motions that not only depend on local conditions but also on the state at farther position. More precisely for an ionized gas to qualify as a plasma three criteria should be fulfilled:-

- Debye length\(^1\) \((\lambda_D)\ll\) system length\((L)\) (for quasi neutrality)
- \(N_D \gg 1\) where \(N_D\) Number of electrons per Debye sphere (for collective behavior)
- \(\omega\tau > 1\) where \(\omega\) is frequency of plasma oscillations and \(\tau\) is mean time between collisions with neutral atoms. (for plasma confinement)

Properties of plasma is different from three states of matter like solid, liquid and gas, that is why it is also called the fourth state of matter. Plasma consists of ionized gas and electrons which provides very large conductivity. Plasma is the most frequently observed state of matter. DC glow discharge is one of the easiest way to produce plasma at low pressure of gas. In nature we can observe formation of plasma at different places due to potential difference. DC glow discharge plasma has a large applications as in tube lights, spectral tubes etc. DC glow discharge system has already been studied by several researchers earlier also. While pursuing plasma physics we often come across several kinds of plasma instabilities and some unwanted fluctuations due to the non-linear behavior. Generally we neglect those fluctuations of tune our parameters to some different parameters to get rid of these instabilities. Here in this experiment we are trying to have a close look at those fluctuations by using nonlinear time series analysis. Till now in most of the papers non-linear studies of DC glow discharge\(^{[18,19,20]}\) has been reported but not much analysis was carried out in presence of external magnetic field. Here we are introducing external magnetic by using copper coils which leads to interesting nonlinear behavior.

\(^1\)Debye length is the radius of sphere outside of which charges are screened.
Fluctuations in floating potential \(^2\) have been analyzed by using nonlinear time series analysis. This analysis can help us to understand instabilities in the linear plasma used in space for giving thrust to satellite.

## 2 Waves in Plasmas

In a plasma electric field and magnetic field are determined by position and motion of charges. A large assembly of charges leads to complicated problem in which we have to determine particle trajectories and field patterns in such a way that the fields produced by them keeps charges in their exact orbits. This assembly of charges leads to certain oscillations and turbulent behavior in plasma leading to various kinds of instabilities which opens the door of non-linear dynamics. In unmagnetized plasma two types of waves are observed *electron plasma waves* and *ion acoustic waves*. While in magnetized plasma in addition to those waves *electron cyclotron waves* are also observed which behaves differently in the direction parallel and perpendicular to magnetic field. Frequency of electron plasma waves is known as plasma frequency. It can be determined by \(^3\) –

\[
\frac{f_p}{2\pi} = \frac{1}{2\pi} \left( \frac{n_0 e^2}{\epsilon_0 m_e} \right)^{1/2} \approx 9\sqrt{n_0} \approx 0.9 \text{GHz}
\]

For this experimental set up magnetic field is in the range of 25 -50 gauss. For that field electron cyclotron frequency is given by :-

\[
\frac{f_c}{2\pi m} = \frac{eB}{2\pi m} \approx 0.53 \text{GHz}
\]

## 3 Description of Experimental Set up

### 3.1 Plasma producing part

- **Electrode system** – Anode is a solid cylindrical rode made of stainless steel which is grounded. Cathode is a disk shaped sheet made of stainless steel and insulated from outside by a nylon cylinder. Cathode is biased to provide discharge voltage.

- **Argon Gas supply, DC power supply**

- **Pirani gauge** - For pressure measurement

- **Rotary pump** - To create vacuum

- **Vacuum chamber**

- **A 10 Kohm resistance connected to anode in order to reduce current in the system so that the power supply may not get damaged.**

- **Copper coils (wrapped over one of the arm)** - For generating magnetic field.

### 3.2 Data acquisition system

- **Langmuir probe** – Langmuir probe is a conductor introduced in plasma system to measure plasma density and fluctuations in floating potential. Here we are using of a cylindrical tungsten wire of diameter 0.5 mm of length \(\approx 20\) mm for probe. A teflon coated stainless steel wire has been soldered to the tungsten wire, and it has been fitted inside a glass tube. The part of wire which is exposed to fluctuation is nearly 2 mm in length. When we insert the probe inside the plasma

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\(^2\)Floating potential is the potential created in Langmuir probe due to fluctuations in plasma (when probe is not biased to some potential)

\(^3\)Under certain assumptions given in Ref. “Introduction to plasma physics and controlled fusion” F.F. Chen Chap. 4 Sec. 4.3
electrons and ions fall on it as electrons have higher mobility than ions this leads to accumulation of electrons on the surface of probe to set up an electric field and potential known as floating potential.

4 Some Nonlinear time series analysis techniques

Time series is the collection of some state variables at some intervals of time keeping a few parameters fixed. In this experiment floating potential is the states variable and its fluctuations are taken at fixed sampling time. Large sampling time is easier to analyze in comparison to small sampling time, however very large sampling time leads to irrelevant data. Analysis of time series involves —

- Determining trend in data
- Determining correlation in data sets

For all kinds nonlinear time series (presented here) I developed my scripts in MATLAB
• Testing for chaoticity of data sets
• Determining periodicity of data

Some techniques are described here:-

4.1 Phase Space

For a time series phase space is the space created by state variable and its derivative at this point. In phase space information of time is lost. For the time series of floating potential its derivative is plotted. As original signal is disturbed by noise, therefore a low pass filter of proper cutoff frequency is used to plot phase spaces. (In fig. 8, 9, 11, 12)

4.2 Fourier Spectrum

Fourier spectrum transforms time series from time domain to frequency domain. In discrete fourier transform we construct time series by adding orthogonal sine and cosine functions. Correlation of each frequency of sine and cosine function to original signal gives us amplitude as a function of frequency. Mathematically it can be written as-

$$ F(\omega) = \frac{1}{N} \sum_{\tau=0}^{\tau=N-1} f(\tau) e^{-i2\pi\omega\tau/N} $$

Here interval of $\omega$ is normalized properly in order to relate frequency to time interval (in which data sets are taken). Here for fourier transform fast fourier transform algorithm is used which is more efficient than direct calculation of transform. (In fig. 8, 9, 11, 12)

4.3 Correlation dimension

Correlation dimension is measure of the complexity of system. A system involving more number of parameters is said to be more complex. For determination of correlation dimension Grassberger-Procaccia algorithm $^5$ is used. For determination of correlation dimension phase space is reconstructed by using concepts of embedding dimension and time lag. In original time series trajectories will be correlated (i.e. trajectory taking one of the point as starting point will disturb by the trajectories starting from some other points), that is why time lag is chosen in such a way that the autocorrelation function has at least one minima between two points. Autocorrelation function for lag $k$ is defined as-

$$ r_k = \frac{1}{N-k} \sum_{i=1}^{i=N-k} (x_i - \bar{x})(x_{i+k} - \bar{x}) $$

$$ \sum_{i=1}^{N} (x_i - \bar{x})^2 $$

It is shown in the figure(3). Embedding dimension indicates the complexity of system. Here in each case we are taking time lag as $(2 * \tau)$ in order to ensure minima of autocorrelation function$^6$.

After reconstructing phase space we determine correlation sum for different values of the disc radius taken from any point chosen as attractor. Then the fraction of points lying inside the radius is taken as correlation sum which is defined as-

$$ C(r) = \frac{2}{N(N-1)} \sum_{i<j}^{N} \theta(r - |X_i - X_j|) $$

$^5$More on phase space reconstruction refer reference 2,3,4.
And correlation dimension is defined as –

\[ d = \lim_{r \to 0} \frac{\log(C(r))}{\log(r)} \]

So correlation dimension is found out by taking the slope of log-log plot of \( C(r) \) and \( r \) in the increasing part of graph.

### 4.4 Largest Lyapunov exponent

Lyapunov exponent is the measure of chaoticity of system. For determining largest lyapunov exponent Rosenstein’s algorithm is used which is really powerful and reliable for small data sets. In reconstructed phase space the nearest neighbor of each point on the trajectory is located. The nearest neighbor, \( X_j \), is found out by looking for point which has minimum distance from reference point \( X_j \)

\[ d_j(0) = \min |X_j - X_j| \]

where \( d_j(0) \) is the initial distance from the \( j \)th point to its nearest neighbour. Similarly nearest neighbor is found out for other points also. Then after \( i \) iterations distance between reference point and nearest point is found out which is denoted by as \( d_j(i) \). According to sano sawada’s \([4]\) algorithm lyapunov exponent can be written as

\[ \lambda(i) = \frac{1}{i(M - i)\Delta t} \sum_{j=1}^{M-1} \ln \frac{d_j(i)}{d_j(0)} \]

Rosenstein \([2]\) gave a different way to find out largest lyapunov exponent which was by fining out the slope of \( \log < \text{divergence} > \) and iteration graph in increasing part. In this algorithm lyapunov exponent can be written as

\[ y(i) = \frac{\ln(d_j(i))}{i\Delta t} \]

This algorithm gives very accurate results for chaotic data but for periodic data it does not show saturation. It involves less computation as well.
4.5  Rescaled range analysis

Rescaled range analysis is a statistical method to analyze long term memory in natural phenomena. In this analysis method two factors are used one is range R which is difference between maximum and minimum cumulative sum of the time series and the other one is standard deviation S of the time series. First of all original times series is divided of length N is divided into partial series of length \( n = N/2, N/3, N/4, \ldots \) Then for each value of n we can calculate R/S ratio is calculated (Average is taken for series with same length). For large values of R/S follows power law–

\[
\frac{R(n)}{S(n)} = Cn^H
\]

as \( n \to \infty \). H is called hurst exponent. Value of H lies between 0 and 1. For random processes \( H = 0.5 \). If \( 0.5 < H < 1 \) time series shows long range autocorrelation which means if high value in series will be followed by another high value. If \( 0 < H < 0.5 \) then series show s long range anticorrelation which means it will be switching between high and low values as time will pass.

Figure 4: Plot for determining hurst exponent for at 596V signal at 0.4mB pressure and 46 gauss magnetic field, slope=0.5230

4.6 0-1 test for chaos

For testing chaoticity standard test is to check for exponential divergence by lyapunov exponent. But for a finite time series calculation of lyapunov exponent involves phase space reconstruction and other computation complications. 0-1 test for chaos works directly on time series and gives a way to check for divergence from initial conditions. This involves first constructing a phase space of two variables \( p(n) \) and \( q(n) \)–

\[
p_c(n) = \sum_{j=1}^{n} x(j)\cos(jc)
\]
Here $c$ is randomly chosen from interval $(0, \pi)$. Then mean square displacement $M(n)$ is calculated for different values of $n$ by:

$$M(n) = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} [(p_c(j + n) - p_c(j))^2 + (q_c(j + n) - q_c(j))^2]$$

Then slope of log($M(n)$) and log($n$) plot is calculated if it comes out to be 1 then the system is said to be chaotic and if comes out to be 0 it is non chaotic. However increasing trend of $M(n) v/s n$ is enough to test for chaos. Here it is done for the data in fig. 5.

Figure 5: For 0-1 test at $c=1$ $M(n)$ showing increasing trend

5 Results and analysis

5.1 Paschen curve

A gas in the atmosphere at room temperature behaves almost like an insulator. For a gas discharge a high voltage is required. But in a vacuum chamber with optimum distance between electrodes and low enough pressure of gas, gas discharge can occur at lower potential difference between electrodes to produce plasma. According to Paschens law\textsuperscript{[13]} breakdown voltage depends on pressure ($p$) and electrode distance($d$):

$$V = \frac{a(pd)}{\ln(pd) + b}$$

where $a$ and $b$ are constants which depends on the gas used.
It can be seen by the observation of Paschen curve (figure 6) here that at lower values of pd breakdown occur at higher voltages because there are less number of atoms per unit volume, and it requires more voltage to discharge gas. With increasing pd value break down voltage decreases but only upto a threshold value after that again starts increasing. The reason for increment can be given by decreased mean free path (as more atoms are present). Due to that the atoms lose most of their energies in thermal collision and do not get ionized. But that increase is not as steep as the other part of curve. That is because of not much significant effect of thermal motion on breakdown voltage. Here I have plotted different paschen curves for different distances because there may be other than pd, there can be other factors also affecting breakdown voltage. Base pressure for the system was kept at 0.02 milli-Bar.

5.2 **Langmuir probe I-V characteristic**

For langmuir probe I-V characteristic langmuir probe is biased from a negative potential to positive potential and corresponding output current is measured by ammeter.

When langmuir probe is biased to negative potential electrons will be repelled and ions will be attracted towards probe but as mobility of ions is very small in comparison to electrons, there will be very small ion current. For very large negative voltage ion current gets saturated which is known as ion saturation current.

When probe is biased to positive potential really high value of current is observed. That is because of high mobility of electrons. If electron distribution is taken to be maxwellian then we can write–

\[ I_e = I_0 e^{\frac{V_0}{kT_e}} \]

9
Therefore if we plot I and V in log-log plot we can determine plasma temperature from the slope of linear fit. Here in this experiment at 956 volt and pressure 0.4mB it was found to be 0.7103eV. I-V characteristic is shown in the fig(7).

Figure 7: Langmuir probe I-V characteristic

5.3 Floating potential observations

5.3.1 For varying discharge voltages

At fixed voltage 596V and pressure 0.4mB when we are increasing the magnetic field, the floating potential fluctuations first moves from periodic and regular behavior to chaotic (at 596V) and then again to relaxation oscillations (periodic). Movement from chaotic to periodic is a kind of SOC (self organized critically) behavior which happens via homoclinic bifurcation.

Correlation dimension decreases with increasing voltage which tells us that system complexity decreases with increasing potential. (fig. 10)
Figure 8: Signal, phase space and FFT for increasing voltages. For phase space a low pass filter is used which cuts off frequency higher than $6 \times 10^4$ Hz.

Figure 9:
5.3.2 For varying magnetic field

In presence of magnetic field in addition to electron waves and ion acoustic waves, electron ion cyclotron waves are also formed. These waves changes the behavior of system upto a large extent. At fixed discharge voltage 596V with increasing magnitude of magnetic field floating potential fluctuations are found to go from period to chaotic. Initially at when no magnetic field is applied it shows only one frequency i.e. 23kHz, but as magnetic field is increased to 9 gauss it shows period 2, after that it goes to more than two period at 28 gauss and then finally it becomes chaotic at 46 gauss. Lyapunov exponent is found to be positive for 46 gauss magnetic field. Largest lyapunov exponent is found to be 0.1324

Correlation dimension increases with increasing voltage which tells us that system complexity increases with increasing potential.

\[0.1225 \text{ from the code I developed}\]
Figure 11: Signal, phase space and FFT for increasing magnetic field (For phase space a low pass filter is used which cuts off frequency higher than $6 \times 10^4$Hz)

Figure 12:
Figure 13: Correlation dimension plot
### Table 1: Hurst exponent for changing discharge voltages at magnetic field 46 gauss

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Hurst Exponent</th>
<th>Behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>508</td>
<td>0.5383</td>
<td>Long range correlation</td>
</tr>
<tr>
<td>537</td>
<td>0.7098</td>
<td>Long range correlation</td>
</tr>
<tr>
<td>596</td>
<td>0.5230</td>
<td>Long range correlation</td>
</tr>
<tr>
<td>616</td>
<td>0.6756</td>
<td>Long range correlation</td>
</tr>
</tbody>
</table>

### Table 2: Hurst exponent for changing magnetic field at 596V

<table>
<thead>
<tr>
<th>Magnetic field (gauss)</th>
<th>Hurst Exponent</th>
<th>Behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5462</td>
<td>Long Range correlation</td>
</tr>
<tr>
<td>9</td>
<td>0.7906</td>
<td>Long Range correlation</td>
</tr>
<tr>
<td>28</td>
<td>0.6048</td>
<td>Long Range correlation</td>
</tr>
<tr>
<td>46</td>
<td>0.5230</td>
<td>Long Range correlation</td>
</tr>
</tbody>
</table>

Figure 14: Largest Lyapunov exponent by using TISEAN [17] software (slope = 0.1324)

Figure 15: Largest Lyapunov exponent by the algorithm I developed in MATLAB (slope = 0.1225)

### 6 Conclusion and further works

Here I presented a non linear analysis of the floating potential in DC glow discharge plasma. Main focus of study was to apply different techniques of time series analysis to the fluctuation. However modeling the system using differential equations will give a better insight of the physical phenomena occurring in plasma. This work is yet to be done. Further some modern techniques like ‘structure function analysis’, ‘wavelet analysis’, ‘deterened fluctation analysis’ etc can also be applied to understand nonlinear behavior more accurately. A bifurcation diagram for the behavior with amplitude can provide a global picture of nonlinear behavior. Reliability of 0 – 1 test for chaos is still to be checked from an improved version of this test.
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17. http://www.mpiiks-dresden.mpg.de/~tisean/

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