

A LARGER THAN NAIVE CUT-OFF IN A SIMPLE MODEL*

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We find the dependence of the maximum cut-off on the fermion mass in a nonrenormalizable field theory of a single massive vector boson with an axial vector coupling to a fermion with a small but non-zero mass.

The quantum field theory of a single massive vector boson with mass M with an axial vector coupling to a fermion with mass μ is a simple example of a nonrenormalizable theory. The Lagrangian is

$$-\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + \frac{M^2}{2}A^\alpha A_\alpha + \bar{\psi}(i\not{\partial} + g\not{A}\gamma_5)\psi - \mu\bar{\psi}\psi. \quad (1)$$

Because this is a nonrenormalizable theory, there is some maximum possible cut-off, Λ , above which this theory does not make sense. How does Λ depend on μ ? Naively, we might say that the cut-off scales with M/g , the apparent scale of gauge symmetry breaking. However, because this gauge symmetry is abelian, this need not be correct. Indeed, in this theory, it seems clear that $\Lambda \rightarrow \infty$ as $\mu \rightarrow 0$, because in this limit, the massive gauge boson couples to a conserved current.^a We would like to find how Λ depends on μ for $\mu \ll M/g$.

One way to approach this issue is to try to find an explicit UV completion of the nonrenormalizable theory. Then we can examine in detail how the cut-off appears. In this short note, we will build such a UV completion and explicitly see how Λ can grow at least as fast as

$$\Lambda \propto \frac{M}{g} \sqrt{\log \frac{M}{g\mu}}. \quad (2)$$

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^a The current in the simple model with a single fermion is anomalous, but we could eliminate this by doubling the fermion without any essential change in the model.

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The strategy will be to produce the massive vector boson in a spontaneously broken gauge theory but to generate the gauge boson mass with a field with very small charge, which can therefore have a very large VEV. Specifically, we consider a scalar field ξ with $U(1)$ charge

$$\frac{g}{n} \quad (3)$$

for large n , and a VEV

$$\langle \xi \rangle = n v \quad \text{where} \quad v = M/g. \quad (4)$$

The VEV (4) lowers the scale of symmetry breaking, but it also prevents us from coupling ξ to a fermion with charge 1, and thus we cannot directly generate a majorana mass for the fermion. We deal with this problem by this by adding additional fermions with the charge differences chosen to allow Yukawa coupling of ξ between neighboring fermion states. Suppose we add $m \approx n$ additional fermions to get an $m+1$ dimensional fermion field ψ , with fermion charges that look like this:

$$Q = g \begin{pmatrix} \frac{2m-1}{2n} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \frac{2m-1}{2n} - \frac{2}{n} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \frac{2m-1}{2n} - \frac{4}{n} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{2m-1}{2n} - \frac{2m-4}{n} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{2m-1}{2n} - \frac{2m-2}{n} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \frac{2m-1}{2n} - \frac{2m}{n} \end{pmatrix}$$

$$= g \begin{pmatrix} \frac{2m-1}{2n} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \frac{2m-5}{2n} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \frac{2m-9}{2n} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{2m-7}{2n} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -\frac{2m-3}{2n} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & -\frac{2m+1}{2n} \end{pmatrix}. \quad (5)$$

The fermion in the first row has charge of order 1, but as we go down the fermion multiplet, the charges decrease gradually.

This charge assignment allows Yukawa couplings of the form (for $m > 5$)

$$-\bar{\psi}_c(H\xi + L\xi^*)\psi, \quad (6)$$

where the symmetric matrices H and L have the form

$$H = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & -h_{1,m+1} \\ 0 & 0 & 0 & \cdots & 0 & -h_{2,m} & 0 \\ 0 & 0 & 0 & \cdots & -h_{3,m-1} & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & -h_{m-1,3} & \cdots & 0 & 0 & 0 \\ 0 & -h_{m,2} & 0 & \cdots & 0 & 0 & 0 \\ -h_{m+1,1} & 0 & 0 & \cdots & 0 & 0 & 0 \end{pmatrix}, \quad (7)$$

$$L = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \ell_{2,m+1} \\ 0 & 0 & 0 & \cdots & 0 & \ell_{3,m} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \ell_{m,3} & \cdots & 0 & 0 & 0 \\ 0 & \ell_{m+1,2} & 0 & \cdots & 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

where $h_{j,k} = h_{k,j}$ and $\ell_{j,k} = \ell_{k,j}$. There are constraints on the size of these Yukawa couplings from the requirement that the theory be stable under radiative corrections. Anticipating that we will be interested in the case where all the non-zero entries in (7) and (8) are of the same order of magnitude, and that we will be interested in large n , we will assume that the Yukawa couplings go like $1/\sqrt{n}$, so that the anomalous dimension of the ξ field is not large for any n . Then the factor of n in the VEV, (4) produces a fermion mass matrix that scales like \sqrt{n} for large n .

Thus we expect a fermion mass matrix of the form

$$M_f = 4\pi v \sqrt{n} \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & -a_{1,m+1} \\ 0 & 0 & 0 & \cdots & 0 & -a_{2,m} & b_{2,m+1} \\ 0 & 0 & 0 & \cdots & -a_{3,m-1} & b_{3,m} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & -a_{m-1,3} & \cdots & 0 & 0 & 0 \\ 0 & -a_{m,2} & b_{m,3} & \cdots & 0 & 0 & 0 \\ -a_{m+1,1} & b_{m+1,2} & 0 & \cdots & 0 & 0 & 0 \end{pmatrix} \quad (9)$$

where the entries $a_{j,k} = a_{k,j}$ and $b_{j,k} = b_{k,j}$ are less than or of order 1.

It is very easy to analyze the consequences of (9) in the limit that the a entries are smaller than the b s. In the limit $a \rightarrow 0$, the fermion in the first row decouples from the rest and is massless. The b term describes a

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majorana mass matrix for the m additional fermions, which have masses

$$4\pi v\sqrt{n}b_{j+1,m+2-j} \quad \text{for } j = 1 \text{ to } m . \quad (10)$$

When the a terms are small but non-zero, they generate a mass for the light fermion state

$$\mu \approx 4\pi v\sqrt{n} \prod_{j=1}^{m+1} a_{j,m+2-j} \bigg/ \prod_{j=1}^m b_{j+1,m+2-j} . \quad (11)$$

This generates the low energy theory we are interested in. The low energy theory breaks down at the energy scale where we start to encounter the additional fermion states. Thus the cut-off Λ is of the order of the smallest of these masses. To maximize Λ , we then want to choose all the b s of order 1, in which case

$$\Lambda \approx 4\pi v\sqrt{n} \approx 4\pi\sqrt{n} \frac{M}{g} . \quad (12)$$

We can now begin to see where (2) comes from. For any fixed bound $a_{j,k} < a_0 < 1$ on the a values, the light fermion mass μ is suppressed exponentially in n ;

$$\mu < 4\pi v\sqrt{n} a_0^m \quad (13)$$

so that for large $m \approx n$

$$\log \frac{M}{g\mu} > n \log 1/a_0 \quad (14)$$

and thus

$$\Lambda < \frac{4\pi}{\sqrt{\log 1/a_0}} \frac{M}{g} \sqrt{\log \frac{M}{g\mu}} . \quad (15)$$

The trouble with this argument is that it is not obvious how to choose a_0 . We would expect that as n increases for fixed μ , a would be driven to 1. Things get complicated in this limit because we can no longer separate the mass matrix into a mass matrix for the extra fermions plus a separate light fermion. Instead, we have to diagonalize the full mass matrix. The effective cut-off will then be the mass of the **second lightest fermion**. In general, this would be quite difficult, but we expect from the analysis above that we are interested in the situation in which all the b s are 1 and all the

a are equal. This mass matrix can be analyzed fairly easily.^b It looks like this:

$$M_f = 4\pi v\sqrt{n} \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & -a \\ 0 & 0 & 0 & \cdots & 0 & -a & 1 \\ 0 & 0 & 0 & \cdots & -a & 1 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & -a & \cdots & 0 & 0 & 0 \\ 0 & -a & 1 & \cdots & 0 & 0 & 0 \\ -a & 1 & 0 & \cdots & 0 & 0 & 0 \end{pmatrix}. \quad (16)$$

It is useful to consider the positive definite mass-squared matrix

$$M_f^\dagger M_f = 16\pi^2 v^2 n \begin{pmatrix} a^2 & -a & 0 & \cdots & 0 & 0 & 0 \\ -a & 1+a^2 & -a & \cdots & 0 & 0 & 0 \\ 0 & -a & 1+a^2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+a^2 & -a & 0 \\ 0 & 0 & 0 & \cdots & -a & 1+a^2 & -a \\ 0 & 0 & 0 & \cdots & 0 & -a & 1+a^2 \end{pmatrix}. \quad (17)$$

We will now find (at least approximately) the eigenvectors and eigenvalues of this matrix, and then see what the consequences are for the cut-off. We will first find the eigenvalues purely mathematically. However, this mass-squared matrix has an interesting mechanical analog in a system of coupled oscillators. This may help some readers to get a feeling for the structure. It is described on page 2103, and readers who get bored with the mathematical derivation may wish to flip forward to this more intuitive approach.

It is straightforward to verify that the eigenvectors of (17) have the form

$$\begin{pmatrix} \sin(m+1)t \\ \sin mt \\ \vdots \\ \sin 2t \\ \sin t \end{pmatrix}, \quad (18)$$

where a , and t are related by

$$a = \frac{\sin(m+1)t}{\sin(m+2)t} \quad (19)$$

^b In fact, for small n and relatively heavy fermions, one can increase the cut-off slightly by allowing the a 's to be different, but this is not important for very light fermions.

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and the eigenvalue can be written as

$$16\pi^2 v^2 n \frac{\sin^2 t}{\sin^2(m+2)t}. \quad (20)$$

For the lowest state, the sines become hyperbolic functions,^c with $t = i\tau$, and the eigenvector has the form

$$\begin{pmatrix} \sinh(m+1)\tau \\ \sinh m\tau \\ \vdots \\ \sinh 2\tau \\ \sinh \tau \end{pmatrix}. \quad (21)$$

Everywhere in the region of interest (that is where the low energy theory contains a light fermion with with appropriate coupling) $m\tau$ is large enough that we can approximate

$$a = \frac{\sinh(m+1)\tau}{\sinh(m+2)\tau} \approx e^{-\tau}. \quad (22)$$

for $\tau > 0$.

So we fix the mass of the light fermion by setting it to the lowest eigenvalue

$$\mu^2 = 16\pi^2 v^2 n \frac{\sinh^2 \tau}{\sinh^2(m+2)\tau} \approx 16\pi^2 v^2 n (a^{-1} - a)^2 a^{2(m+2)} \quad (23)$$

and the corrections to the approximations in (22) and (23) are down by a factor of about

$$e^{-2m\tau} \approx a^{2m}. \quad (24)$$

Now we can find the higher eigenstates, at least approximately, for large n . The τ and t must satisfy

$$a = \frac{\sin(m+1)t}{\sin(m+2)t} = \frac{\sinh(m+1)\tau}{\sinh(m+2)\tau} \approx e^{-\tau} \quad (25)$$

and there will be m solutions for t , which we call t_k for $k = 1$ to m . We can write the t_k s as

$$t_k = \frac{k\pi}{m+1} + \frac{\theta_k}{(m+1)^2}. \quad (26)$$

^c At least we are interested in the region of parameter space in which this is true, $a < (m+1)/(m+2)$.

Here we should think of $k/(m+1)$ as order 1 because it can be at the top of the tower of massive fermion states. And we expect $\theta_k \propto k$. But the second term in (26) is small for large n . Then for large n we can write

$$\begin{aligned}\sin t_k &\approx \sin \frac{k\pi}{m+1} + \frac{\theta_k}{(m+1)^2} \cos \frac{k\pi}{m+1}, \\ \sin(m+1)t_k &\approx (-1)^k \sin \frac{\theta_k}{m+1}, \\ \sin(m+2)t_k &\approx (-1)^k \left(\sin \frac{\theta_k + k\pi}{m+1} + \frac{\theta_k}{(m+1)^2} \cos \frac{\theta_k + k\pi}{m+1} \right).\end{aligned}\quad (27)$$

Then (25) implies

$$a \approx \sin \frac{\theta_k}{m+1} \bigg/ \sin \frac{\theta_k + k\pi}{m+1}.\quad (28)$$

We can now find the t_k s as follows:

$$\frac{1}{a} = \cos \frac{k\pi}{m+1} + \cot \frac{\theta_k}{m+1} \sin \frac{k\pi}{m+1},\quad (29)$$

$$\frac{1}{a} \sec \frac{k\pi}{m+1} = 1 + \cot \frac{\theta_k}{m+1} \tan \frac{k\pi}{m+1},\quad (30)$$

$$\tan \frac{\theta_k}{m+1} = \frac{\tan \frac{k\pi}{m+1}}{\frac{1}{a} \sec \frac{k\pi}{m+1} - 1} = \frac{a \sin \frac{k\pi}{m+1}}{1 - a \cos \frac{k\pi}{m+1}},\quad (31)$$

$$\cot \frac{\theta_k}{m+1} = \frac{1 - a \cos \frac{k\pi}{m+1}}{a \sin \frac{k\pi}{m+1}},\quad (32)$$

$$\sin \frac{\theta_k}{m+1} = \frac{a \sin \frac{k\pi}{m+1}}{\sqrt{1 + a^2 - 2a \cos \frac{k\pi}{m+1}}}\quad (33)$$

and thus

$$\begin{aligned}\sin(m+2)t_k &= \frac{1}{a} \sin(m+1)t_k \approx (-1)^k \frac{\sin \frac{k\pi}{m+1}}{\sqrt{1 + a^2 - 2a \cos \frac{k\pi}{m+1}}}, \\ \cos \frac{\theta_k}{m+1} &= \frac{1 - a \cos \frac{k\pi}{m+1}}{\sqrt{1 + a^2 - 2a \cos \frac{k\pi}{m+1}}}.\end{aligned}\quad (34)$$

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The eigenvalues in this approximation are

$$M_k^2 = 16\pi^2 v^2 n \frac{\sin^2 t_k}{\sin^2(m+2)t_k} \approx 16\pi^2 v^2 n \left(1 + a^2 - 2a \cos \frac{k\pi}{m+1} \right). \quad (35)$$

This approximation becomes exact both for large m and for small a (we will discuss this further when we discuss the mechanical analog on page 2103 below) It is a good approximation even for rather small m , as illustrated in figures 1 and 2.

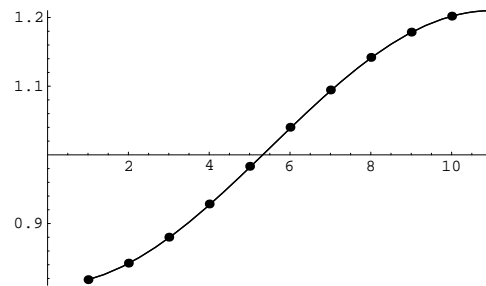


Figure 1. The large eigenvalues of (17) and the approximate function (35) in units of $16\pi^2 v^2 n$ for $m = 10$ and $a = .1$

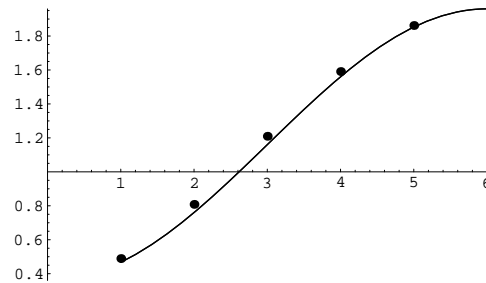


Figure 2. The large eigenvalues of (17) and the approximate function (35) in units of $16\pi^2 v^2 n$ for $m = 5$ and $a = .4$

The mass-squared matrix (17) This has an amusing mechanical analog.

We can write (17) as

$$\begin{pmatrix} -\kappa^2 + K/m & -K/m & 0 & \cdots & 0 & 0 & 0 \\ -K/m & \omega_0^2 + 2K/m & -K/m & \cdots & 0 & 0 & 0 \\ 0 & -K/m & \omega_0^2 + 2K/m & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \omega_0^2 + 2K/m & -K/m & 0 \\ 0 & 0 & 0 & \cdots & -K/m & \omega_0^2 + 2K/m & -K/m \\ 0 & 0 & 0 & \cdots & 0 & -K/m & \omega_0^2 + 2K/m \end{pmatrix} \quad (36)$$

where

$$\frac{K}{m} = 16\pi^2 v^2 n a, \quad \omega_0^2 = 16\pi^2 v^2 n (1 - a)^2, \quad \kappa^2 = 16\pi^2 v^2 n a(1 - a). \quad (37)$$

This could be the $M^{-1}K$ matrix for a system of oscillators coupled with spring constant K . The last coupling spring is attached to a fixed wall (the $2K/m$ rather than K/m in the lower right-hand corner includes the contribution of a spring connecting the last oscillator to the wall). All but the first of the oscillators have uncoupled angular frequency ω_0 , but the first is actually in unstable equilibrium in the absence of coupling. One could build such a thing (for example) with rigid light pendulum rods connected to coupled masses as shown in Figure 3. The left pendulum is inverted to give an unstable equilibrium. Physically, we can think of this unstable

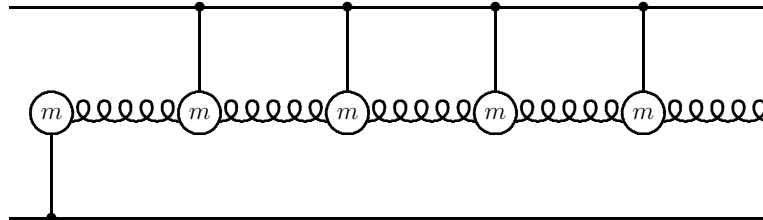


Figure 3. A mechanical system for which the frequencies of the normal modes are proportional to the eigenvalues of the fermion mass-squared matrix (17) (for $m = 4$ in this case). The \bullet s are frictionless pivots. The pendulum rods are rigid, so the pendulum on the left would be in unstable equilibrium in the absence of the coupling springs.

equilibrium as driving one eigenvalue to be much smaller than the others. If the coupling (proportional to a) is small, the normal mode with the small frequency will involve primarily the inverted pendulum. That means that for

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small a , the inverted pendulum will be nearly at rest in all the other modes (because the eigenvectors must be orthogonal). Thus all but the lowest mode can be described approximately by a simpler system in which the inverted pendulum is replaced by a fixed wall, as shown in Figure 4. This is the approximation that leads to (35). Evidently, this approximation becomes exact for small a for any n .

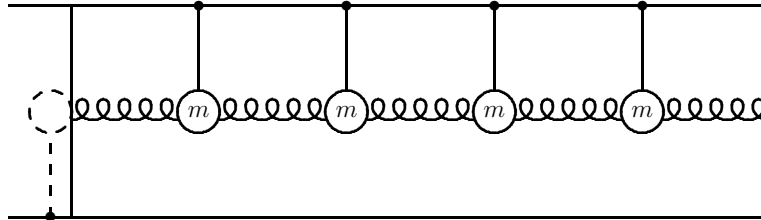


Figure 4. A mechanical system that gives a good approximate description of the large eigenvalues (35) when the coupling is small or the number of block is large.

The eigenvectors for the system of Figure 3 for $a = 0.1$ are depicted in Figure 5.

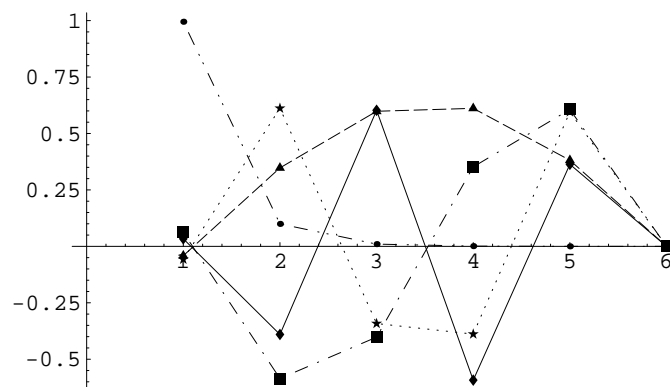


Figure 5. The eigenvectors for the system of Figure 3 for $a = 0.1$. An additional point (labeled 6) has been added at the position of the fixed wall to guide the eye.

Now having understood the structure of the mass matrix, we can return

to the issue of the cut-off. Let us first recall (23),

$$\mu^2 = 16\pi^2 v^2 n \frac{\sinh^2 \tau}{\sinh^2(m+2)\tau} \approx 16\pi^2 v^2 n (a^{-1} - a)^2 a^{2(m+2)}. \quad (23)$$

Once we specify the precise relation of m to n , this gives μ as a function of a and n . For definiteness, we will take

$$m = n + 1 \quad (38)$$

because this will give a reasonable value of the fermion coupling to the vector boson in the low energy theory. Then (23) becomes

$$\mu^2 = 16\pi^2 v^2 n \frac{\sinh^2 \tau}{\sinh^2(m+2)\tau} \approx 16\pi^2 v^2 n (a^{-1} - a)^2 a^{-2(n+3)} \quad (39)$$

or

$$\mu = 4\pi v \sqrt{n} (1 - a^2) a^{(n+1)}. \quad (40)$$

The cut-off corresponds roughly to the bottom of the tower of massive states, which from (35) is

$$\Lambda^2 \approx 16\pi^2 v^2 n \left(1 + a^2 - 2a \cos \frac{\pi}{n+2} \right). \quad (41)$$

Now for large n , the cosine is 1 and we can write

$$\Lambda^2 \approx 16\pi^2 v^2 n (1 - a)^2. \quad (42)$$

To get a quick sense of what is going on, we can make a scatter plot of

$$(x, y) = \left(\log \left[1 / \log \frac{4\pi v}{\mu} \right], \log \frac{\Lambda}{4\pi v} \right). \quad (43)$$

This is shown in Figure 6, and shows clearly the dependence we anticipated in (2).

We can find the boundary either by holding μ fixed and maximizing Λ or by holding Λ fixed and maximizing μ . The latter is somewhat simpler because it is easy to solve (42) for n ,

$$n = \frac{\Lambda^2}{16\pi^2 v^2 (1 - a)^2} \quad (44)$$

and then

$$\mu = \Lambda (1 + a) a^{1 + \Lambda^2 / 16\pi^2 v^2 (1 - a)^2}. \quad (45)$$

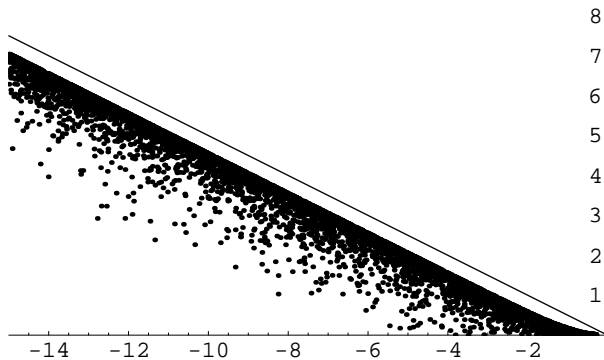
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Figure 6. Scatter plot of (43) showing that the boundary of the allowed region has slope $-1/2$, in agreement with (2). The straight line is $y=-x/2$.

It is easiest to look at the log,

$$\log \mu = \log \Lambda + \log(1+a) + \log a - \frac{\Lambda^2}{16\pi^2 v^2} \frac{\log 1/a}{(1-a)^2}. \quad (46)$$

For large Λ , the last term in (46) is the important one (the others are either small or independent of a), so the maximum value of μ occurs at fixed a , at the minimum of

$$\frac{\log 1/a}{(1-a)^2} \quad (47)$$

which is for

$$a = a_0 \equiv 0.2847 \quad \text{for which} \quad \frac{\log 1/a_0}{(1-a_0)^2} = 2.4554. \quad (48)$$

Thus at the maximum

$$\log \frac{\mu}{4\pi v} \approx -\frac{\Lambda^2}{16\pi^2 v^2} \frac{\log 1/a_0}{(1-a_0)^2} \quad (49)$$

or

$$\Lambda \approx 0.6382 \times \frac{M}{g} \sqrt{\log \frac{M}{g\mu}}. \quad (50)$$

This is shown as the straight white line just below the boundary in Figure 7.

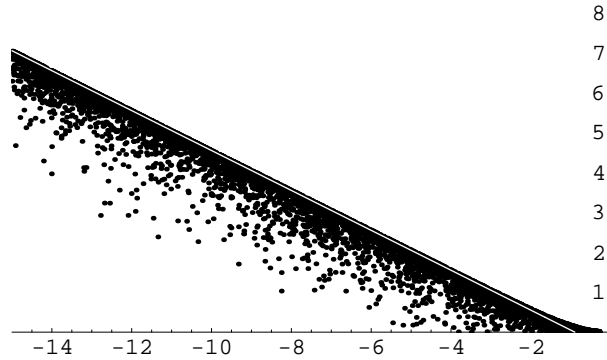


Figure 7. Scatter plot of (43). The straight white line is (50).

The coupling of the fermion in the low energy theory for the maximum cut-off can now be calculated approximately from the fermion charge matrix, (5) and the explicit form (21) for the light eigenstate, using the fact that now from (25),

$$e^\tau = a = a_0. \quad (51)$$

The result for the charge is

$$q_{\text{eff}} \approx g \left(1 + \frac{0.324}{n} + \dots \right). \quad (52)$$

Thus we have demonstrated in a simple model how the cut-off in a non-renormalizable theory may be parametrically larger than the obvious dimensional scale in the model. This raises a number of questions that we hope to address in a future publication.

- How does the maximum cut-off we found in this explicit UV completion compare with what we would expect from unitarity arguments?
- Is there any vestige of the large cut-off in a non-abelian theory where the scale of gauge symmetry breaking has a physical meaning?
- Are there other classes of non-renormalizable theories in which the maximum cut-off is parametrically larger than expected from naive dimensional analysis?

We hope that the answers to some of these questions may shed light on the fine-tuning issues that we seem to face in particle physics.