MIRROR SYMMETRY AND CLOSED STRING TACHYON CONDENSATION

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We study closed string tachyon condensation using the RG flow of the worldsheet theory. In many cases the worldsheet theory enjoys $\mathcal{N}=2$ supersymmetry, which provides analytic control over the flow, due to non-renormalization theorems. Moreover, Mirror symmetry sheds light on the RG flow in such cases. We discuss the relevant tachyon condensation in the context of both compact and non-compact situations which lead to very different conclusions. Furthermore, the tachyon condensation leads to non-trivial dualities for non-supersymmetric probe theories.

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1. Introduction

A deeper understanding of non-supersymmetric string dynamics seems to be the most fundamental obstacle to overcome in connecting string theory with real world. For non-supersymmetric backgrounds, by varying moduli, one typically ends up in a situation with tachyons. Thus the question gets related to the fate of the closed string tachyons upon their condensation.

Recently the question of closed string tachyons has been addressed in a number of situations [1,2,3]. The aim of this note is to show that despite the fact that the target is non-supersymmetric, very often the worldsheet in the NSR formulation *is* supersymmetric, and for a wide class of examples admits $\mathcal{N}=2$ worldsheet supersymmetry. Thus one can bring powerful techniques developed in the context of 2d QFT's with $\mathcal{N}=2$ supersymmetry to bear on the question of tachyon condensation in superstring theories in non-supersymmetric backgrounds. In particular the RG flow of the $\mathcal{N}=2$ theories, whose F-terms are protected by non-renormalizations theorems, suggest what the fate of the closed string tachyons are in many cases.

Here we make an assumption about the nature of string field theory, which has proven to be rather successful in the context of open string theories [4,5]. Namely we view the relevant space for closed string field theory to be the space of 2d QFT's and that the RG flow in such spaces as indicative of dynamics of strings. In this context we identify the relevant 2d QFT's and deformations corresponding to tachyon condensation and assume that there is an evolution in *physical time* which can be identified with the *RG time on the worldsheet*. This is of course consistent with the fact that fixed points of RG flow are stationary solutions for classical strings.

The organization of this paper is as follows. In Section 2 we review the gauged linear sigma model construction [6] and its mirror description [7] and point out its relevance to tachyons of orbifold theories. In Section 3 we apply the ideas of Section 2 to non-compact orbifolds with tachyons. We consider examples with complex dimensions 1, 2 and 3. In Section 4 we consider compact orbifolds with tachyons and consider the dynamics of tachyon condensation in such cases. For illustrative purposes we consider the case of complex dimension 1 (orbifolds of T^2) and complex dimension 3 (orbifolds of compact CY 3-folds). In all such compact cases the internal theory loses some degrees of freedom as one would expect from c-theorem of Zamolodchikov. In Section 5 we show how the consideration of probes in such theories would lead to non-trivial non-supersymmetric dualities. We give some examples of non-chiral 4d non-supersymmetric dualities which follows from this picture. In Section 6 we suggest some directions for future work.

2. Linear Sigma Model for Non-Compact Targets and its Mirror

Worldsheet sigma models with $\mathcal{N}=2$ supersymmetry have a powerful description in terms of gauged linear sigma models [6]. Let us consider such a theory with a single $U(1)$ with charged matter fields $(X_0, X_1, X_2, .., X_r)$ with charges given by

$$Q = (-n, k_1, k_2, ..., k_r)\,.$$

We take n, k_i to be positive. The theory is asymptotically free when $n < \sum_i k_i$, and flows to a conformal theory when $n = \sum_i k_i$. The FI term for the $U(1)$ is naturally complexified, by combining it with the θ-angle to form a complex parameter t. When the theory is not conformal, t can be traded with the scale. In particular the UV fixed point corresponds to $t \to \infty$ and the IR fixed point corresponds to $t \to -\infty$. A closely related theory is where we consider $Q \to -Q$. This theory is the same as the above, except with $t \to -t$ with the role of UV, IR exchanged. In particular suppose Q corresponds to a theory which is not asymptotically free. Then $-Q$ corresponds to a theory which is asymptotically free. In this context $t \to -\infty$ of the original model is the UV fixed point.

The geometry of the linear sigma model with a given t can be analyzed by solving the D-term constraints

$$-n|X_0|^2 + \sum_i k_i |X_i|^2 = t \tag{2.1}$$

modulo the $U(1)$ action

$$\big(X_0, ..., X_r\big) \to \big(X_0\, e^{-in\theta}, \dots, X_r\, e^{ik_r\theta}\big)$$

which is the Higgs branch of the GLSM. To be precise, the real part of t appears in the above equation. Note that if we consider the limit $t << 0$ the equation (2.1) implies that X_0 takes a large vev. In such a case the $U(1)$ is spontaneously broken to Z_n. The fields X_i correspond to massless fields which transform according to $\exp(2\pi i k_i/n)$ under the unbroken Z_n gauge symmetry. Thus we have that in the $t \to -\infty$ limit the GLSM is equivalent to the $\mathbf{C^r}/Z_n$ orbifold

$$\big(X_1, ..., X_r\big) \sim \big(\omega^{k_1} X_1, ..., \omega^{k_r} X_r\big)\,,$$

where $\omega = \exp(2\pi i/n)$. On the other hand in the limit $t \to \infty$ the equation (2.1) requires that not all X_i, with $i = 1, ..., r$ are zero. The equation (2.1) together with the $U(1)$ gauge symmetry implies that the Higgs vacuum with $X_0 = 0$ corresponds to the weighted projective space $WP_{k_1,...,k_r}$

$$\big(X_1, ..., X_r\big) \sim \big(\lambda^{k_1} X_1, \ldots, \lambda^{k_r} X_r\big)$$

with $\lambda \neq 0$ (the phase of λ corresponds to the $U(1)$ action and its magnitude is related to the choice of t). Note that in this limit t plays the role of the size (kahler class) for $WP_{k_1,...,k_r}$. The X_0 direction corresponds geometrically to a non-compact bundle over this space which is denoted by $O(-n)$. Thus the target space is identified with the total space of this bundle over the weighted projective space. In other words the total space can be viewed in the limit of large $t >> 0$ as the r dimensional complex space given by

$$\big(X_0, X_1, ..., X_r\big) \sim \big(\lambda^{-n} X_0, \lambda^{k_1} X_1, \ldots, \lambda^{k_r} X_r\big)$$

with $\lambda \neq 0$ and not all $X_1, ..., X_r$ are zero at the same time.

There is a convenient mirror description for this theory which effectively sums up the gauge theory instantons [7], roughly by dualizing the phases of the fields X_i. One obtains twisted chiral fields Y_i which are periodic variables $Y_i \sim Y_i + 2\pi i$ and are related to X_i by

$$|X_i|^2 = ReY_i \tag{2.2}$$

and the theory becomes equivalent to a LG theory with

$$W = \sum_{i=0}^{r} \exp(-Y_i) = \sum_{i=0}^{r} y_i$$

(with $y_i = e^{-Y_i}$) subject to

$$y_0^{-n} \prod_{i=1}^{r} y_i^{k_i} = e^{-t}$$

(compare the absolute value of this equation with (2.1)). We define u_i by

$$u_i = y_i^{1/n}\,. \tag{2.3}$$

From which we deduce

$$y_0 = e^{t/n} \prod_{i=1}^{r} u_i^{k_i}\,. \tag{2.4}$$

Note however that the change of variables (2.3) is well defined as long as we identify u_i with a Z_n phase multiplication (since Y_i are periodic). However since the phase of y_0 is well defined, Eq. (2.4) implies that the group we have to mod out by is a subgroup of $(Z_n)^r$ preserving the monomial $\prod_{i=1}^r u_i^{k_i}$, which is thus a group $G = (Z_n)^{r-1}$. Thus we have found that in terms of u_i, after eliminating y_0 from the superpotential in terms of the u_i the theory is equivalent to

$$W = \Big[\sum_{i=1}^{r} u_i^n + e^{t/n} \prod u_i^{k_i}\Big]//G\,, \tag{2.5}$$

where $G = (Z_n)^{r-1}$ is the maximal group preserving all the monomials. This result was derived exactly as presented here in [7]. Note that we have to be careful to consider U_i as the natural variables, where $u_i = e^{-U_i}$. If $\sum k_i = n$ the theory is conformal and that is reflected in the fact that the above superpotential admits an R-symmetry in this case. The RG flow corresponds to $W \to \Lambda^{-1} W$ where Λ denotes the energy scale; this is due to non-renormalization of F-terms which implies that the scaling is given by the naive classical scaling given by the dimension $\int d^2x d^2\theta$. By a field redefinition

$$u_i \to \Lambda^{1/n} u_i \tag{2.6}$$

this gives a running for t given by

$$t(\Lambda) = t + \Big(\sum_{i=1}^{r} k_i - n\Big) \log \Lambda\,. \tag{2.7}$$

This in particular implies that if $\sum_{i=1}^r k_i$ is less than (greater than) n the UV fixed point is equivalent to $t \to -\infty$ ($t \to +\infty$) and the IR fixed point is equivalent to $t \to +\infty$ ($t \to -\infty$).

2.1. *The Orbifold Point*

As discussed before, for $t \to -\infty$ the theory is equivalent to $\mathbf{C}^r/Z_n$, which is a conformal theory. The mirror theory becomes equivalent in this limit to the LG theory with superpotential

$$\Big[W = \sum_{i=1}^{r} u_i^n\Big]//G\,,$$

where the only information about the k_i is encoded in the action of G on the fields–it acts on each field by multiplication with an n-th root of unity subject to preserving the chiral field $T = \prod_{i=1}^r u_i^{k_i}$. The theory $\mathbf{C}^r/Z_n$ will have $n-1$ twisted sectors, each of which gives rise, in its lowest state to a twist field, which is also an $\mathcal{N}=2$ chiral field [8] (see also [9]). The first twist field is identified on the mirror LG theory with $T=\prod_{i=1}^{r} u_i^{k_i}$ and that generates the chiral fields associated with the other twisted sectors. Namely in the l-th sector we get T^l as the corresponding twist field. Since u_i^n has to have $\mathcal{N}=2$ charge 1 the charge of u_i is $1/n$ and that of the twist field T is

$$Q_T=\sum_{i=1}^{r}\frac{k_i}{n}$$

which is the expected result for the charge of the twist field in the first twisted sector. Note that $\mathcal{N}=2$ superconformal algebra implies that the left and right dimension of T is $h_T=\frac{1}{2}Q_T$. A generic deformation by all twist fields is given by

$$\Big[W=\sum_{i=1}^{n}u_i^n+\sum_{l=1}^{n-1}t_lT^l\Big]//G$$

for some complex parameter t_l representing the strength of the deformation for the ground state in the l-th twisted sector and $T=\prod_{i=1}^{r}u_i^{k_i}$. In the context of type II superstrings GSO projection restricts the allowed t_l as will be discussed below, to be compatible with a $W\to -W$ symmetry. In particular one needs a definition of an order 2 operator $(-1)^{F_L}$. For the action to be invariant, we need $W\to -W$ because $\int d\theta_L d\theta_R W$ should be invariant and θ_L is odd under $(-1)^{F_L}$. Also what one means by T^l is $\prod u_i^{[lk_i]}$ where $0\leq [lk_i]<n$ and is equal to lk_i mod n, this is to make the relevant chiral field to be the lowest dimension twist operator.

In the context of type II superstrings, the mirror LG model we have presented changes IIA/IIB if the complex dimension of the space is odd, and otherwise maps IIA and IIB back to themselves. This correlates with the number of times T-duality has been applied.

Now we are ready to discuss some examples.

3. Examples of Tachyon Condensation in Non-Compact Targets

As an illustration of the models we will consider some non-compact examples including the ones discussed in [10] and [3]. We will consider tachyons in 1, 2 and 3 complex non-compact directions.

3.1. *Tachyon Condensations in C/Z_n*

Consider the linear sigma models with charges $(-n, k)$. As $t \to -\infty$ this theory is given by the orbifold of C/Z_n given by $x \to e^{2\pi i k/n} x$.

In this case the mirror description (2.5) is given by the LG model with superpotential

$$W = u^n + e^{t/n} u^k \,.$$

Note that here G is trivial. We take $k < n$ and for simplicity restrict attention to the case where k, n are relatively prime. As discussed above, the deformation by u^k is equivalent to condensation of tachyon in the 1st twisted sector. Note that this orbifold C/Z_n is equivalent to $x \to e^{2\pi i/n} x$ since k, n are relatively prime, which would be given by a linear sigma model with charges $(-n, 1)$[a]. However what is the 1st twisted sector in the $(-n, k)$ linear sigma model corresponds to the k-th twisted sector of the $(-n, 1)$ model. Thus from the perspective of the $(-n, 1)$ theory, the FI-term deformation of the $(-n, k)$ theory is equivalent to tachyon condensation in the k-th twisted sector. The infrared flow of this theory, as follows from (2.7) takes us to the theory with $W = u^k$ which is equivalent to C/Z_k. This is exactly the pattern of flow discovered in [3]. In the case considered in [3] avoidance of tachyon in the bulk restricts n to be odd. The beautiful arguments in [3] were based on the analysis in two regimes: near the orbifold point, by studying the deformation D-brane probes feel, and in the IR where the target gravity regime was relevant. Here we have found a framework which interpolates between the two regimes in a way that we have analytic control over the flow. For example we can now recover the RG evolution of the shell picture on the complex plane of the changing of the deficit angle discussed in [3]. Namely the space is given by

$$-n|X_0|^2 + k|X_1|^2 = t + (k - n) \log \Lambda$$

modulo the $U(1)$ action, where we have included the running of t given by (2.7). So in the UV, where $\Lambda \to \infty$, we have a C/Z_n geometry and in the IR, where $\Lambda \to 0$ we have a C/Z_k geometry. This is the same shell picture proposed in [3].

In the context of type II superstrings, the existence of left/right independent GSO projection further restricts the allowed k. In particular one needs a definition of an order 2 operator $(-1)^{F_L}$. For the action to be invariant,

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[a] The fact that $W = u^n$ is the mirror of C/Z_n can be obtained also from taking the large k limit of Kazama-Suzuki $SL(2)/U(1)$ models, as noted in [11].

we need an involution symmetry under which $W \to -W$. This symmetry acts on $u \to -u$ takes $W \to -W$ when n and k are both odd. In the context of type II strings this mirror LG description exchanges type IIA strings with type IIB strings. We can also translate this geometric description to the variables of mirror LG theory using (2.2) and (2.3).

We can also consider deformations by all tachyon operators from various twisted sectors at once. This would correspond to LG theory deformation with superpotential

$$W = u^n + \sum_{i=1}^{n-1} t_i u^i \,,$$

where t_i is the deformation in the direction of the tachyon coming from the i-th twisted sector. Here we are using mirror symmetry to connect different linear sigma models for different values of k into deformations of a single LG theory. In the context of type II superstrings, as discussed above, we restrict to the case where n is odd and only odd sector tachyons are turned on (so that $u \to -u$ sends $W \to -W$).

3.2. *Tachyon Condensation in C^2/Z_n*

Next we consider the linear sigma model with charge given by

$$(-n, k_1, k_2)$$

which in the $t \to -\infty$ goes over to the C^2/Z_n orbifold generated by

$$\big(X_1, X_2\big) \to \big(\omega^{k_1} X_1, \omega^{k_2} X_2\big)$$

where $\omega = \exp(2\pi i/n)$. The mirror theory is given by (2.5)

$$\Big[W = u_1^n + u_2^n + e^{t/n} u_1^{k_1} u_2^{k_2}\Big] // Z_n \,, \tag{3.1}$$

where Z_n acts as n-th roots of unity on u_i preserving $T = u_1^{k_1} u_2^{k_2}$. In the orbifold limit the LG superpotential is given by $[W = u_1^n + u_2^n]//Z_n$ and the only input about (k_1, k_2) comes from the action of Z_n which preserves $T = u_1^{k_1} u_2^{k_2}$. For the case $(k_1, k_2) = (1, 1)$ or $(n-1, 1)$ the theory is equivalent to the target supersymmetric A_{n-1} singularity, whose twist field should give rise to a conformal theory. However the above superpotential (3.1) seems to be conformal (i.e. admit an R symmetry) only for $(k_1, k_2) = (n-1, 1)$. The point is that in the Z_n orbifold leading to spacetime supersymmetric A_{n-1}, in the twisted NS sector there is a tachyon which is projected out by the GSO projection. The difference between the two cases here is exactly the choice of the GSO projection in the twisted sector. We can put this also directly in the language of worldsheet conformal theory: From the perspective of the conformal theory we should see a tachyonic deformation, even for the $(n-1,1)$ theory. How is that realized in the above LG theory? This is realized by noting that there is a field $\overline{u}_1 u_2$ which is invariant under the Z_n action for the $(n-1,1)$ theory, but it is not chiral in the canonical realization of the $\mathcal{N}=2$ algebra. However in the orbifold limit we have two decoupled $\mathcal{N}=2$ systems and redefining the $U(1)$ charge of the first theory, gives an $\mathcal{N}=2$ theory for which $\overline{u}_1 u_2$ is a chiral field. This would then be equivalent to the deformations by $u_1 u_2$ in the $(1,1)$ theory.

Just as in the C/Z_n example we can consider tachyon condensation in various twisted sectors and that corresponds to further addition of the operator T^l to the superpotential. Or, equivalently we can think of this as the first twisted sector of the GLSM given by charges $(-n, lk_1, lk_2)$, whose twist field corresponds to the FI term deformation (or RG flow). Let us consider first the simplest case with $(k_1,k_2)=(1,1)$. In this case we have charges given by $(-n,1,1)$ and from the GLSM we know that in the IR the FI-term flows (2.7) so that $t\to\infty$. The target space geometry in this case will be the total bundle of $O(-n)$ over P^1, as discussed before ($WP_{1,1}=P^1$). Also the P^1 volume is given by t and in the IR it becomes infinitely large. Thus we obtain C^2 as the end point of tachyon condensation in this case.

For a more complicated example let us consider general (k_1,k_2) with k_1,k_2 relatively prime (the case $(1,k)$ is the case discussed in [3] with some shift $k\to n-k$ in notation).

In this case the GLSM analysis suggests that in the IR the geometry is given by the total space of an $O(-n)$ bundle over the weighted projective space WP_{k_1,k_2}. This is the space which can be viewed as the complex space

$$\big(X_0,X_1,X_2\big)\sim\big(\lambda^{-n}X_0,\lambda^{k_1}X_1,\lambda^{k_2}X_2\big)\,,$$

where $\lambda\neq 0$, and X_1,X_2 cannot both be zero. The compact part corresponds to the X_1,X_2 directions at $X_0=0$, and the IR limit corresponds to making it infinitely large. Except that there are some orbifold points. In particular at $X_1=X_0=0$ the geometry is locally $\mathbf{C^2}/Z_{k_2}$, as can be seen by considering $\lambda=e^{2\pi i/k_2}$. The orbifold action is given by

$$\big(X_0,X_1\big)\to\big(\exp(-2\pi i n/k_2)X_0,\exp(2\pi i k_1/k_2)X_1\big)\,.$$

Similarly at $X_2=X_0=0$ we have another orbifold point which locally is given by C^2/Z_{k_1} given by

$$\big(X_0,X_2\big)\to\big(\exp(-2\pi i n/k_1)X_0,\exp(2\pi i k_2/k_1)X_2\big)\,.$$

Thus the endpoint of the Tachyon condensation is given by a space which has two orbifold singularities given by the two orbifold actions above. Moreover these two points are infinitely far away in space as the volume of the X_1, X_2 space is going to infinity (and roughly speaking they correspond to the north and south poles of a sphere). Considering tachyon condensation in the l-th twisted sector corresponds to replacing k_1, k_2 above by lk_1, lk_2. For generic flows this would end in non-supersymmetric orbifold fixed points, but for special choices it would have a supersymmetric endpoint (for example if $n-k_1 = 0 \bmod k_2$ or if $n-k_2 = 0 \bmod k_1$). Of course at the new fixed points, in case they are non-supersymmetric and include tachyons, we can also use another tachyon direction to flow. Since the order of the orbifold group has gone down, i.e. $k_i < n$, this procedure, i.e. continuing to condense tachyons if the corresponding points are non-supersymmetric, would iteratively end up at a supersymmetric orbifold point or free space.

The geometry of the IR fixed point above can also be seen in the mirror LG description. We have

$$\Big[W = u_1^n + u_2^n + e^{t/n} u_1^{k_1} u_2^{k_2}\Big]//Z_n \,.$$

In the IR limit the last term dominates, but this does not uniquely fix the IR R-charge of the fields. In order to do that we have to go to regions in field space where either u_1 is small or u_2 is small, which would give limiting LG by dropping one of the first two terms. This is the mirror of the two orbifold points we found above. In particular if we consider the regime $u_2 \sim 0$ and $u_1^n \sim e^{t/n} u_1^{k_1} u_2^{k_2}$ (which can be arranged since $t >> 0$) then we have

$$\Big[W \sim u_1^n + e^{t/n} u_1^{k_1} u_2^{k_2}\Big]//Z_n \,.$$

It is convenient to define

$$v_1 = u_1^{n/k_2} \,, \quad v_2 = e^{t/nk_2} u_1^{k_1/k_2} u_2 \,.$$

The single valuedness of v_i induces the Z_n action above. However the single valuedness of u_1^n and $u_1^{k_1} u_2^{k_2}$ implies that $v_1,\ v_2$ are orbifolded by Z_{k_2} while preserving $v_1^{k_1} v_2^{-n}$, i.e., in this limit we have

$$\Big[W = v_1^{k_2} + v_2^{k_2}\Big]//Z_{k_2} \,,$$

where Z_{k_2} preserves $v_1^{k_1} v_2^{-n}$. This is the mirror of C^2/Z_{k_2} with action $(X_1, X_0) \to (\exp^{2\pi i k_1/k_2} X_1, \exp^{-2\pi i n/k_2} X_0)$ obtained above near $X_1 \sim X_0 \sim 0$. Similarly by analyzing the limit where $u_1 \sim 0$ and $u_2^n \sim u_1^{k_1} u_2^{k_2} e^{t/n}$ we obtain the other orbifold point.

Just as in the C/Z_n analysis we can generalize the shell picture. This can also be stated in the original variables of the linear sigma model, namely we consider the space of solutions

$$-n|X_0|^2 + k_1|X_1|^2 + k_2|X_2|^2 = t + (k_1 + k_2 - n)\log\Lambda$$

modulo the $U(1)$ action, where we have plugged in the RG flow of t in accordance with (2.7).

Considering the flow by the tachyon in the l-th twisted sector, as noted before replaces $(-n, k_1, k_2) \to (-n, lk_1, lk_2)$ to which the above analysis applies. We can also consider a linear combination of fields in the twisted sector, for which the mirror description is the most suitable one and given by a LG theory with superpotential

$$\Big[W = u_1^n + u_2^n + \sum_l t_l u_1^{lk_1} u_2^{lk_2}\Big]//Z_n\,.$$

Note however, if $lk_i \geq n$ then the corresponding twist field is not the lightest state in that tachyon sector. To obtain the lowest state, we should replace $lk_i \to [lk_i]$ where $[lk_i] = lk_i \bmod n$ and $0 \leq [lk_i] < n$. In the following we shall not bother putting brackets around lk_i but that is what is implied. Note that $u_1^{[lk_1]} u_2^{[lk_2]}$ is also Z_n invariant because u_1^{mn} and $u_2^{m'n}$ are Z_n invariant for all integers m and m'.

In embedding this worldsheet theory in type II superstrings, we should also consider GSO projection. If n is odd, then the $(-1)^{F_L}$ can be taken to act as $u_i \to -u_i$, and we can, without loss of generality assume $k_1 + k_2$ is odd (if not, as discussed before we can change one of them to $k \to n-k$ and this would then be satisfied). Then $T = u_1^{k_1} u_2^{k_2}$ is also odd under $(-1)^{F_L}$, and so is all the odd powers T^l for l odd. In this case the deformations are restricted to odd l above. If n is even, we can assume k_i are not both even (otherwise we divide all three by factors of 2). If k_i are both odd, then we define the $(-1)^{F_L}$ to act as $u_1 \to e^{i\pi(n-k_2)/n} u_1$ and $u_2 \to e^{i\pi k_1/n} u_2$; this is an order 2 operation on the invariant fields which are generated by u_i^n and $u_1^{k_1} u_2^{k_2}$. In this case, again we can deform by all odd powers of T, compatible with $W \to -W$ symmetry. The case when one of the k_i's is odd and the other even, will lead to a tachyon in the bulk which we are excluding. Note that the mirror LG theory we have obtained here is for the same IIA or IIB superstrings (since we have applied T-duality twice).

3.3. C^3/Z_n Tachyon Condensation

Clearly we can consider many other cases. For example consider in complex dimension 3 the GLSM with charges

$$(-n, 1, 1, 1)\,.$$

If $n = 3$ this is conformal and corresponds to the $O(-3)$ geometry over P^2 or its orbifold limit as $t \to -\infty$ given by C^3/Z_3. For $n > 3$ the theory corresponds to a non-conformal theory. It corresponds to C^3/Z_n with a tachyon which condenses. Again in the IR the theory has a growing P^2 with infinite volume. Thus in the IR it flows to flat C^3. The mirror description is given by

$$\Big[W = u_1^n + u_2^n + u_3^n + e^{t/n} u_1 u_2 u_3\Big] // (Z_n \times Z_n)\,,$$

where the Z_n acts as n-th roots of unity on each field preserving all the monomials. Similarly we can consider more general charges $(-n, k_1, k_2, k_3)$ which similarly to the C^2/Z_n case would end up with flat C^3 modulo three points infinitely far away each of which is an orbifold of the type C^3/Z_{k_i}. For example for the C^3/Z_{k_1} action it is given by phase action on X_0, X_2, X_3 given by

$$\big(X_0, X_2, X_3\big) \to \big(\omega^{-n} X_0, \omega^{k_2} X_2, \omega^{k_3} X_3\big)$$

with $\omega^{k_1} = 1$. Again we can trace in the mirror language where the corresponding regions come from, as in the C^2/Z_n case.

In the context of superstrings we can again consider the condition of being able to define $(-1)^{F_L}$, as in complex dimensions 1 and 2 discussed above. We leave this as an exercise to the reader.

4. Compact Tachyons

In the previous section we have discussed tachyon condensation in the context of non-compact theories, and this has given us another conformal theory with the same central charge. This can also be seen from the relation between the central charge of LG theory with the charges of the chiral fields [12][13] $\widehat{c} = \sum_i (1 - 2q_i)$. In the case at hand the relevant fields Y_i have zero charge (more precisely they have a "logarithmic charge" because they appear as e^{-Y_i} in the superpotential which has charge 1) so the naive counting of the fields gives the complex dimension of the theory.[b] In particular the IR

[b] If one considers the GLSM with charges $(-n, 1)$ and add a superpotential $X_0 X_1^n$ to the original sigma model, then according to the arguments of [7] the mirror superpotential will still be given

flow discussed above does not change the central charge of the theory. This was noted in [3] where it was emphasized that the non-compactness allows one to evade the Zamolochikov's c-theorem [14]. In the compact case, as noted in [3] one would expect the IR flow to have a smaller central charge, and in the generic case to become a purely massive theory. Here we will show that this is indeed the case. Furthermore we can also address the RG flow for such cases using mirror symmetry techniques.

Let us consider the compact examples of dimensions 1 and 3.

4.1. T^2/Z_n Tachyon Condensation

Consider conformal theory for T^2/Z_n for $n = 3, 4, 6$. The complex moduli of these tori are frozen, due to the existence of the symmetry to be $\tau = e^{2\pi i/3}, i, e^{2\pi i/3}$ respectively. The Kahler moduli is free to vary. These theories will have tachyon fields in the twisted sector and we can condense them. We are interested to study the RG flow of the worldsheet theory upon such tachyon condensations.

Let us first discuss T^2/Z_3. This is mirror to N=2, LG theory with

$$W = x^3 + y^3 + z^3 + axyz\,,$$

where a is mirror to the Kahler structure of T^2/Z_3 and there is no analog of the Kahler structure on the mirror because complex structure is frozen. To see this note that the LG/CY correspondence [15][16] states that

$$\big[W = x^3 + y^3 + z^3 + axyz\big]/Z_3 = T^2\,,$$

where the complex structure of the T^2 is related to a and Kahler structure is fixed to be at the point with the Z_3 symmetry [8]. Or, using the Kahler/Complex structure exchange of T^2 this implies that this is equivalent to T^2 with complex structure frozen at the Z_3 symmetric point and where a is related to the Kahler parameter. This is the way we wish to view the above LG theory. Now as usual one can mod out by quantum symmetry on both sides [8]. Quantum symmetry on left gives back $x^3 + y^3 + z^3 + axyz$. On the right (i.e. the mirror) the quantum symmetry is a classical symmetry (this is due to the usual winding/momentum exchange) and so we obtain T^2/Z_3 which is the proof of the statement made above. Note that the fields x, y, z in the LG theory map to the three tachyon fields of T^2/Z_3 at the three

in the UV by $W = u^n$ but now the good variable is u. This would be the n-th minimal model of $\mathcal{N}$=2 SCFT [12,13].

fixed points of the Z_3 action. Adding them to the W we have

$$W = x^3 + y^3 + z^3 + axyz + (t_1 x + t_2 y + t_3 z)$$

which will flow in the IR to a massive theory. Thus the T^2 formally should disappear, unlike the non-compact case. Note that here x, y, z are the good variable, whereas in the non-compact case Y_i's are the good variable, where $y_i = e^{(-Y_i)}$ appears in the superpotential. In the context of type II superstrings, we can define left/right independent GSO projection by having $(x, y, z) \to -(x, y, z)$ under $(-1)^{F_L}$, which is compatible with the above deformation.

Similarly the other two orbifolds of T^2 are mirror to

$$W = x^4 + y^4 + z^2 + axyz \qquad \text{for} \qquad T^2/Z_4\,,$$

$$W = x^6 + y^3 + z^2 + axyz \qquad \text{for} \qquad T^2/Z_6\,,$$

where the twist fields are identified with x, y, z. Note that this is consistent with the geometric fact that T^2/Z_4 has two Z_4 fixed points and one Z_2 fixed point, and T^2/Z_6 has one Z_6, one Z_3 and one Z_2 fixed point. Just as above one can consider tachyon condensation for these theories. However in embedding in type II strings, since we have even order modding out, this will lead to tachyon in the bulk.

4.2. *Tachyon Condensation on Orbifolds of CY 3-fold*

Consider the LG theory given by

$$W = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 + \psi x_1 x_2 x_3 x_4 x_5\,.$$

If we modded this out by a Z_5 acting simultaneously on all 5 fields by multiplication with $e^{2\pi i/5}$, it would be mirror to the $(Z_5)^3$ orbifold of the quintic [17]. However we can undo the extra Z_5 in here as in the T^2 case, which on the mirror corresponds to taking a $(Z_5)^4$ orbifold of the quintic not preserving the holomorphic 3-form. Thus we have a mirror symmetry on quintic modded out by $(Z_5)^4$ with the above LG model without any modding out. This leads to tachyonic modes in the geometry which just as in the T^2 case can be identified with the chiral fields x_i of the above LG model. Again generic deformations by them will give a purely massive theory. However, we can also obtain intermediate situations. For example if we deform by GSO allowed term $\sum_i x_i^3$ (i.e. by using tachyons in the triply twisted sector) we obtain a conformal theory in the IR with complex dimension $\widehat{c} = 5/3$. This has a piece which can be identified with the mirror of T^2/Z_3. Or if we deform further by $x_4 + x_5$ it would be exactly a T^2/Z_3 theory in the IR. So we will have eaten up two complex dimensions in this flow.

5. Consequences for Non-Supersymmetric Dualities on Brane Probes

Brane probes, possibly wrapped over non-trivial cycles of compactification geometry, have been a source of a great deal of interaction between field theory results and string theory. In particular in [18,19] it was argued how studying wrapped branes for type IIA, B strings on Calabi-Yau threefold leads to Seiberg duality. The idea there is to study the field theory living on the brane as Calabi-Yau moduli vary. This has been extended to more non-trivial geometries recently [20] (see also the related work [21,22]).

One can also follow the same idea in this context, namely we consider probes in non-supersymmetric theories and ask what we can learn about them from the analysis we did, about the fate of the tachyon condensation. We make an assumption here, which we find plausible, but cannot justify. We assume that *the IR dynamics of the probe theory is the same as the theory on the probe after tachyon condensation in the target corresponding to the IR limit of the worldsheet theory.*

One can provide many examples of such dualities. To illustrate the point, however, we will limit ourselves to a single class of examples which we develop in detail. Consider the orbifold C^3/Z_n which we studied in Section 3, namely the one corresponding to GLSM with charges $(-n, 1, 1, 1)$. As $t \to -\infty$ this is the orbifold theory and the quiver theory on it can be deduced as discussed in the general context in [23] and in the present case in [24,25]. We will assume that $n = 3k$ with k odd. The charges are given by $(-3k, 1, 1, 1)$. This can also by viewed as a Z_k orbifold of the supersymmetric C^3/Z_3 orbifold.

Let us first consider the supersymmetric case $(-3, 1, 1, 1)$. If we consider N D3 branes on this singularity, this gives an $\mathcal{N}=1$ theory studied in [26] with $U(N)^3$ gauge symmetry with three chiral matter fields A^i_{12} A^i_{23} and A^i_{31}, where $1, 2, 3$ denote the three gauge groups and the ordered subscripts on the fields represent which bifundamental representation they transform as, and i runs over three flavor values. There is in addition a superpotential given by

$$W = \epsilon_{ijk} \mathrm{Tr} A^i_{12} A^j_{23} A^k_{31} \, .$$

As we go away from the orbifold point, i.e. from $t \to -\infty$ towards positive t, as it passes through 0, the probe theory changes to its Seiberg dual as argued in [20]. In particular suppose node 1 corresponds to the cycle which shrinks. Then the dual probe theory will again be $\mathcal{N}=1$ and will have gauge group given by $U(2N)_1 \times U(N)_2 \times U(N)_3$, and with matter fields $B^i_{21}, B^i_{13}, B^{ij}_{32}$, where $i,j=1,2,3$ are flavor indices and B^{ij}_{32} is symmetric in its ij indices (i.e. there are 6 such fields). The superpotential for this dual theory is given by

$$W = \mathrm{Tr}\, B^{ij}_{32} B^i_{21} B^j_{13}\,.$$

This corresponds to dualizing the first gauge group.

Now we consider the non-supersymmetric case corresponding to $(-3k,1,1,1)$ (we take k to be odd to avoid tachyons in the bulk). The probe theory on the orbifold point for N D3 branes is easy to write down, and it will correspond to $U(N)^{3k}$ gauge group with matter fields and interactions that can be written down easily. This theory preserves no supersymmetry. Under the RG flow, as discussed in Section 3, the bulk is expected to pass through $t=0$ after which case we would expect to get a new theory on the probe. This theory on the probe should be a dual description of the non-supersymmetric theory. The main question is how to find what this probe theory is *after* this non-trivial transition.

In order to answer this question, recall that we can also view the bulk theory as the Z_k orbifold of the supersymmetric C/Z_3 as noted before. At the level of the brane the Z_k symmetry acts by

$$g = e^{2\pi i(3R)/k}\,,$$

where R represents the R charge, which for all the A^i fields is $2/3$. We can also obtain the quiver theory at the orbifold point by acting on the $(-3,1,1,1)$ quiver theory by Z_k by the usual rules discussed in [23], which of course yields the same answer as we would obtain by considering C^3/Z_{3k}. But now, we also have a natural proposal for what the dual non-supersymmetric quiver theory is: If we start with the duality in the supersymmetric case for $t<0$ going to $t>0$, and if we mod out on both sides by Z_k, we may expect the duality to continue in the non-supersymmetric case. Examples of this have been seen in the context of large N duals of non-supersymmetric quivers (including the one under discussion) where the AdS/CFT duality commutes with non-supersymmetric orbifolds. In particular evidence in this direction was presented where for example it was shown in [27] that the small 't Hooft coupling analysis of large N conformality, agrees with the large 't Hooft coupling analysis of [26] on the AdS side. One thus expects the same to be true here.

Note that the dual theory has R charges of chiral fields given by $R_{B^i_{13}} = R_{B^i_{21}} = 1/3$, $R_{B^{ij}_{32}} = 4/3$, and using this we can find the Z_k orbifold of this quiver, which we identify with the dual non-supersymmetric quiver theory on the non-supersymmetric probe. It would be interesting to check the validity of this non-supersymmetric duality. More generally one is tempted to use this idea to generate many more non-supersymmetric dual quiver theories by modding out a supersymmetric pair of dual quiver theories with an R symmetry on both sides. In fact this idea has already been considered in [28] and successfully tested at large N.

6. Open Problems

One could apply the ideas in this paper to a number of different situations. For example, one can apply it to the question of tachyon condensations in the context of fluxbrane. This seems to yield a nice picture [29]. One can also consider more elaborate geometries than considered in this paper, and study the corresponding dualities for non-supersymmetric theories on the probe.

So far we have mainly concentrated on aspects of worldsheet theory and concentrated on the RG flow of the worldsheet and assumed that some target string dynamics represents this flow. However, even within the framework we have studied it is natural to ask if there is a full conformal theory representing the internal RG flow. In particular can we write down a theory with a fixed central charge as a fibered QFT where on the fiber the theory appears not to be conformal but the totality of the theory is conformal with the critical central charge for string theory? For example one could look for solutions where the compact Calabi-Yau in one region is smoothly connected to a region where the CY has been eaten up! It is natural to look for such conformal theories which preserve $\mathcal{N}=2$ superconformal symmetry on the worldsheet. In fact, it turns out that it is possible to make a canonical construction along these lines by promoting the complexified RG parameter to a chiral field. For example, we consider

$$\Big[W = \sum_{i=1}^{r} u_i^n + e^X \prod u_i^{k_i}\Big]//G$$

in the non-compact case, or say

$$W = \sum_{i=1}^{5} x_i^5 + e^X \sum_{i=1}^{5} x_i$$

in the case of compact quintic orbifold. We can view X as a non-compact extra complex space and one can assign an R-charge to e^X so that W can flow to an $\mathcal{N}=2$ CFT. This is basically the same as introducing the $\mathcal{N}=2$ axion field to the system in the language of GLSM. The central charge is given by $\sum(1-2q_i)$ and is fixed. This is not the temporal dynamics one usually looks for in the Tachyon condensation, but nevertheless it is a well defined conformal theory and a consistent background for superstrings. In such a description for $Re(X) << 0$ we are at one "internal" CFT and for $Re(X) >> 0$ we are at a very different one. In a sense this is a kind of "Euclidean instanton" corresponding to tachyon condensation.

One may also expect the 2d QFT to be part of a consistent string field theory. This would be clearly rather desirable to develop. In such a framework we should have in particular a notion of what a tachyon potential is. In connection with what a right notion of tachyon potential may be in the context of $\mathcal{N}=2$ supersymmetric worldsheet theories one has a natural thought which is currently under study [30]. The idea is to relate the axial charge of chiral fields (which changes under the RG flow) to the tachyon potential. The axial charge is precisely what gives the $-m^2$ of the tachyon potential at the conformal points and so its flow should be naturally related to the tachyon potential. Luckily the axial charge can be exactly computed in $\mathcal{N}=2$ theories in terms of tt^* geometry developed in [31]. This is generally characterized by solutions to an integrable system. One would thus expect to compute the tachyon potential in terms of solutions to this integrable system.

Acknowledgments

I would like to thank E. Witten for many valuable discussions. I would also like to thank M. Aganagic, N. Arkani-Hamed, A. Cohen, A. Dabholkar, M. Gutperle, K. Hori, K. Intriligator, S. Katz and S. Minwalla for comments on this work.

This research is supported in part by NSF grants PHY-9802709 and DMS-0074329.

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